Fall 24 152 WIR 3

	Disks/washers	Shells
→ (rotate about horizontal axis)	$\int dx$	$\int dy$
↑ (rotate about vertical axis)	$\int dy$	$\int dx$

TEXAS A&M UNIVERSITY
College of Arts & Sciences

Math 152 - Week-In-Review 3

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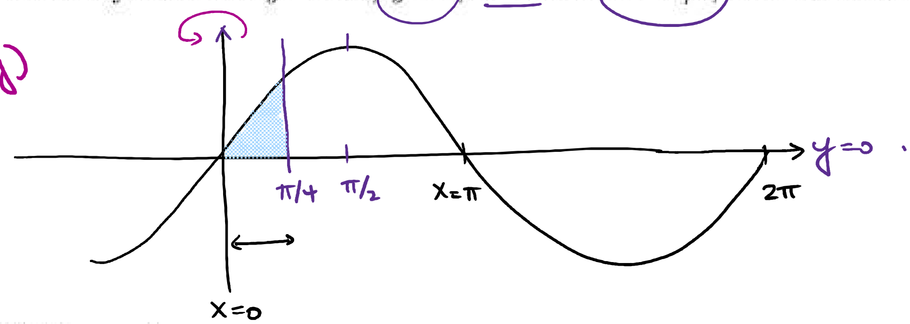
1. Set up the integral(s) to find the volume of the solid obtained by rotating the region bounded by the curves $y = \sin x$, $y = 0$, $x = 0$ and $x = \pi/4$, about the following:

$y = \sin(x)$
$x = \arcsin(y)$



(a) the y-axis.

Washers → $\int dy$

✓Shells → $\int dx$

$2\pi r h$

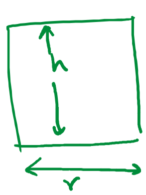
$h \to T - B$

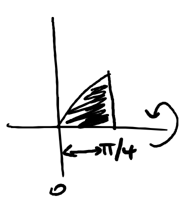
$h = \sin(x) - 0$

$r = x$

$$V = 2\pi \int_0^{\pi/4} (x)(\sin x)\, dx$$

(b) the x-axis.





✓Disks → $\int dx$

Shells → $\int dy$

πr^2

$r \to T - B$

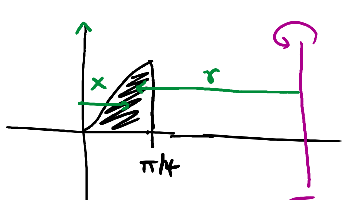
$r = \sin(x) - 0$

$$V = \pi \int_0^{\pi/4} [\sin(x)]^2\, dx$$

(c) the line $x = \pi$.

$x + r = \pi$

$r = \pi - x$



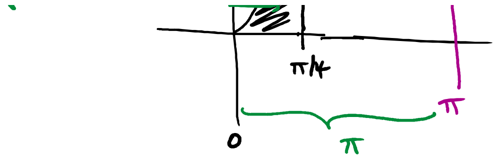
washers → $\int dy$

✓Shells → $\int dx$

$h = \sin(x) - 0$

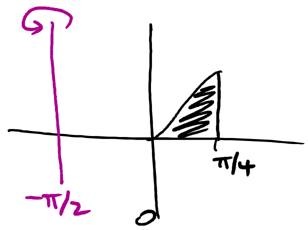
$r = \pi - x$

$$V = 2\pi \int^{\pi/4} (\pi - x)(\sin x)\, dx$$



$$V = 2\pi \int_0^{\pi/4} (\pi - x)(\sin x)\,dx \quad \Big\{ \; r = \pi - x$$

(d) the line $x = -\pi/2$.



Shells $\to \int dx$

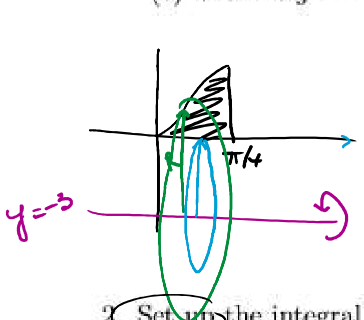
$h = \sin(x) - 0$

$r = x - \left(-\frac{\pi}{2}\right)$

$= x + \frac{\pi}{2}$

$$V = 2\pi \int_0^{\pi/4} \left(x + \frac{\pi}{2}\right)(\sin x)\, dx$$

(e) the line $y = -3$.



Washers $\to \int dx$ (T-B) Shells $\to \int dy$

$\pi(R^2 - r^2)$

$R = \sin(x) - (-3) = \sin x + 3$

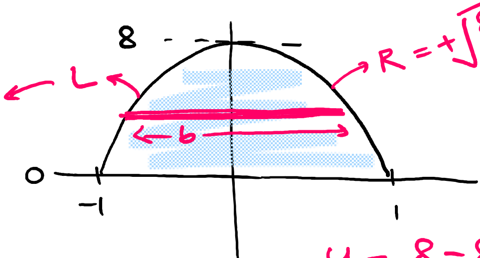
$r = 0 - (-3) = 3$

$$V = \pi \int_0^{\pi/4} \left[(\sin x + 3)^2 - 3^2\right] dx$$

2. Set up the integral to find the volume of a solid whose base is the region bounded by the parabola $y = 8 - 8x^2$ and the x-axis and where the cross sections perpendicular to the y-axis are isosceles triangles with base equal to height.

$8 - 8x^2 = 0$

$x = \pm 1$



Method of Slices

① Shape of base

② Shape of x-sec

③ Orientation of x-sec

$\perp$ y-axis $\to \int dy$

(R-L)

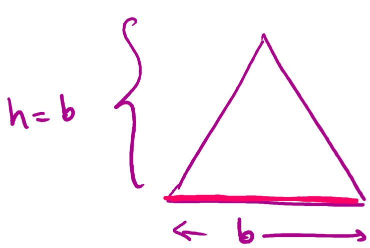
$y = 8 - 8x^2$

$8x^2 = 8 - y$

$x^2 = \frac{8-y}{8}$

$x = \pm\sqrt{\frac{8-y}{8}}$

$(b = R - L)$



$b = \sqrt{\frac{8-y}{8}} - -\sqrt{\frac{8-y}{8}}$

$b = 2\sqrt{\frac{8-y}{8}}$

$A = \frac{1}{2} bh = \frac{1}{2} b^2$

$A = \frac{1}{2} \cdot \left(2\sqrt{\frac{8-y}{8}}\right)^2$

$= \frac{1}{2} \cdot \cancel{4} \left(\frac{8-y}{\cancel{8}_2}\right) \Rightarrow \boxed{A = \frac{8-y}{4}}$

$\frac{4}{16} = \frac{1}{4}$

$$V = \int_0^8 \frac{8-y}{4} \cdot dy$$

$V = 8$

① force $= kx$

② work $= \frac{1}{2}kx^2 \Big|_{a-l}^{b-l}$

every spring has l, k

$l = 20\text{cms} = \frac{20}{100}\text{m} = \frac{1}{5}\text{m}$

Metric

3. A spring has a natural length of 20 cms. If a force of 25N is required to keep it stretched to a length of 30 cms, how much work would be required to stretch the spring from 25 cms to 40 cms?

Step 1. $f = 25 = kx = k\left(\frac{30-20}{100}\right) \Rightarrow 25 = k\cdot\frac{1}{10}$

$k = 250$

Step 2 $W = \frac{1}{2}kx^2 \Big|_{\frac{25-20}{100}}^{\frac{40-20}{100}} = \frac{1}{2}(250)\,x^2 \Big|_{\frac{5}{100}}^{\frac{20}{100}}$

$= \frac{1}{2}(250)\left[\left(\frac{20}{100}\right)^2 - \left(\frac{5}{100}\right)^2\right]$

$W = 4.6875$ Nm or J

$l = 0$

FPS

12 inches = 1 foot

4. A force of 10 pounds is required to hold a spring stretched 4 inches beyond its natural length. How much work is required to stretch the spring from its natural length to 6 inches beyond itsnatural length?

$a = 0$

$b = \frac{6}{12}$

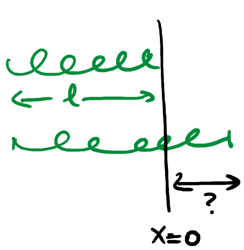
Step 1 $f = kx \longrightarrow 10 = k\left(\frac{4}{12} - 0\right) = k\,\frac{4}{12}$

$k = \frac{10(12)}{4} = 30$

Step 2 $W = \frac{1}{2}kx^2 \Big|_0^{\frac{6}{12}} = \frac{1}{2}(30)\,x^2 \Big|_0^{1/2}$

$= \frac{1}{2}(30)\left[\frac{1}{4} - 0\right]$

$W = \frac{15}{4}$ ft-lb



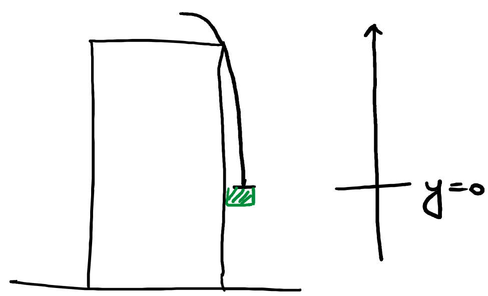
Ropes: $W = g\int_0^a (\text{Initial wt} - \rho y)\,dy$

FPS
no g

ignore

5. A uniform cable 40 feet long is hanging over the side of a building that is ~~500~~ feet tall. The cable weighs 60 pounds.

(a) How much work would be required to pull up 10 feet of cable?



Initial wt = 60 lb

$\rho = \frac{60}{40} = \frac{3}{2}$ lb/ft

$b = 10$

$$W = \int_0^{10} \left(60 - \tfrac{3}{2}y\right)dy$$

$= 525$ ft-lb.

(b) If there is a weight of 80 pounds attached to the bottom of the cable, how much work would be required to pull up 25 feet of cable?

Initial wt = 60 + 80 = 140 lb

$\rho = \frac{3}{2}$ lb/ft

$b = 25$

$$W = \int_0^{25} \left(140 - \tfrac{3}{2}y\right)dy = 3031.25 \text{ ft-lb}.$$

$W = \frac{1}{2}kx^2 \Big|_{a-\ell}^{b-\ell}$

metric

6. If 6J of work are required to stretch a spring from 10 cms to 12 cms and 10J of work are required to stretch the spring from 12 cms to 14 cms, find the natural length of the spring.

$\frac{10}{100}$ m, $\frac{12}{100}$ m, $\frac{14}{100}$ m

$\ell = ?$ cms.

① $6 = \frac{1}{2}kx^2 \Big|_{\frac{10-\ell}{100}}^{\frac{12-\ell}{100}}$

② $10 = \frac{1}{2}kx^2 \Big|_{\frac{12-\ell}{100}}^{\frac{14-\ell}{100}}$

$6 = \frac{1}{2}k\left[\left(\frac{12-\ell}{100}\right)^2 - \left(\frac{10-\ell}{100}\right)^2\right]$

$20 = k\left[\left(\frac{14-\ell}{100}\right)^2 - \left(\frac{12-\ell}{100}\right)^2\right]$

$k = \dfrac{12}{\left(\frac{1}{100}\right)^2\left[\left(\frac{12-\ell}{\cancel{100}}\right)^2 - \left(\frac{10-\ell}{\cancel{100}}\right)^2\right]}$ $=$ $k = \dfrac{20}{\left(\frac{1}{100}\right)^2\left[\left(\frac{14-\ell}{\cancel{100}}\right)^2 - \left(\frac{12-\ell}{\cancel{100}}\right)^2\right]}$

$12[196 - 28\ell + \cancel{\ell^2} - 144 + 24\ell - \cancel{\ell^2}] = 20[144 - 24\ell + \cancel{\ell^2} - 100 + 20\ell - \cancel{\ell^2}]$

$\overset{3}{\cancel{12}}[52 - 4\ell] = \overset{5}{\cancel{20}}[44 - 4\ell]$

$156 - 12\ell = 220 - 20\ell$

$8\ell = 64$

$\ell = 8$

natural length is 8 cms.

metric
need g.

7. A rope 20 meters long weighing 2 kilograms per meter is used to lift up 360 kilograms of coal from the bottom of a mine shaft. How much work is required to pull up the coal to the top of the mine?

Initial weight $= \underset{\text{rope}}{(20\times 2)} + \underset{\text{coal}}{(360)} = 400$ kg

$\rho = 2$ kg/m

$b = 20$

$W = g\int_0^{20} (400 - 2y)\,dy$

$= 7600\,g$ Nm or J.

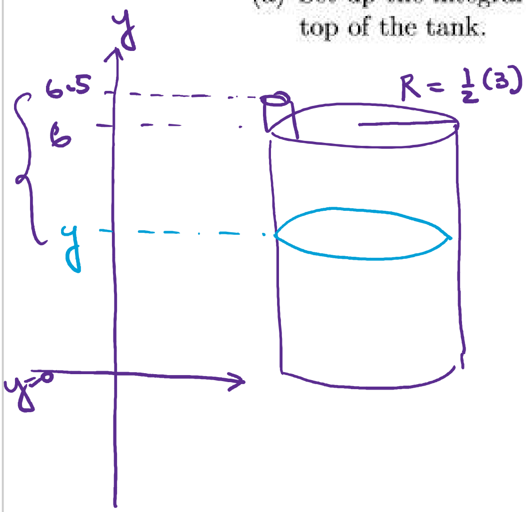
$$W = \rho g \int_a^b (A_s)\underbrace{(T-y)}_{\text{displacement}}\,dy$$

$\frac{1}{2}(6) = 3 = b$

metric need g.

8. A cylindrical tank 6 meters tall with a diameter of 3 meters is half full of water. There is a 0.5 meter spout at the top of the tank.

(a) Set up the integral to find the work required to pump all the water out from the top of the tank.



① $A_s \Rightarrow$ circle $\longrightarrow \pi R^2$

$A_s = \pi\left(\frac{3}{2}\right)^2 = \frac{9\pi}{4}$

② $T = 6.5$

③ $b = 3$

④ $a = 0$

$$W = \rho g \int_0^3 \pi \frac{9}{4}(6.5 - y)\,dy$$

$b = 6$

(b) If the tank is full of olive oil which has a density of 960 kilograms per cubic meter, how much work would be required to empty half the tank?

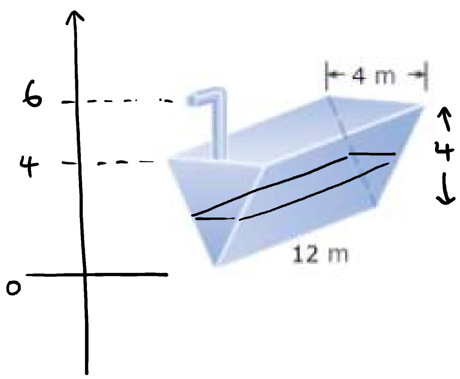
$a = \frac{1}{2}(6) = 3$ $\qquad \rho = 960\ \text{kg/m}^3$

$$W = (960)\,g\,\pi \int_3^6 \frac{9}{4}(6.5 - y)\,dy$$

metric need g.

$b = 4$

9. A 12 meter long tank in the shape of a triangular trough is full of water. Its vertical cross sections are isosceles triangles with base equal to its height of 4 meters. There is a 2 meter spout at the top of the tank. Set up the integral to find the work required to pump out the top 1.5 meters of water from the tank.

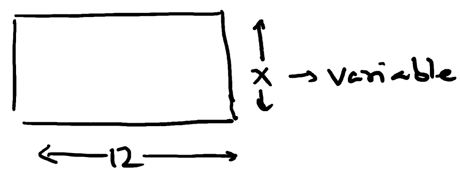


$T = 4+2 = 6\text{ m}$

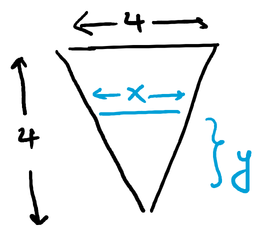
$b = 4$

$a = 4 - 1.5 = 2.5$

x-sec of tank



Use similar triangles



$\frac{4}{4} = \frac{x}{y}$

$x = \frac{4y}{4} = y$

$A_{(x\text{-sec})} = 12x$

$= 12y$

$$W = \rho g \int_{2.5}^{4} (12y)(6-y)\,dy$$

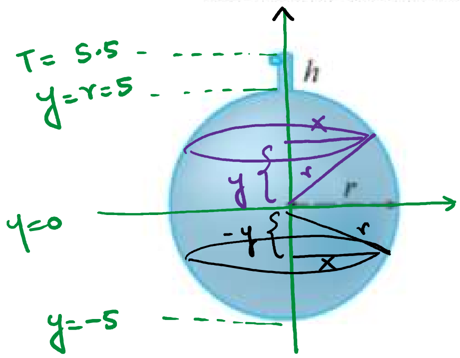
metric $\rightarrow$ Kg, m, s

FPS $\rightarrow$ ft, lb, s

metric use g.

b = 5

10. A spherical tank with a radius r of 5 meters is completely full of water. The tank has a 0.5 meter spout h at the top.



Cross section $\rightarrow$ circle

$A_s = \pi x^2$

Pythagorus th.

$\boxed{x^2+y^2=r^2}$

$x^2 = r^2 - y^2$

$x^2 = 25 - y^2$

$A_s = \pi(25-y^2)$

$T = 5.5$

(a) Set up an integral to find the work required to empty the full tank of water.

$$W = \rho g \int_{-5}^{5} \pi (25-y^2)(5.5-y)\,dy$$

(b) Set up an integral to find the work required to empty only half the tank of water. $\rightarrow y=0$.

$$W = \rho g \int_{0}^{5} \pi (25-y^2)(5.5-y)\,dy$$

(c) If you initially started out with only half a tank of water, set up an integral to find the work required to empty the tank.

$$W = \rho g \int_{-5}^{0} \pi (25-y^2)(5.5-y)\,dy$$

IBP $\rightarrow$ 2 types of functions $\rightarrow$ polynomials, trig, exp, log, inverse trig.

11. Evaluate the indefinite integral $\int xe^{-2x}\,dx$.

$u = x$ $\qquad dv = e^{-2x}\,dx$

$du = 1 \cdot dx$

$v = \int dv$

$= \int e^{-2x}\,dx$

$= \frac{e^{-2x}}{-2} \rightarrow v$

Identify -

u $\qquad$ dv

$\downarrow$ $\qquad$ $\downarrow$

derivative $\qquad$ integrate

$uv - \int v\,du$

By IBP $\int x e^{-2x} dx = (x)\left(\frac{e^{-2x}}{-2}\right) - \int \frac{e^{-2x}}{-2} \cdot dx$

$= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \int e^{-2x}\,dx$

$= -\frac{1}{2} x e^{-2x} + \frac{1}{2}\left(\frac{e^{-2x}}{-2}\right) + C$

$\int x e^{-2x} dx = -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C$

choose u

I $\rightarrow$ Inv. trig

L $\rightarrow$ Log

A $\rightarrow$ algebraic

T $\rightarrow$ Trig

E $\rightarrow$ Exp

12. Evaluate the indefinite integral $\int e^{3x} \cos x\,dx$.

13. Evaluate the indefinite integral $\int x^3 \ln x \, dx$.

14. Evaluate the definite integral $\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right) dx$.

15. Evaluate the definite integral $\int_0^{\pi} e^{\cos t} \sin 2t \; dt$.