Math 140: Week-in-Review 6 (4.2, 4.3, 4.4 & Chapters 3 & 4 review)

1. An Easter egg is chosen from a box of containing 4 yellow, 2 green, 1 blue, 5 red, and 3 orange eggs and its color is noted.

   (a) Write the probability distribution for this experiment.

   (b) Does this experiment have a uniform sample space? Explain why or why not.

   (c) What is the probability that a yellow or a blue egg is chosen?

   (d) What is the probability that an orange egg is not chosen?

2. A pair of fair **five-sided** dice are rolled noting the uppermost numbers.

   (a) Write an appropriate sample space for this experiment.

   (b) Is this a uniform sample space? Explain why or why not.

   (c) Write the event, E, that a sum of less than 5 is rolled.

   (d) Determine $P(E)$.

   (e) Determine the probability that a sum of 3 and at least one 4 is rolled.

(f) Determine the probability that a product of 12 or a 5 is rolled on at least one of the dice.

(g) Determine the probability that a sum of 7 is rolled or a double is rolled.

(h) Determine the probability that a 2 is not rolled on either die.

(i) Draw a probability distribution for the outcome, X, representing the sum of the numbers rolled.

(j) Determine the expected value for the sum, X.

3. The students attending an Investment Club meeting were surveyed concerning the courses they were taking in the Fall. The findings were gathered in the table below

	Math (M)	Economics (E)	Accounting (A)	Other (O)	Totals
Freshman (F)	17	10	27	5	59
Sophomore (H)	12	7	13	9	41
Totals	29	17	40	14	100

For each question below, re-write the question being asked using probability notation and then determine the numerical answer.

Determine the probability that a randomly surveyed student at the Investment Club meeting

(a) Is a sophomore?

(b) Is taking a Math class?

(c) Is a sophomore and taking a Math class?

(d) Is a sophomore or taking a Math class?

(e) Is not taking an Accounting class?

(f) Is a freshman or is not taking an Accounting class?

4. Let $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ be the sample space for an experiment with the following probability distribution.

Outcome	s_1	s_2	s_3	s_4	s_5	s_6
Probability	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{5}{16}$	$\frac{1}{16}$	$\frac{3}{16}$

(a) Determine if the probability distribution is uniform. Explain why or why not.

(b) Assume that $A = \{s_1, s_4, s_6\}$, $B = \{s_2, s_4, s_5\}$ and $C = \{s_1, s_3, s_6\}$. Determine the following probabilities.

(i) $P(A)$

(ii) $P(B)$

(iii) $P(A \cup B)$

(iv) $P(B \cap C)$

Outcome	s_1	s_2	s_3	s_4	s_5	s_6
Probability	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{5}{16}$	$\frac{1}{16}$	$\frac{3}{16}$

(v) $P(B \cup C)$

(vi) $P(A^C)$

(vii) $P(B^C \cup A)$

5. Suppose that $P(E) = 0.45$, $P(F) = 0.55$, and $P(E \cup F) = 0.8$. Calculate the following.

   (a) $P(E \cap F)$

   (b) $P(E^C)$

   (c) $P(E^C \cap F^C)$

   (d) $P(E^C \cup F)$

6. 150 students were surveyed about the sports they play. 100 students play tennis, 85 play basketball, and 40 don't play either sport. Determine the probability that a randomly selected student

   (a) Plays both tennis and basketball.

   (b) Plays only basketball

   (c) Does not play tennis?

7. Assume $P(A) = 0.38$ and $P(B^C) = 0.59$

(a) If $P(A \cap B) = 0.27$, determine

(i) $P(A \cup B)$

(ii) $P(A^C \cap B)$

(b) If A and B are mutually exclusive, determine

(i) $A \cap B$

(ii) $P(A \cap B)$

(iii) $P(A \cup B)$

(vi) $P(A^C \cup B^C)$

8. Consider the probability distribution below

X	-2	-1	0	1	2	3
$P(X)$	0.2	0.1	0.25	0.05		0.15

(a) Fill in the missing probability in the probability distribution above.

(b) Draw a histogram to represent the probability data for X.

(c) Calculate the following

(i) $P(X < 0)$

(ii) $P(-2 < X < 3)$

(iii) $P(X = 1.5)$

(d) Determine the expected value of X.

9. An artwork is insured for \$15,000 in the case that it is stolen and \$7,500 in the case that it is damaged over the next year. There is a 0.5% chance that the artwork will be stolen in the next year, and a 2% chance it will be damaged in the next year. The annual premium charged by the insurance company is \$$p$

   (a) Write down the probability distribution for the insurance company's profit on this policy

   (b) Determine the minimum premium charged by the insurance company to break-even.

10. You pay \$2 to play a game where you roll a fair **four sided die** and toss a **fair coin**. If the coin comes up heads you win twice the amount shown on the die in dollars. If the coin comes up tails and you roll an odd number, you win the amount shown on the die in dollars. Otherwise you win nothing.

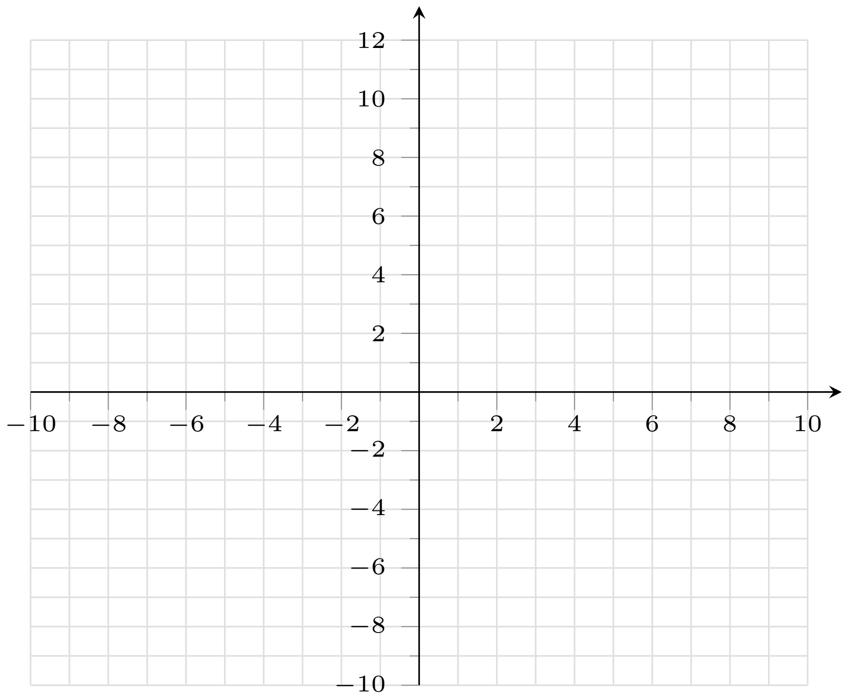
    (a) Draw a probability distribution table to describe your **net** winnings.

    (b) Determine your expected net winnings.

    (c) Is the game fair or unfair? Explain.

11. The following inequalities are constraints in a linear programming problem. Graph the inequalities and determine if the region is bounded or unbounded. Determine all corner points.

$$\begin{aligned} x + y &\le 8 \\ 2x + y &\le 12 \\ 2x - y &\ge -2 \\ x \ge 0,\ & y \ge 0. \end{aligned}$$



(a) At what point(s) is the objective function $z = 4x + 2y$ maximized on this region, and what is the maximum value? (If there is no maximum value, explain why not.)

(b) At what point(s) is the objective function $z = 4x + 2y$ minimized on this region, and what is the minimum value? (If there is no minimum value, explain why not.)

12. A company makes souvenirs of type A and B that they sell to tourists. Each A is sold for \$20 and each B is sold for \$17. It takes 3 hours and \$12 to make each A and 1 hour and \$15 to make each B. If the company has a total of 90 hours and \$600 available for making the souvenirs, and the company can not make more than 20 type A souvenirs because of limited demand, how many of each souvenir should the company make in order to maximize their revenue? What is the maximum revenue? Is anything leftover at the optimal production level?

    (a) Formulate and then solve the given linear programming using the Simplex Method.

(b) Solve the same problem using the Method of Corners and compare your answers.

(c) The demand for souvenirs of type B increases during summer. If the company needs to produce at least twice as many B as A, what additional constraint would have been added to the set-up of the problem?

13. Consider the sample space $S = \{p, q, r\}$.

(a) Determine the total number of events associated with this experiment.

(b) List all the events of this experiment.

(c) How many of these events are simple events? List one.

(d) Give an example of two events that are mutually exclusive.

14. Suppose there is an experiment with sample space $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e\}$ and events

$A =$ the event an even number is drawn.
$B = \{1, 3, 4, 5\}$
$C = \{1, 3, 5, 8\}$
$D =$ the event a letter from the phrase "fall break" is drawn.

Answer each of the following.

(a) $(A \cap B) \cup C$

(b) $A^C \cap B$

(c) $A \cap [(B \cup C)^C]$

(d) Determine the outcomes of event D

(e) Verbally describe the event A^C