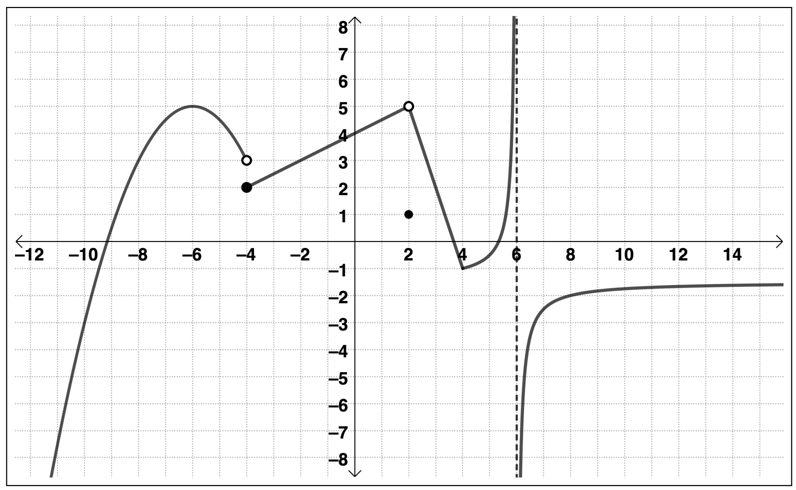
Session 1: Sections 1-1 and 1-2

The notation $\lim_{x\to c} f(x)$ (for a real number c) means we need to find the value $f(x)$ approaches when x is near, *but not necessarily equal to* c. The function must approach the same value (i.e., L) from both the left and right side of $x = c$ for $\lim_{x\to c} f(x) = L$

1. A graph of $f(x)$ is given below. Use the graph to find each limit below. If a limit does not exist, state so and use limit notation to describe any infinite behavior.



(a) $\lim_{x\to -4^-} f(x)$

(b) $\lim_{x\to -4^+} f(x)$

(c) $\lim_{x\to -4} f(x)$

(d) $\lim_{x\to 2} f(x)$

(e) $\lim_{x\to 6^+} f(x)$

(f) $\lim_{x\to 6} f(x)$

2. Given $f(x) = \dfrac{5(x+3)}{x^2+5x+6}$, complete the tables below and then use the table to estimate the given limits

(a) $\lim_{x \to -3} f(x)$

left-hand limit / **right-hand limit**

x	$f(x)$	x	$f(x)$
-3.1		-2.9	
-3.01		-2.99	
-3.001		-2.999	
-3.0001		-2.9999	

(b) $\lim_{x \to -2} f(x)$

left-hand limit / **right-hand limit**

x	$f(x)$	x	$f(x)$

Direct Substitution Property For Polynomial and Rational Functions

If P and Q are polynomials and c is any real number, then

$$\lim_{x\to c} P(x) = P(c) \qquad \text{and} \qquad \lim_{x\to c} \frac{P(x)}{Q(x)} = \frac{P(c)}{Q(c)}$$

as long as $Q(c)$ is nonzero.

Cases for a Ratio of Two Functions

Given two functions $f(x)$ and $g(x)$, and any real number c, use the cases below when finding $\lim_{x\to c} \frac{f(x)}{g(x)}$.

- **Case 1:** (L is any real number and $M \neq 0$)

  If $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$, then $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{L}{M}$.

- **Case 2**

  If $\lim_{x\to c} f(x) \neq 0$ and $\lim_{x\to c} g(x) = 0$, then $\lim_{x\to c} \frac{f(x)}{g(x)}$ **does not exist**.

- **Case 3**

  If $\lim_{x\to c} f(x) = 0$ and $\lim_{x\to c} g(x) = 0$, then $\lim_{x\to c} \frac{f(x)}{g(x)}$ cannot be determined (i.e., is indeterminate) and further algebraic manipulation is necessary to convert the limit to an expression in which Case 1 or Case 2 applies.

3. Given $h(x)$ below, find the following limits algebraically, check your results graphically.

$$h(x) = \begin{cases} 2x + 10 & x \leq -7 \\ \dfrac{3x-8}{4-x} & -7 < x < 1 \\ \dfrac{2x^2 + x - 3}{x^2 - 5x + 4} & x \geq 1 \end{cases}$$

(a) $\lim_{x\to 1} h(x)$

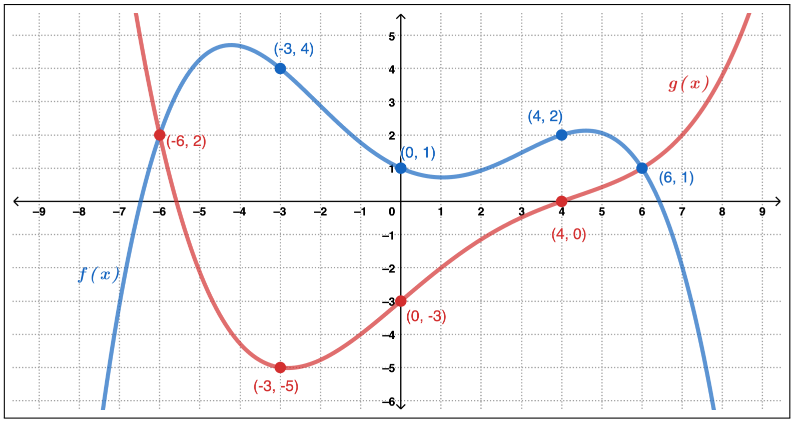
$$h(x) = \begin{cases} 2x + 10 & x \leq -7 \\ \dfrac{3x - 8}{4 - x} & -7 < x < 1 \\ \dfrac{2x^2 + x - 3}{x^2 - 5x + 4} & x \geq 1 \end{cases}$$

(b) $\lim_{x \to 4} h(x)$

(c) $\lim_{x \to -7^-} h(x)$

(d) $\lim_{x \to 0} h(x)$

4. Given the graph of $f(x)$ and $g(x)$ below find $\lim_{x \to -3} \left(2f(x) + \frac{g(x)}{x^2} + 8 \right)$.



5. Find the limits below algebraically.

(a) $\lim_{x \to -5} [\ln(6 + x) - 2x]$

(b) $\lim_{x \to 4} \frac{x^2 - 8}{x + 4}$

(c) $\lim_{x \to 4} \dfrac{x-4}{x+4}$

(d) $\lim_{x \to 4} \dfrac{x+4}{x-4}$

(e) $\lim_{x \to 1^-} \dfrac{\frac{8}{x+5} - \frac{4}{x+2}}{x - 1}$

(f) $\lim_{x \to -1/2} f(x)$ given $f(x) = \begin{cases} \dfrac{2x^2 - 3x - 2}{2x + 1} & x < -\frac{1}{2} \\ 2x + 7 & x > -\frac{1}{2} \end{cases}$