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Math 151
Week-In-Review 2
Appendix J.3, 1.5, and 2.2
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Problem Statements

Section J.3

1. Eliminate the parameter to find a Cartesian Equation of the curve. Sketch the curve and indicate the direction of the curve as the parameter increases.

(a) $x = \sqrt{t+2}, y = 6 - t$

$x^2 = t + 2$

$t = x^2 - 2$

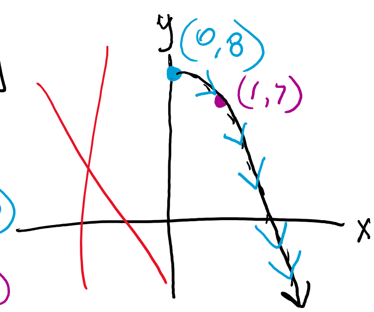
$y = 6 - (x^2 - 2) = 6 - x^2 + 2$

$$\boxed{y = 8 - x^2}$$

$t + 2 \geq 0$

$t \geq -2$

t	x	y	
-2	0	8	$(0,8)$
-1	1	7	$(1,7)$



(b) $x = 2 + 3\cos(\theta), y = -1 - \sin(\theta)$

$x - 2 = 3\cos\theta$ $\qquad$ $y + 1 = -\sin\theta$ $\qquad$ $\sin^2\theta + \cos^2\theta = 1$

$\frac{x-2}{3} = \cos\theta$ $\qquad$ $-(y+1) = \sin\theta$

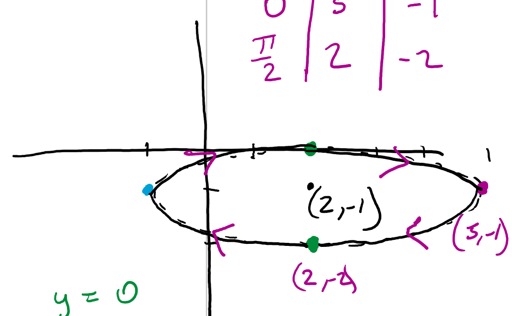
$\left(\frac{x-2}{3}\right)^2 = \cos^2(\theta)$ $\qquad$ $[-(y+1)]^2 = \sin^2(\theta)$

$$\boxed{(y+1)^2 + \left(\frac{x-2}{3}\right)^2 = 1}$$

Ellipse $(2,-1)$

Center: $x = 2, y = -1$

θ	x	y
0	5	-1
$\frac{\pi}{2}$	2	-2



when $x = 2$ $\quad (y+1)^2 = 1$ $\quad y + 1 = 1$ $\quad y = 0$

$y + 1 = -1$ $\quad y = -2$

when $y = -1$ $\quad \left(\frac{x-2}{3}\right)^2 = 1$ $\quad \frac{x-2}{3} = 1$ $\quad x - 2 = 3$ $\quad x = 5$

$\frac{x-2}{3} = -1$ $\quad x - 2 = -3$ $\quad x = -1$

(c) $\mathbf{r}(t) = \langle t^2 + 2, 6 - t \rangle$ $\qquad$ $x = t^2 + 2$, $y = 6 - t$

$x - 2 = t^2$

(c) $\boldsymbol{r}(t) = \langle t^2 + 2, 6 - t \rangle$

$x = t^2 + 2, \quad y = 6 - t$

$x - 2 = t^2$

$t = \pm\sqrt{x-2}$

$\cancel{y = 6 \pm \sqrt{x-2}}$

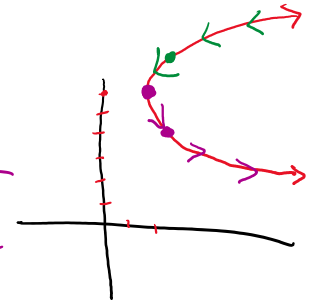
$y = 6 - t$

$t = 6 - y$

$\boxed{x = (6-y)^2 + 2}$

$x = (y-6)^2 + 2$

t	x	y
0	2	6
1	3	5
-1	3	7



$[-(x-6)]^2 + 2$

$y = (6-x)^2 + 2 = (x-6)^2 + 2$

(d) $\boldsymbol{r}(\alpha) = \langle \sin(\alpha), \cos^2(\alpha) \rangle$

$x = \sin(\alpha)$

$y = \cos^2(\alpha)$

$x^2 = \sin^2(\alpha)$

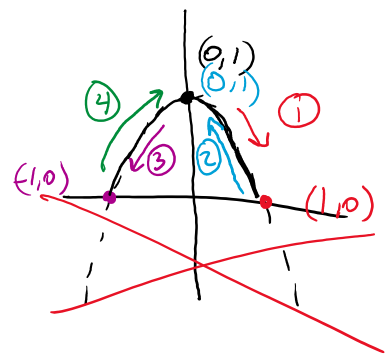
Note: y is always positive

$\sin^2(\alpha) + \cos^2(\alpha) = 1$

$\boxed{x^2 + y = 1}$

$\boxed{y = 1 - x^2}$

α	x	y
0	0	1
$\frac{\pi}{2}$	1	0
π	0	1
$\frac{3\pi}{2}$	-1	0
2π	0	1



2. Find a vector equation of the line through that passes through the points $(1, 8)$ and $(-3, -4)$. Find parametric equations of the same line.

Line: Point and Slope

Initial Vector

Slope Vector

Slope: $m = \frac{\Delta y}{\Delta x}$

$\vec{m} = \langle \Delta x, \Delta y \rangle = \langle x_2 - x_1, y_2 - y_1 \rangle$

$= \langle -3-1, -4-8 \rangle = \langle -4, -12 \rangle$

$\vec{r}_0 = \langle 1, 8 \rangle$

$\vec{r}(t) = \vec{r}_0 + t\,\vec{m}$

$y = mx + b$

$\vec{r}_0 = \langle 1, 8 \rangle$

$\vec{r}(t) = \vec{r}_0 + t\,\vec{m}$

$y = b + x \cdot m$

$y = mx + b$

Vector Eq:

$$\boxed{\begin{aligned}\vec{r}(t) &= \langle 1, 8\rangle + t\langle -4, -12\rangle \\ \vec{r}(t) &= \langle 1-4t,\ 8-12t\rangle\end{aligned}}$$

Parametric Eq:

$$\boxed{\begin{aligned}x &= 1-4t \\ y &= 8-12t\end{aligned}}$$

$x = x_0 + at$
$y = y_0 + bt$

3. Find parametric equations of the line parallel to $\mathbf{r}(t) = \langle \cancel{2} - 11t, \cancel{4} + 3t\rangle$ that passes through the point $(-1, 3)$. Find parametric equations of the line perpendicular to this line that passes through the point $(-2, 4)$.

Line: Point + Slope $\rightarrow$ $\vec{m} = \langle -11, 3\rangle$

$\vec{r}_0 = \langle -1, 3\rangle$

$\vec{r}(t) = \langle -1, 3\rangle + t\langle -11, 3\rangle = \langle -1-11t,\ 3+3t\rangle$

Parametric Eq:

$$\boxed{\begin{aligned}x &= -1-11t \\ y &= 3+3t\end{aligned}}$$

$m = \frac{\Delta y}{\Delta x}$

$\vec{m} = \langle -11, 3\rangle$

$m^{\perp} = -\frac{\Delta x}{\Delta y}$

Line: Point + Slope $\vec{r}_0 = \langle -2, 4\rangle$

$\vec{m}^{\perp} = \langle -3, -11\rangle$ or $\langle 3, 11\rangle$

$\vec{m} = \langle \Delta x, \Delta y\rangle$

$\vec{m}^{\perp} = \langle -\Delta y, \Delta x\rangle$

$\vec{r}(t) = \langle -2, 4\rangle + t\langle -3, -11\rangle$

Parametric Eq:

$$\boxed{\begin{aligned}x &= -2-3t \\ y &= 4-11t\end{aligned}}$$

4. Determine if the lines are parallel, perpendicular, or neither. If they are not parallel, find their point of intersection.
$r_1(t) = \langle 2+4t, -8-t \rangle$, $r_2(s) = (3s)i + (9+12s)j$

$\vec{r}_2(s) = \langle 3s, 9+12s \rangle$

$\vec{m}_1 = \langle 4, -1 \rangle$ $\quad \vec{m}_2 = \langle 3, 12 \rangle$

Perpendicular? $\vec{m}_1 \cdot \vec{m}_2 = 0$?

$4(3) + (-1)(12) = 12 - 12 = 0$ ✓

$\boxed{\vec{r}_1 \text{ and } \vec{r}_2 \text{ are perpendicular}}$

x: $2+4t = 3s \Rightarrow 4t - 3s = -2$

y: $-8-t = 9+12s \Rightarrow -t - 12s = 17$

$4t - 3s = -2$
$(-t - 12s = 17)\,4$

$(4t - 3s = -2)\,{-4}$
$-t - 12s = 17$

$4t - 3s = -2$
$+\; -4t - 48s = 68$
$-51s = 66$
$s = \frac{66}{-51}$

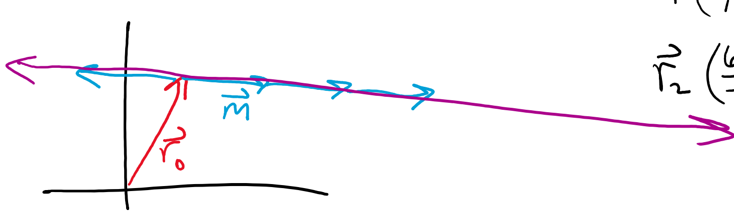
$-16t + 12s = 8$
$+\; -t - 12s = 17$
$-17t = 25$
$t = \frac{-25}{17}$

(x, y)

Point of Intersection

$\vec{r}_1\left(\frac{-25}{17}\right) = \langle 2 + 4\left(\frac{-25}{17}\right), -8 - \left(\frac{-25}{17}\right) \rangle$

$\vec{r}_2\left(\frac{66}{-51}\right) = \langle 3\left(\frac{66}{-51}\right), 9 + 12\left(\frac{66}{-51}\right) \rangle$



$\arcsin(\#) = \theta \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\arccos(\#) = \theta \quad 0 \le \theta \le \pi$

$\arctan(\#) = \theta \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

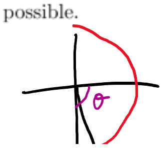
Section 1.5

$\sin(\theta) = -\frac{\sqrt{3}}{2} \qquad \theta = \arcsin\left(-\frac{\sqrt{3}}{2}\right)$

5. Evaluate the following expressions, if possible.

(a) $\arcsin\left(\frac{-\sqrt{3}}{2}\right) = \theta$

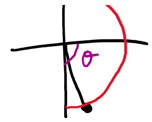
$\sin(\theta) = -\frac{\sqrt{3}}{}$



$\boxed{\theta = \frac{-\pi}{3}}$

(a) $\arcsin\left(\frac{-\sqrt{3}}{2}\right) = \theta$

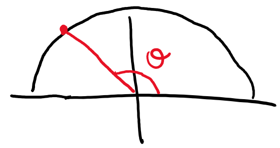
$\sin(\theta) = -\frac{\sqrt{3}}{2}$



$\boxed{\theta = -\frac{\pi}{3}}$

(b) $\arccos\left(\frac{-\sqrt{2}}{2}\right) = \theta$

$\cos(\theta) = -\frac{\sqrt{2}}{2}$

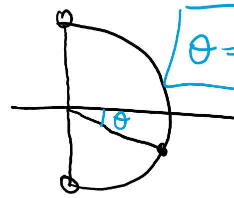


$\boxed{\theta = \frac{3\pi}{4}}$

(c) $\arctan\left(\frac{-1}{\sqrt{3}}\right) = \theta$

$\tan\theta = \frac{-1}{\sqrt{3}}$

$\frac{\sin\theta}{\cos\theta} \quad \frac{-1/2}{\sqrt{3}/2}$



$\boxed{\theta = -\frac{\pi}{6}}$

$\sin^{-1}(x)$
$\parallel$
$\arcsin(x)$

$\frac{1}{\sin(x)} = \csc(x)$

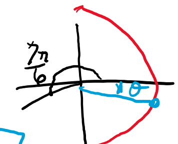
(d) $\arccos 2 = \theta$

$\cos(\theta) = 2$ ✗ $\boxed{\text{DNE}}$

(e) $\sin^{-1}\left(\sin\left(\frac{7\pi}{6}\right)\right) = \sin^{-1}\left(-\frac{1}{2}\right) = \theta$

$\sin(\theta) = -\frac{1}{2}$

$\boxed{\theta = -\frac{\pi}{6}}$



(f) $\cos^{-1}\left(\sin\left(\frac{-7\pi}{3}\right)\right) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \theta$

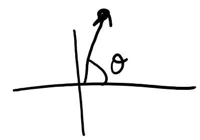
$\boxed{\theta = \frac{5\pi}{6}}$

$\cos(\theta) = -\frac{\sqrt{3}}{2}$

(g) $\sin(\underbrace{\tan^{-1}(\sqrt{3})}_{\theta}) = \sin(\theta) = \boxed{\frac{\sqrt{3}}{2}}$

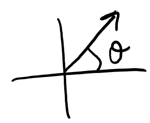
$\tan(\theta) = \sqrt{3}$

$\frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}/2}{1/2}$

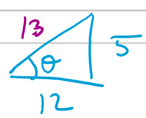


(h) $\csc(\underbrace{\arctan(1)}_{\theta}) = \cos(\theta) = \cos\left(\frac{\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}}$

$\tan(\theta) = 1 \qquad \frac{\sin\theta}{\cos\theta} = 1 \qquad \theta = \frac{\pi}{4}$



$\tan(\theta) = \frac{5}{12}$



$\cos(\theta) = \frac{12}{13}$

Section 2.2

Section 2.2 is a little different. The answers to the questions we're asking are usually very simple, so it is easy to get a correct answer. However, the goal here is to make sure we understand some subtle ideas, so we may have longer discussions about why we arrive at those answer. We will incorporate algebra and more typical solving methods into our examples starting in Section 2.3.

6. Use a calculator to estimate the limits. Note: We are using the calculator to perform (a lot of) arithmetic. Do not just graph the function. That defeats the purpose of what we're accomplishing with these examples.

(a) $\lim_{x\to 3} x^2 - x + 1 = 7$

$\lim_{x\to 3^-} x^2 - x + 1 \overset{?}{=} 7$

$\lim_{x\to 3^+} x^2 - x + 1 \overset{?}{=} 7$

x	f(x)
2	3
2.9	6.51
2.99	6.9501
2.999	6.995001

x	f(x)
4	
3.1	
3.01	
3.001	

$f(3) = 7$

$$\lim_{x \to 3^+} x^2 - x + 1 \doteq 7$$

2.999	6.995001	3.001
2.9999	6.99950001	3.0001

(b) $\lim_{x\to 3} \underbrace{\frac{x^3 - 4x^2 + 4x - 3}{x - 3}}_{f(x)} = 7$

$$\lim_{x \to 3^-} f(x) = 7$$

$$\lim_{x \to 3^+} f(x) = 7$$

x	$f(x)$
2.9	
2.99	
2.999	

x	$f(x)$
3.1	
3.01	
3.001	

$f(3)$ DNE

(c) $\lim_{t\to 0} \frac{\sqrt{t^2+9} - 3}{t^2} = \frac{1}{6}$

7. Use a calculator to estimate the limits.

(a) $\lim_{x \to 3} \frac{5}{(x-3)^2}$ DNE Limit is not approaching a single #.

$\lim_{x \to 3} \frac{5}{(x-3)^2} = \infty$

Getting really big
growing without bound.

(b) $\lim_{x \to 3^-} \frac{5}{x-3} = -\infty$ (DNE)

(c) $\lim_{x \to 3^+} \frac{5}{x-3} = \infty$ (DNE)

(d) $\lim_{x \to 3} \frac{5}{x-3}$ $\boxed{\text{DNE}}$

x	f(x)
3.1	$\frac{5}{3.1-3} = \frac{5}{0.1} = 50$
3.01	$\frac{5}{3.01-3} = \frac{5}{0.01} = 500$
3.001	$\frac{5}{3.001-3} = \frac{5}{0.001} = 5000$

8. Find the vertical asymptotes of the following functions. Use a calculator as needed.

(a) $\frac{x^2+x}{x^2-5x+6}$

Could have a vertical asymptote where denominator is equal to zero

$x^2-5x+6=0$

$(x-3)(x-2)=0$

$x-3=0 \quad x-2=0$

$x=3 \quad x=2$

Vertical Asymptotes:

$\boxed{x=2}$ $\boxed{x=3}$

(b) $\frac{x^2-6x-9}{x^2-5x+6}$

$x^2-5x+6=0$

$(x-2)(x-3)=0$

$x=2,\ \cancel{x=3}$

Vertical Asymptote

$\boxed{x=2}$

9. Use the graph below to evaluate one-sided and two-sided limits at each of the following x−values: $x = -6, x = -4, x = -1, x = 0, x = 1, x = 2, x = 3, x = 5$.

$\lim_{x \to 6^-} f(x) = 4$

$\lim_{x \to 6^+} f(x) = 4$

$\lim_{x \to 6} f(x) = 4$

$\lim_{x \to 0^-} f(x) = 3$

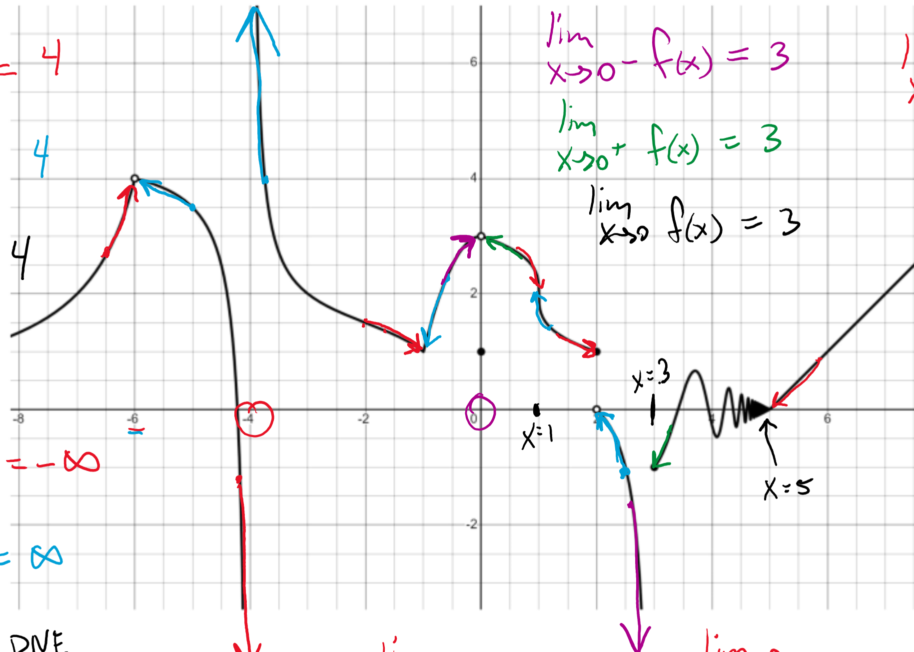
$\lim_{x \to 0^+} f(x) = 3$

$\lim_{x \to 0} f(x) = 3$

$\lim_{x \to 2^-} f(x) = 1$

$\lim_{x \to 2^+} f(x) = 0$

$\lim_{x \to 2} f(x)$ DNE



$\lim_{x \to -4^-} f(x) = -\infty$

$\lim_{x \to -4^+} f(x) = \infty$

$\lim_{x \to -4} f(x)$ DNE

$\lim_{x \to 3^-} f(x) = -\infty$

$\lim_{x \to 3^+} f(x) = -1$

$\lim_{x \to 3} f(x)$ DNE

$\lim_{x \to -1^-} f(x) = 1$

$\lim_{x \to -1^+} f(x) = 1$

$\lim_{x \to -1} f(x) = 1$

$\lim_{x \to 1^-} f(x) = 2$

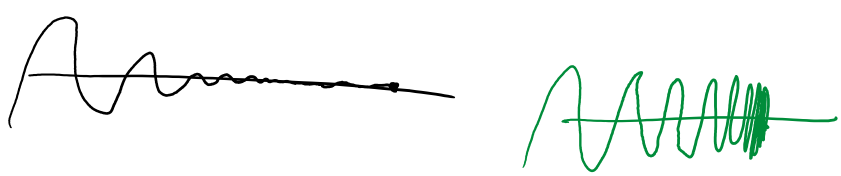
$\lim_{x \to 1^+} f(x) = 2$

$\lim_{x \to 1} f(x) = 2$

$\lim_{x \to 5^+} f(x) = 0$

$\lim_{x \to 5^-} f(x) = 0$

$\lim_{x \to 5} f(x) = 0$

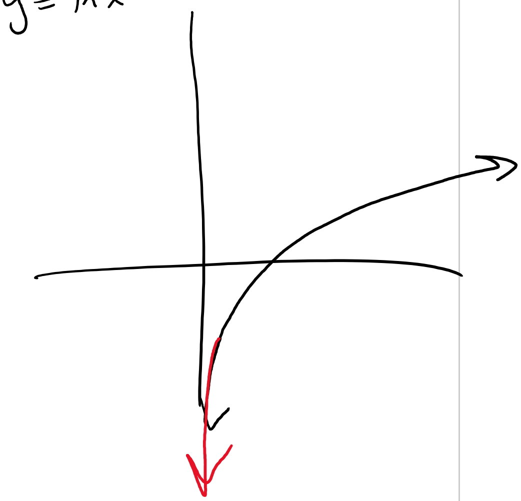


10. Evaluate the limits.

(a) ~~$\lim_{x\to 0^+} \ln x$~~

$$\lim_{x\to 0^+} \ln(x) = -\infty$$

$y = \ln x$



(b) ~~$\lim_{x\to 5^+} \ln(\sin(x-5))$~~

$$\lim_{x\to 5^+} \ln(\sin(\underbrace{x-5}_{0}))$$

$\sin(x-5) = t$

As $x \to 5^+$

$t \to 0^+$

$\underbrace{\sin 0}_{0}$

$$\lim_{t\to 0^+} \ln(t) = -\infty$$