Math 151
Week-In-Review 10

4.2, 4.3

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Problem Statements

1. Determine the value(s) of c that satisfies the conclusion of the Mean Value Theorem for the given function on the interval.

   (a) $f(x) = \dfrac{1}{x}$ on [1,5]

   (b) $f(x) = e^{x/2}$ on [2,4]

2. Let $f(x) = (x-2)^{-2}$. Show there is no value of c in $(1,4)$ such that $f(c) = \dfrac{f(4)-f(1)}{4-1}$. Does this contradict the Mean Value Theorem?

3. A few understanding questions:

   (a) What does an increasing function look like? What does a decreasing function look like?

   (b) What does it mean for a function to be increasing/decreasing?

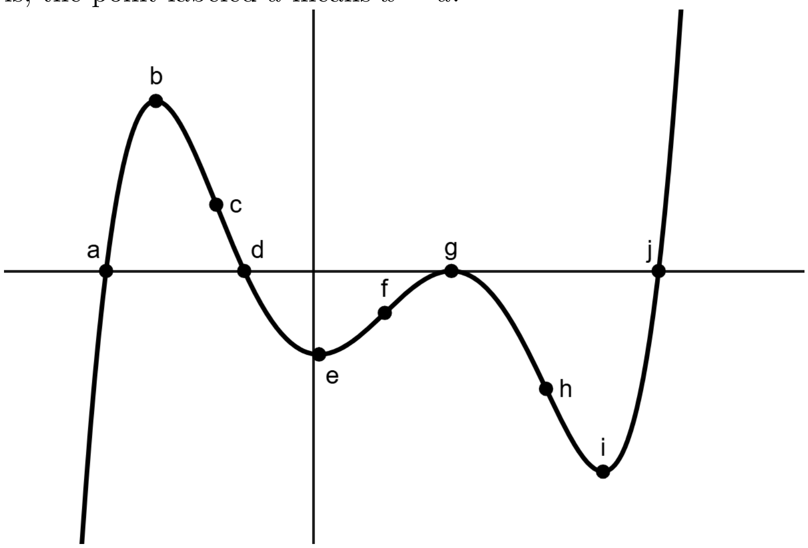
   (c) How do we determine when a function is increasing/decreasing?

   (d) What does a concave up function look like? What does a concave down function look like?

   (e) What does it mean for a function to be concave up/down?

   (f) How do we determine when a function is concave up/down?

4. Consider the graph of $\boldsymbol{f(x)}$ below. Assume each label corresponds with an $x-$value. That is, the point labeled a means $x = a$.



(a) Determine the critical numbers of $f(x)$, that, is where $f'(x)$ DNE or $f'(x) = 0$.

(b) Determine the intervals where $f(x)$ is increasing/decreasing.

(c) Determine the location of any local extrema of $f(x)$.

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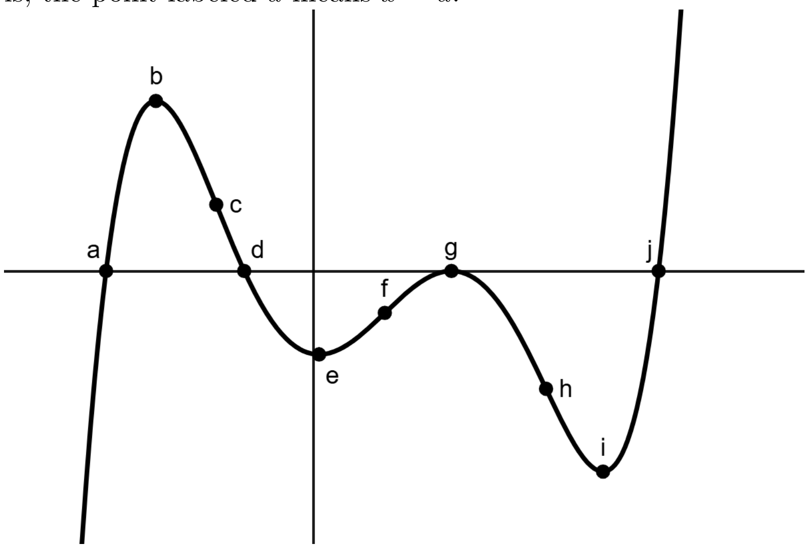

(a) Determine the critical numbers of $f'(x)$, that is, where $f''(x)$ DNE or $f''(x) = 0$.

(b) Determine the intervals where $f(x)$ is concave up/down.

(c) Determine the location of any inflection points of $f(x)$.

(d) Determine the intervals where $f'(x)$ is increasing/decreasing.

(e) Determine the location of any local extrema of $f'(x)$

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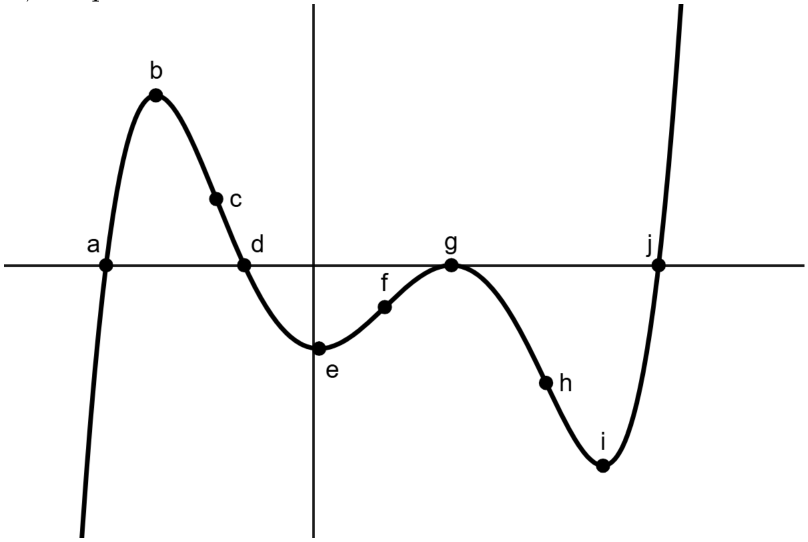

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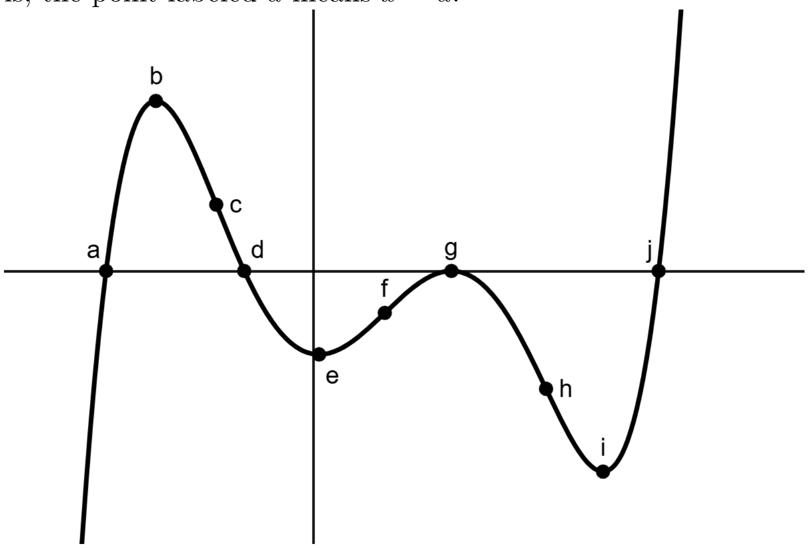

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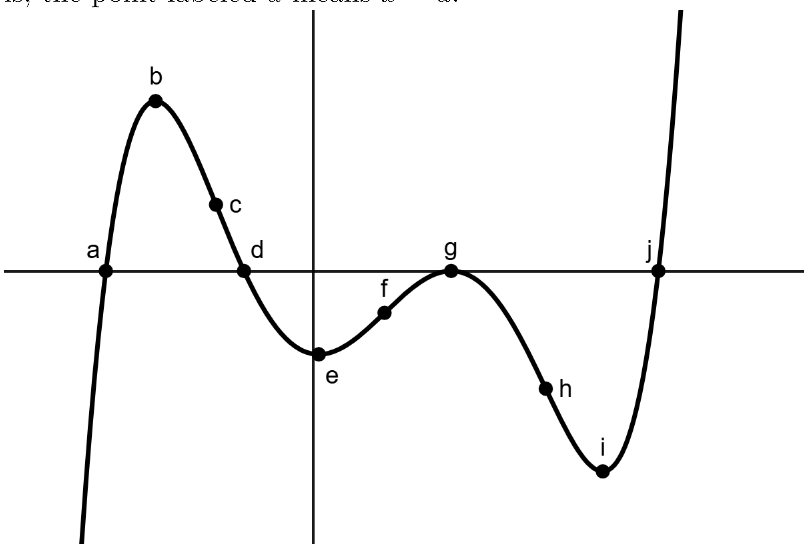

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(e) Determine the location of any local extrema of $f'(x)$

9. Consider the function $f(x) = 5x^4 - \frac{40}{3}x^3$.

   (a) Determine the intervals where $f(x)$ is increasing and where $f(x)$ is decreasing, as well as the $x-$values of any local extrema.

   (b) Determine the intervals where $f(x)$ is concave up and where $f(x)$ is concave down, as well as the $x-$values of any local extrema.

   (c) Sketch the function.

10. Consider the function $f(x) = x^{1/3}(6-x)^{2/3}$.

(a) Determine the intervals where $f(x)$ is increasing and where $f(x)$ is decreasing, as well as the $x-$values of any local extrema.

(b) Determine the intervals where $f(x)$ is concave up and where $f(x)$ is concave down, as well as the $x-$values of any local extrema.
Note: $f''(x) = \dfrac{-8}{(6-x)^{4/3}x^{5/3}}$

(c) Sketch the function. Note: $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$.

11. Let $f(t)$ be the temperature at time t where you live, and suppose that at time $t = 5$ you are uncomfortably hot. How would you feel about the given data in each case?

    (a) $f'(5) = 2, f''(5) = 4$

    (b) $f'(5) = 2, f''(5) = -4$

    (c) $f'(5) = -2, f''(5) = 4$

    (d) $f'(5) = -2, f''(5) = -4$

    (e) $f'(5) = 0, f''(5) = 4$

    (f) $f'(5) = 0, f''(5) = -4$