Week 6

Monday, September 30, 2024 7:30 PM

TEXAS A&M UNIVERSITY
Math Learning Center

Math 151
Week-In-Review 6
3.3, 3.4, 3.5, 3.6
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Problem Statements

1. Find $f'(x)$ if $f(x) = \sin(x) + 2\cos(x) - 3\sec(x) + 4\tan(x) + 5\csc(x) - 6\cot(x)$.

$$f'(x) = \cos(x) + 2(-\sin(x)) - 3\sec(x)\tan(x) + 4\sec^2(x) + 5(-\csc(x)\cot(x)) - 6(-\csc^2(x))$$

$$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$$

$$\frac{d}{dx}[\tan(x)] = \sec^2(x)$$

2. Find the equation of the tangent line to the curve $g(x) = 10\sec(x)$ when $x = \frac{\pi}{3}$.

Point: $g\left(\frac{\pi}{3}\right) = 10\sec\left(\frac{\pi}{3}\right) = \dfrac{10}{\cos\left(\frac{\pi}{3}\right)} = \dfrac{10}{1/2} = 10(2) = 20 \quad \left(\frac{\pi}{3}, 20\right)$

Slope: $g'(x) = 10\sec(x)\tan(x)$

$$g'\left(\frac{\pi}{3}\right) = 10\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right)\cdot\frac{\sin(\pi/3)}{\cos(\pi/3)} = 10\left(\frac{1}{1/2}\right)\cdot\frac{\sqrt{3}/2}{1/2} = 20\sqrt{3}$$

$$\boxed{y - 20 = 20\sqrt{3}\left(x - \frac{\pi}{3}\right)}$$

3. Find all values of θ for which the equation $h(\theta) = \theta + \cos(\theta)$ has a horizontal tangent line.

Slope $= 0$

$h'(\theta) = 1 - \sin\theta = 0$

$1 = \sin\theta$

$\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

$$\boxed{\theta = \frac{\pi}{2} + 2\pi \cdot k}$$

$$\frac{d}{dx}\left[(x^2+1)^{100}\right] = 100(x^2+1)^{99}\cdot(2x)$$

$$\frac{d}{dx}\left[(x^2+1)^{100}\right] = 100(x^2+1)^{99}\cdot(2x)$$

4. Find the derivative of the following functions. Do not worry about simplifying.

(a) $f(x) = (x^6 - 15x - 5)^{12} + 4^{117x+23}$

$$f'(x) = 12(x^6-15x-5)^{11}\cdot(6x^5-15) + 4^{117x+23}\cdot \ln(4)\cdot(117)$$

$$e^{(\ln 4^x)} = e^{x\ln(4)}$$

$$\frac{d}{dx}[4^x] = \frac{d}{dx}\left[e^{x\ln(4)}\right] = e^{x\ln(4)}\cdot \ln(4)$$

$$\frac{d}{dx}[e^x] = e^x\cdot \ln(e) = e^x \qquad = 4^x\cdot\ln(4)$$

(b) $h(x) = 8\cot^{10}(e^{2x+3\sin(-5x)})$

$$\cot^{10}(\ \ ) = [\cot(\ \ )]^{10}$$

$$h'(x) = (8)10\cot^9\left(e^{2x+3\sin(-5x)}\right)\left(-\csc^2\left(e^{2x+3\sin(-5x)}\right)\right)\cdot\left(e^{2x+3\sin(-5x)}\right)\left(2+3\cos(-5x)(-5)\right)$$

$$\sqrt{3x} = (3x)^{1/2}$$

Quotient Rule: $\dfrac{(\text{Bot})(\text{Top})' - (\text{Top})(\text{Bot})'}{(\text{Bot})^2}$

Product Rule: $(1^{st})(2^{nd})' + (2^{nd})(1^{st})'$

(c) $g(t) = \dfrac{\sec(5x)\cdot 3^{2x+1}}{e^{\sqrt{3x}}}$

Product Rule

$$g'(t) = \frac{e^{\sqrt{3x}}\left[\sec(5x)\cdot 3^{2x+1}\ln(3)(2) + 3^{2x+1}\sec(5x)\tan(5x)\cdot(5)\right] - \sec(5x)\,3^{2x+1}e^{\sqrt{3x}}\cdot\frac{1}{2}(3x)^{-1/2}\cdot 3}{\left(e^{\sqrt{3x}}\right)^2}$$

$\frac{d}{dx}[y] = \frac{d}{dx}[x^2]$

$\frac{dy}{dx} = 2x$

$\frac{d}{dx}[(y)^7] = 7y^6 \cdot \frac{dy}{dx}$

5. Find $\frac{dy}{dx}$ for the following equations.

(a) $x^5 + y + y^7 = e^y - 11$

$\frac{d}{dx}[x^5 + y + y^7] = \frac{d}{dx}[e^y - 11]$

$5x^4 + \frac{dy}{dx} + 7y^6 \cdot \frac{dy}{dx} = e^y \cdot \frac{dy}{dx} - 0$

$(1)\frac{dy}{dx} + 7y^6 \frac{dy}{dx} - e^y \cdot \frac{dy}{dx} = -5x^4$

$\frac{dy}{dx}[1 + 7y^6 - e^y] = -5x^4$

$$\boxed{\frac{dy}{dx} = \frac{-5x^4}{1 + 7y^6 - e^y}}$$

$3x \cdot (y^{-1})$

$x^2 \cdot y^3$

$\frac{dy}{dx} = y'$

(b) $2y^3 - \frac{3x}{y} = \csc(x) + \sin(y) - e^{x^2y^3}$

$6y^2 \cdot \frac{dy}{dx} - \left[\frac{y \cdot (3) - (3x)\frac{dy}{dx}}{y^2}\right] = -\csc(x)\cot(x) + \cos(y) \cdot \frac{dy}{dx} - e^{x^2 \cdot y^3}\left[x^2 \cdot 3y^2 \cdot \frac{dy}{dx} + y^3 \cdot 2x\right]$

$6y^2 \frac{dy}{dx} + \frac{3x}{y^2}\frac{dy}{dx} - \cos(y)\frac{dy}{dx} + e^{x^2y^3} \cdot 3x^2y^2 \frac{dy}{dx} = \frac{3y}{y^2} - \csc(x)\cot(x) - e^{x^2y^3} \cdot 2xy^3$

$$\boxed{\frac{dy}{dx} = \frac{3/y - \csc(x)\cot(x) - e^{x^2y^3} \cdot 2xy^3}{6y^2 + 3x/y^2 - \cos(y) + e^{x^2y^3} \cdot 3x^2y^2}}$$

6. If $y = J(x)$ satisfies the equation $xy'' + y' + xy = 0$, find $J''(0)$ if $J(0) = 1$ and $J'(0) = 0$.

$x \cdot y''' + y''(1) + y'' + x \cdot y' + y(1) = 0$

Replace $y = J(x)$, $y' = J'(x)$, ... and Plug in $x = 0$

$\cancel{0 \cdot J'''(0)} + J''(0) + J''(0) + \cancel{0 \cdot J'(0)} + J(0) = 0$

$2J''(0) + J(0) = 0$

$2J''(0) + 1 = 0$

$2J''(0) = -1$

$$\boxed{J''(0) = \frac{-1}{2}}$$

$\frac{dy}{dx}$ = Slope

7. Determine the slope of the tangent line to the curve $xy = 1 + \cos(x)$ at the point $\left(2\pi, \frac{1}{\pi}\right)$.

$$x \cdot \frac{dy}{dx} + y \cdot 1 = 0 - \sin(x)$$

$$x \cdot \frac{dy}{dx} = -\sin(x) - y$$

$$\frac{dy}{dx} = \frac{-\sin(x) - y}{x}$$

$$\left.\frac{dy}{dx}\right|_{(2\pi, \frac{1}{\pi})} = \frac{-\sin(2\pi) - \frac{1}{\pi}}{2\pi}$$

$$= \frac{0 - \frac{1}{\pi}}{2\pi/1}$$

$$= \frac{-1}{\pi} \cdot \frac{1}{2\pi} = \boxed{\frac{-1}{2\pi^2}}$$

8. Show the derivative of $f(x) = \arccos(x)$ is $f'(x) = \dfrac{-1}{\sqrt{1 - x^2}}$.

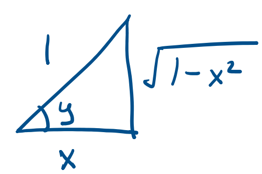
$$y = \arccos(x)$$

$$\frac{d}{dx}\left[\cos(y)\right] = \frac{d}{dx}[x]$$

$$-\sin(y) \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{-1}{\sin(y)} = \boxed{\frac{-1}{\sqrt{1-x^2}}}$$

$$\cos(y) = \frac{x}{1}$$



$$\sin(y) = \frac{\sqrt{1-x^2}}{1}$$

$$\frac{1}{\sin(y)} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$$

9. Show the derivative of $f(x) = \ln(x)$ is $f'(x) = \dfrac{1}{x}$.

$$y = \ln(x)$$

e^ e^

$$\frac{d}{dx}\left[e^y\right] = \frac{d}{dx}[x]$$

$$e^y \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y} = \boxed{\frac{1}{x}}$$

$$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$$

$$\frac{d}{dx}[\ln|x|] = \frac{1}{x}$$

10. Show the derivative of $f(x) = \log_b(x)$ is $\frac{1}{x\ln(b)}$.

$$\frac{d}{dx}\left[\log_b(x)\right] = \frac{d}{dx}\left(\frac{\ln(x)}{\ln(b)}\right) = \frac{1}{x}\cdot\frac{1}{\ln(b)} = \frac{1}{x\ln(b)}$$

$$\frac{d}{dx}\left[b^x\right] = b^x \cdot \ln(b)$$

11. Find the derivative of $h(x) = \arcsin(x) + \arccos(x) - \arctan(x) + \ln(x) + \log_3(x) - \log(x)$.

$$h'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}} - \frac{1}{1+x^2} + \frac{1}{x} + \frac{1}{x\cdot\ln(3)} - \frac{1}{x\cdot\ln(10)}$$

$$\frac{d}{dx}\left[\operatorname{arccot}(x)\right] = \frac{-1}{1+x^2}$$

12. Find the derivative of $g(x) = \log_6(\sin(x)) \cdot \arctan(5^{4x})$.

$$g'(x) = \log_6(\sin x)\cdot\frac{1}{1+(5^{4x})^2}\cdot 5^{4x}\cdot\ln(5)\cdot(4) + \arctan(5^{4x})\cdot\frac{1}{\sin(x)\cdot\ln(6)}\cdot\cos(x)$$

$\cancel{\frac{d}{dx}\left[x^{f(x)}\right] = x^{f(x)} \cdot \ln(x)}$?

$\cancel{\frac{d}{dx}\left[x^{f(x)}\right] = f(x) \cdot x^{f(x)-1}}$?

13. Find the derivative of $y = x^{f(x)}$, assuming $f(x)$ is a differentiable function.

Goal: Find $\frac{dy}{dx}$

$$\ln(y) = \ln\left(x^{f(x)}\right)$$

$$\ln(y) = f(x) \cdot \ln(x)$$

Take Derivative

$$\frac{1}{y} \cdot \frac{dy}{dx} = f(x) \cdot \frac{1}{x} + \ln(x) \cdot f'(x)$$

$$\frac{dy}{dx} = \left[f(x) \cdot \frac{1}{x} + \ln(x) \cdot f'(x)\right] \cdot y$$

$$\boxed{\frac{dy}{dx} = \left[f(x) \cdot \frac{1}{x} + \ln(x) \cdot f'(x)\right] x^{f(x)}}$$

14. Find the derivative of $f(x) = \dfrac{4x^3 e^x \cos(2x)}{\arcsin(x) \cdot \ln(x)}$

$$\ln(y) = \ln\left(\frac{4x^3 e^x \cos(2x)}{\arcsin(x) \cdot \ln(x)}\right)$$

$$\ln(y) = \ln(4) + \underbrace{\ln(x^3)}_{3\ln(x)} + \underbrace{\ln(e^x)}_{x} + \ln(\cos(2x)) - \ln(\arcsin(x)) - \ln(\ln(x))$$

Take derivative:

$$\frac{1}{y} \cdot \frac{dy}{dx} = 0 + 3 \cdot \frac{1}{x} + 1 + \frac{1}{\cos(2x)} \cdot (-\sin(2x)) \cdot 2 - \frac{1}{\arcsin(x)} \cdot \frac{1}{\sqrt{1-x^2}} - \frac{1}{\ln(x)} \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = \left[\frac{3}{x} + 1 - \frac{2\sin(2x)}{\cos(2x)} - \frac{1}{\arcsin(x)\sqrt{1-x^2}} - \frac{1}{x\ln(x)}\right]\left(\frac{4x^3 e^x \cos(2x)}{\arcsin(x) \cdot \ln(x)}\right)$$