Math 151 - Week-In-Review 12

Topics for the week:

4.9 Antiderivatives
5.1 Areas and Distances
5.2 The Definite Integral
Review for Exam 3 (3.10 - 5.2)

4.9 Antiderivatives

1. List the antiderivative rules.

2. Compute the most general antiderivative of the function $f'(x) = x^4 + \dfrac{8}{x} + \dfrac{3}{x^2} + \sqrt[3]{x} + 7$.

3. Compute the most general antiderivative of the function $f(x) = 7e^x + \dfrac{4}{1+x^2} + \sqrt[5]{x^2}$.

4. Compute the antiderivative of the function $f'(x) = \sec^2(x) + \sec(x)\tan(x)$ when $f\left(\frac{\pi}{4}\right) = 0$.

5. Compute the position function $s(t)$, given $a(t) = 5\cos(t) - \sin(t)$ and $v(0) = -1$ and $s(0) = 4$.

6. Find the position function of a particle whose movement can be described with the following information.

   $\mathbf{a}(t) = \left\langle 3\sin(t), 2e^t \right\rangle$, $\mathbf{v}(0) = \langle 6, 3\rangle$, $\mathbf{s}(\pi) = \langle 0, \pi\rangle$

5.1 Areas and Distances

7. The speed, in m, of a rowing team during a race is collected at 3 minute intervals.

t (min)	0	3	6	9	12	15	18
v (m)	0	12	14	17	18	17	20

   Compute the lower and upper estimates for the distance traveled by the rowing team in the first 18 minutes of the race.

8. Estimate the area under the graph of $f(x) = x^3 + 1$ over the interval $[-1, 3]$ using four approximating rectangles and midpoints.

9. Use the definition with right endpoints to express the area under the graph of $f(x) = \dfrac{x\ln(x)}{x^2 - 3}$ over the interval $\left[e^2, e^5\right]$ as a limit.

10. Determine a region whose area is equal to the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{\pi}{4n} \tan\left(\frac{\pi i}{4n}\right)\right)$. Do not evaluate the limit.

5.2 The Definite Integral

11. Express the integral, $\int_2^8 \left(x\sqrt{x^2+4}\right) dx$ as a limit of Riemann sums using left endpoints. Do not evaluate the limit.

12. Use geometry to evaluate $\int_0^3 \left(\sqrt{9-x^2}+x\right) dx$.

13. If $\displaystyle\int_A^B f(x)\ dx = 12$, $\displaystyle\int_B^A h(x)\ dx = 15$ and $\displaystyle\int_A^B [2f(x) - 3g(x) + 5h(x)]dx = 150$, find $\displaystyle\int_A^B g(x)\ dx$.

Review for Exam 3

14. Approximate $\dfrac{1}{\sqrt{4.1}}$.

15. Determine the differential of the function $f(x) = x^2 \ln(3x + 4)$.

16. Determine the absolute extrema for $g(x) = \left(x^3 - 12x\right)^{\frac{1}{3}}$ on the interval $[0, 4]$.

17. Determine the absolute extrema for $h(x) = \begin{cases} 3x + 4 & \text{if} \quad -4 \leq x < -1 \\ x^2 & \text{if} \quad -1 \leq x \leq 0 \\ e^x - 1 & \text{if} \quad 0 < x \leq 2 \end{cases}$

18. Determine any real number(s) c that satisfy the conclusion of the Mean Value Theorem for $f(x) = 2x - 5x^2$ on [-1,3].

19. Given $f(x) = x^2 \ln(x)$, determine the intervals on which $f(x)$ is increasing or decreasing.

20. Given $g(x) = -x^4 + 2x^2 + 5$, determine any local extrema and any inflection points for $g(x)$.

21. Compute the limit, $\lim_{x\to\infty}\left(\frac{3x^3}{3x^2-4} - \frac{x^2}{x+1}\right)$ if possible.

22. Compute the limit, $\lim_{x \to -\infty} \left(\dfrac{e^{-x^3}}{x^2} \right)$ if possible.

23. Compute the limit, $\lim_{x \to 1} \left(\dfrac{5 \ln(x) + 4x - 4}{e^{3x-3} + 7x^2 - 8} \right)$ if possible.

24. A cylindrical storage container has a volume of 108π m^3. The material for the base and top costs \$2 per square meter and the material for the side costs \$1 per square meter. Determine the minimum cost for the storage container.