MATH 140: WEEK-IN-REVIEW 8 (CHAPTER 5.2)

* a_0 any real #

$f(x) = a_n x^n + \cdots + a_1 x + a_0$ * $n = 0, 1, 2, \ldots$ non-negative integer

1. Determine if the given function is a polynomial function. If the answer is yes, state the degree, leading term, leading coefficient, and constant term.

   (a) $f(x) = 5x^{-1} - 7^x + 12x^{2.6}$ ✗ $\boxed{\text{not a polynomial}}$

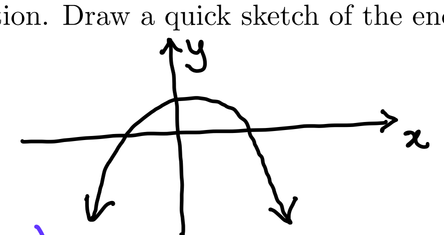
   * $5x^{-1} \rightarrow$ negative power
   * $7^x \rightarrow$ variable power
   * $12x^{2.6} \rightarrow$ power not a positive integer

   (b) $g(r) = 9^8 + \sqrt[3]{10}\, r^2 - 4r^3 + \frac{3}{4}r$ ✓ $\boxed{\text{polynomial}}$

   * degree : $\boxed{3}$
   * leading term : $\boxed{-4r^3}$
   * leading coefficient : $\boxed{-4}$
   * constant term : $\boxed{9^8 = 43046721}$

2. Describe the end behavior of each polynomial function. Draw a quick sketch of the end behavior.

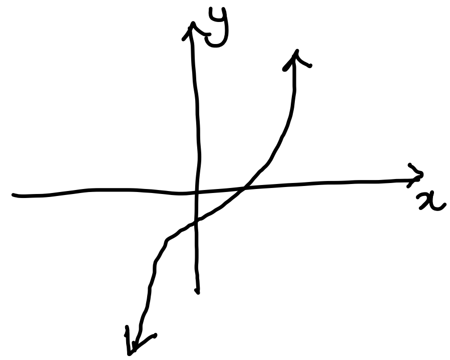
   (a) $f(x) = -23x^6 + 50x^3 - 7x + 1045$

   * leading term : $-23x^6$
   * degree : 6 (even)
   * leading coefficient : -23 (negative)

   $\boxed{f(x) \rightarrow -\infty \text{ as } x \rightarrow \infty \text{ or as } x \rightarrow -\infty}$

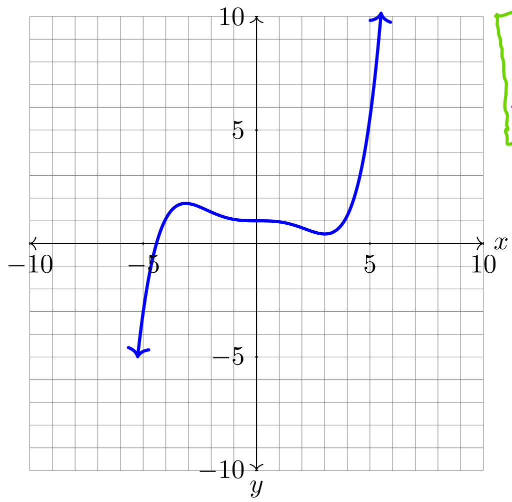
   (b) $g(x) = 15x^4 - 19 + 3x^9 - 6x^2$

   * leading term : $3x^9$
   * degree : 9 (odd)
   * leading coefficient : 3 (positive)

   $\boxed{\begin{array}{l} f(x) \rightarrow \infty \text{ as } x \rightarrow \infty \\ f(x) \rightarrow -\infty \text{ as } x \rightarrow -\infty \end{array}}$

3. Describe the end behavior symbolically for the polynomial function, $f(x)$, graphed below.



$\boxed{\begin{array}{l} * \ f(x) \to \infty \text{ as } x \to \infty \\ * \ f(x) \to -\infty \text{ as } x \to -\infty \end{array}}$

* odd degree polynomial
* leading coefficient : positive

4. State the domain of each polynomial function.

* all polynomials have domain $(-\infty, \infty)$

(a) $h(x) = 6x^{15} + 9x^2 - 30x$

Domain: $\boxed{(-\infty, \infty)}$ since $h(x)$ is a polynomial

(b) $g(r) = 15r^3 - r^4 + 5r^2 - 120$

Domain: $\boxed{(-\infty, \infty)}$ since $g(r)$ is a polynomial

5. Determine all exact real zeros, the x-intercept(s), and y-intercept of each given polynomial function, if possible.

(a) $f(x) = 7(3x+4)(7-5x)$

y-intercept : $\boxed{(0, f(0)) = (0, 196)}$

$f(0) = 7(3\cdot 0+4)(7-5\cdot 0)$
$= 7\cdot 4\cdot 7$
$= 196$

Real zeros : Solve $f(x)=0$ for x

$7(3x+4)(7-5x) = 0$

$\Rightarrow 7 = 0$ X

$3x+4=0 \Rightarrow \frac{3x}{3} = \frac{-4}{3} \Rightarrow \boxed{x = \frac{-4}{3}}$ ✓

$7-5x=0 \Rightarrow \frac{-5x}{-5} = \frac{-7}{-5} \Rightarrow \boxed{x = \frac{7}{5}}$ ✓

x-intercepts : * ordered pairs *

$\boxed{(-\frac{4}{3}, 0)}$ and $\boxed{(\frac{7}{5}, 0)}$

(b) $g(x) = 10x^3 + 15x^2 - 30x = 5x(2x+3)(x-2)$

y-intercept : $(0, g(0)) = (0, 0)$

$g(0) = 5\cdot 0(2\cdot 0+3)(0-2) = 0$

Real zeros : Solve $g(x)=0$ for x

$5x(2x+3)(x-2) = 0$

* $\frac{5x}{5} = \frac{0}{5} \Rightarrow \boxed{x = 0}$ ✓

* $2x+3=0 \Rightarrow \frac{2x}{2} = \frac{-3}{2} \Rightarrow \boxed{x = -\frac{3}{2}}$ ✓

* $x-2=0 \Rightarrow \boxed{x=2}$ ✓

x-intercepts

$\boxed{(0,0)}$, $\boxed{(-\frac{3}{2}, 0)}$, $\boxed{(2,0)}$

(c) $h(r) = 5r^2 - r^3 + 4r - 20$

y-intercept : $(0, h(0)) = \boxed{(0, -20)}$ constant term

$h(0) = 5\cdot 0^2 - 0^3 + 4\cdot 0 - 20$

Real zeros: * factor *

$h(r) = 5r^2 - r^3 + 4r - 20$
$= -r^3 + 5r^2 + 4r - 20$
$= -r^2(r-5) + 4(r-5)$
$= (r-5)(4-r^2)$

Solve $h(r)=0$ for r :

$(r-5)(4-r^2) = 0 \Rightarrow r-5=0 \Rightarrow \boxed{r=5}$

$4-r^2=0 \Rightarrow (2-r)(2+r)=0$

$\Rightarrow \boxed{r = \pm 2}$

x-intercepts

$\boxed{(5,0)}$, $\boxed{(-2,0)}$, $\boxed{(2,0)}$

(d) $k(x) = (x^2+5)(x^2-9)$

y-intercept : $(0, k(0)) = \boxed{(0, -45)}$

$k(0) = (0^2+5)(0^2-9) = (5)(-9) = -45$

Real zeros : Solve $k(x)=0$ for x

$(x^2+5)(x^2-9)=0 \Rightarrow x^2+5=0 \Rightarrow x^2 = -5$ X no real solns

$x^2-9=0 \Rightarrow x^2 = 9 \Rightarrow \boxed{x = \pm 3}$

6. Determine the vertex, axis of symmetry, domain, range, x-intercept(s), y-intercept, maximum value and minimum value for each quadratic function, if they exist. * general form $f(x) = ax^2 + bx + c$

(a) $f(x) = 3x^2 + 9x$

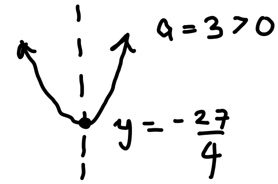
Vertex: $(h,k) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$, $h = \frac{-9}{2 \cdot 3} = \frac{-9}{6} = -\frac{3}{2}$, $k = 3\left(-\frac{3}{2}\right)^2 + 9\left(-\frac{3}{2}\right) = \frac{-27}{4}$

$= \boxed{\left(-\frac{3}{2}, -\frac{27}{4}\right)}$

Axis of symmetry: $\boxed{x = -\frac{3}{2}}$

Domain: $\boxed{(-\infty, \infty)}$ polynomial

Range: $\boxed{\left[-\frac{27}{4}, \infty\right)}$



y-intercept: $(0, f(0)) = \boxed{(0,0)}$

Max: $\boxed{\text{NONE}}$

Min: $\boxed{y = -\frac{27}{4}}$

Real zeros: * factor or use quadratic formula *

$3x^2 + 9x = 3x(x+3) = 0 \Rightarrow \frac{3x}{3} = \frac{0}{3} \Rightarrow \boxed{x = 0}$

$x + 3 = 0 \Rightarrow \boxed{x = -3}$

x-ints: $\boxed{(0,0)}$ and $\boxed{(-3,0)}$

(b) $g(x) = 4x^2 - 3x + 1$

Vertex: $x = \frac{-b}{2a} = \frac{-(-3)}{2 \cdot 4} = \frac{3}{8}$, $y = g(3/8) = 4\left(\frac{3}{8}\right)^2 - 3\left(\frac{3}{8}\right) + 1 = \frac{7}{16}$

$\boxed{(3/8, 7/16)}$

Domain: $\boxed{(-\infty, \infty)}$ polynomial

Axis of symmetry: $\boxed{x = 3/8}$

Range: $\boxed{\left[\frac{7}{16}, \infty\right)}$

Max: $\boxed{\text{none}}$

Min: $\boxed{y = \frac{7}{16}}$

y-int: $(0, g(0)) = \boxed{(0, 1)}$ constant term

Real zeros: * quadratic formula if difficult to factor *

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - (4)(4)(1)}}{2 \cdot 4}$$

$$= \frac{3 \pm \sqrt{9 - 16}}{8} = \frac{3 \pm \sqrt{-7}}{8}$$

no real sols $\Downarrow$ no real zeros

$\boxed{\text{No } x\text{-intercepts}}$

(c) $h(x) = 16 - 25x^2$ $= -25x^2 + 0x + 16$

Vertex: $x = \frac{-b}{2a} = 0$, $y = 16 - 25 \cdot 0 = 16$

$\boxed{(0,16)}$

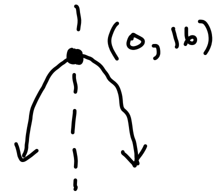
Axis of symmetry: $\boxed{x = 0 \text{ (y-axis)}}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 16]$

Max: $\boxed{y = 16}$

Min: $\boxed{\text{NONE}}$



Real zeros: $16 - 25x^2 = 0$

* factor or use quadratic formula * $(4-5x)(4+5x) = 0$

$4 - 5x = 0 \Rightarrow \frac{4}{5} = \frac{5x}{5} \Rightarrow \boxed{\frac{4}{5} = x}$

$4 + 5x = 0 \Rightarrow \frac{4}{-5} = \frac{-5x}{-5} \Rightarrow \boxed{\frac{-4}{5} = x}$

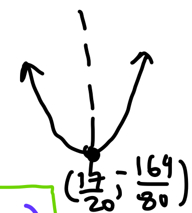
x-int(s)

$\boxed{(-\frac{4}{5}, 0)}$ and $\boxed{(\frac{4}{5}, 0)}$

(d) $j(x) = 5x^2 - \dfrac{17}{2}x + \dfrac{3}{2}$

Vertex: $x = \dfrac{-(-\frac{17}{2})}{2 \cdot 5} = \frac{17}{2} \cdot \frac{1}{10} = \frac{17}{20}$, $y = 5\left(\frac{17}{20}\right)^2 - \frac{17}{2}\left(\frac{17}{20}\right) + \frac{3}{2} = \frac{-169}{80}$

$\boxed{\left(\frac{17}{20}, \frac{-169}{80}\right)}$



Domain: $\boxed{(-\infty, \infty)}$, Range: $\boxed{\left[\frac{-169}{80}, \infty\right)}$

Axis of symmetry: $\boxed{x = \frac{17}{20}}$

Max: $\boxed{\text{NONE}}$, Min: $\boxed{y = -\frac{169}{80}}$

Real zeros

$x = \dfrac{\frac{17}{2} \pm \sqrt{\left(\frac{17}{2}\right)^2 - (4)(5)\left(\frac{3}{2}\right)}}{2 \cdot 5} = \dfrac{\frac{17}{2} \pm \sqrt{\frac{169}{4}}}{10} = \dfrac{\frac{17}{2} \pm \frac{13}{2}}{10} = \frac{15}{10}, \frac{2}{10} = \boxed{\frac{3}{2}, \frac{1}{5}}$

x-ints: $\boxed{(3/2, 0)}$ and $\boxed{(\frac{1}{5}, 0)}$ y-int: $\boxed{(0, 3/2)}$

(e) $k(x) = 4x^2 - 20x + 25$

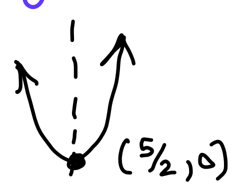
Vertex: $x = \dfrac{-(-20)}{2 \cdot 4} = \frac{20}{8} = \frac{5}{2}$, $y = 4(5/2)^2 - 20(5/2) + 25 = 0$

$\boxed{(5/2, 0)}$

Axis of symmetry: $\boxed{x = 5/2}$

Domain: $\boxed{(-\infty, \infty)}$

Range: $\boxed{[0, \infty)}$



Real zeros:

$x = \dfrac{20 \pm \sqrt{20^2 - (4)(4)(25)}}{2 \cdot 4} = \dfrac{20 \pm \sqrt{400 - 400}}{8} = \frac{5}{2}$ *only one root

x-int: $\boxed{(5/2, 0)}$ y-int: $\boxed{(0, 25)}$

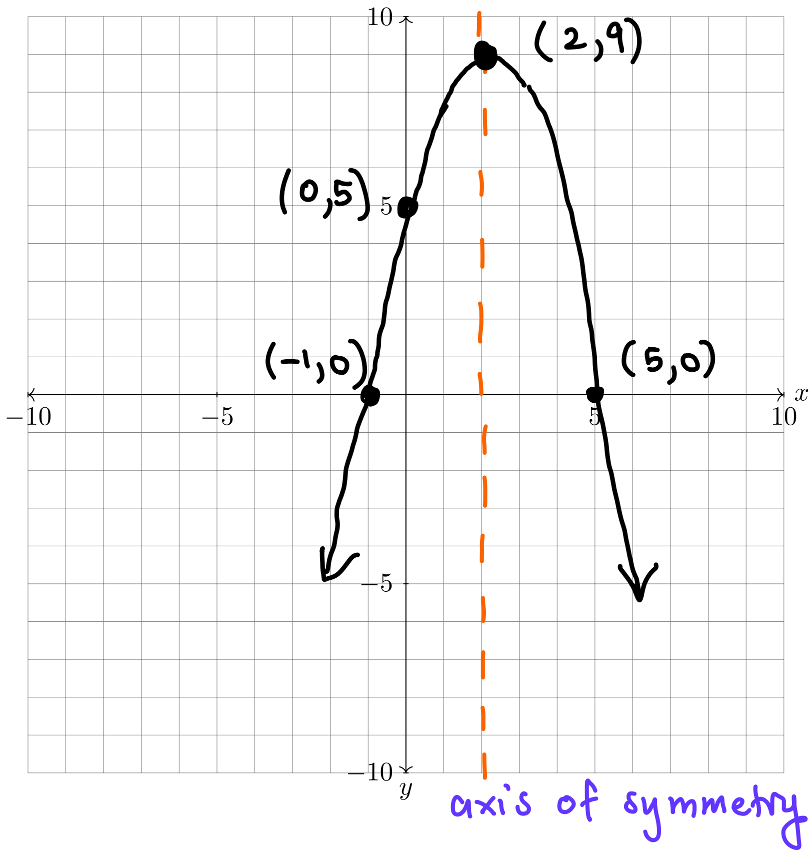
(f) Graph the quadratic function with the following properties

i. As $x \to -\infty$, $f(x) \to -\infty$ and as $x \to \infty$, $h(x) \to -\infty$

ii. $f(x)$ has real zeros at $x = -1,\ 5$.

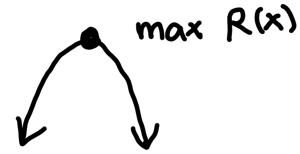
iii. There is a maximum value of 9.

iv. The graph has a y-intercept of $(0, 5)$.



7. Use the given revenue function, $R(x)$, and cost function, $C(x)$, where x is the number of items made and sold, to determine each of the following. Assume both revenue and cost are given in dollars.

   i. The number of items sold when revenue is maximized.
   ii. The maximum revenue.
   iii. The number of items sold when profit is maximized.
   iv. The maximum profit.
   v. The break-even quantity/quantities.



   (a) $R(x) = -0.5x^2 + 100x$ and $C(x) = 40x + 1600$

(i) $x = -\frac{b}{2a} = -\frac{100}{2(-0.5)} = \frac{100}{1} = \boxed{100 \text{ items}}$

max revenue

(ii) $R(100) = -0.5(100^2) + (100)(100) = (0.5)(100^2) = \boxed{\$5000}$

(iii) $P(x) = R(x) - C(x) = -0.5x^2 + 100x - 40x - 1600$

$= -0.5x^2 + 60x - 1600$

* find x to maximize profit *

$x = -\frac{b}{2a} = -\frac{60}{2(-0.5)} = \frac{60}{1} = \boxed{60 \text{ items}}$

(iv) $P(60) = -0.5(60^2) + 60(60) - 1600$

$= 0.5(60^2) - 1600$

$= 1800 - 1600$

$= \boxed{\$200}$ → max profit

(v) $P(x) = 0$ at break-even pnts

$x = \frac{-60 \pm \sqrt{60^2 - (4)(-0.5)(-1600)}}{2(-0.5)} = \frac{-60 \pm 20}{-1}$

$= 60 \mp 20$

$= \boxed{80, 40}$ * break-even quantities

Use the given revenue function, $R(x)$, and cost function, $C(x)$, where x is the number of items made and sold, to determine each of the following. Assume both revenue and cost are given in dollars.

i. The number of items sold when revenue is maximized.
ii. The maximum revenue.
iii. The number of items sold when profit is maximized.
iv. The maximum profit.
v. The break-even quantity/quantities.

(b) $R(x) = -4x^2 + 300x$ and $C(x) = 28x + 500$

(i) $x = -\frac{300}{2(-4)} = \frac{300}{8} = \boxed{37.5}$ * round to 38 if item is a whole

(ii) $R(37.5) = -4(37.5)^2 + 300(37.5) = \boxed{\$5625}$

(iii) $P(x) = R(x) - C(x) = -4x^2 + 300x - 28x - 500$
$= -4x^2 + 272x - 500$

* to maximize profit *

$x = \frac{-b}{2a} = \frac{-272}{2(-4)} = \frac{272}{8} = 34$ items

(iv) $P(34) = -4(34^2) + 272(34) - 500 = \boxed{\$4124}$

(v) $P(x) = 0$

$-4x^2 + 272x - 500 = 0$

$$x = \frac{-272 \pm \sqrt{272^2 - (4)(-4)(-500)}}{2(-4)}$$

$x \approx 1.8908$ or $x \approx 66.109$ * round-off if item is a whole

$\boxed{x = 2}$ or $\boxed{x = 66}$ * approximate break-even quantities

8. The demand function for a calculator is given by $p(x) = -\frac{1}{20}x + 100$ where $p(x)$ is the price in dollars. The fixed costs are \$1,280 and the variable costs are \$80 per calculator made.

(a) Determine the cost, revenue and profit as functions of the number of calculators made and sold.

$$\boxed{C(x) = \underbrace{80x}_{\text{variable costs}} + \underbrace{1280}_{\text{fixed costs}}}$$

$$R(x) = px = \left(-\frac{1}{20}x+100\right)x$$
$$= \boxed{-\frac{1}{20}x^2+100x}$$

$$P(x) = R(x) - C(x) = -\frac{1}{20}x^2 + 100x - 80x - 1280$$

$$\boxed{P(x) = -\frac{1}{20}x^2 + 20x - 1280}$$

(b) How many calculators must be sold to maximize profit?

$$x = \frac{-20}{\left(2 \cdot \frac{-1}{20}\right)} = \frac{20}{\left(\frac{1}{10}\right)} = (20)(10) = \boxed{200}$$

* 200 calculators must be sold to maximize profit

(c) What is the maximum profit?

$$P(200) = -\frac{1}{20}(200)^2 + 20(200) - 1280$$
$$= \boxed{\$720}$$

9. The cost of manufacturing collectible bobble head figurines is given by $C(x) = 30x + 350$, where x is the number of collectible bobble head figurines produced. If each figurine has a price-demand function of $p(x) = -1.2x + 360$, in dollars, determine

   (a) the company's profit function.

$$R(x) = px = (-1.2x + 360)x$$
$$= -1.2x^2 + 360x$$

$$P(x) = R(x) - C(x)$$
$$= -1.2x^2 + 360x - 30x - 350$$
$$\boxed{P(x) = -1.2x^2 + 330x - 350}$$

   (b) how many figurines must be sold in order to maximize revenue?

$$x = \frac{-360}{(2)(-1.2)} = \frac{360}{2.4} = \boxed{150}$$

* 150 figurines must be sold to maximize revenue *

   (c) how many figurines must be sold in order to maximize profit?

$$x = \frac{-330}{(2)(-1.2)} = \frac{330}{2.4} = 137.5$$ * round up to 138 *

* Approximately 138 figurines must be sold to maximize profit

   (d) at what price per figurine will the maximum profit be achieved?

* plug into price equation *

$$p(138) = -1.2(138) + 360 = \boxed{\$194.40}$$