Math 251/221 WEEK in REVIEW 1. Fall 2024.

1. Draw a rectangular box that has the points $P(1,1,2)$ and $Q(3,4,5)$ as opposite vertices and has its faces parallel to the coordinate planes. Find the coordinates of the other six vertices of the box. What is the length of the diagonal of the box?

2. Find the center and radius of the sphere $x^2 + y^2 + z^2 + 4x + 6y - 10z + 2 = 0$

3. Find an equation of the sphere given that it touches the yz- plane and has the center at $(2,1,3)$.

4. Describe in words the region of $\mathbb{R}^3$ represented by the equation or inequality.

   (a) $y = z$

   (b) $y > 2$

   (c) $y^2 + z^2 \leq 4$

   (d) $x^2 + y^2 + z^2 - 2z < 3$

5. Find the lengths of the sides and the medians of the triangle with the vertices $P(1,1,0)$, $Q(1,0,1)$, and $R(0,1,1)$.

6. If $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 3\mathbf{j} - 5\mathbf{k}$, find:

   (a) A unit vector in the direction of $\mathbf{a}$.

   (b) A vector in the direction of $\mathbf{a} + \mathbf{b}$ with length 4.

7. A constant force of $\mathbf{F} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ moves an object along the line segment from $(1,0,2)$ to $(3,4,5)$. Find the work done. What is an angle between the force and the displacement vector?

8. What restrictions must be made on b so that the vector $2\mathbf{i} + b\mathbf{j}$ is orthogonal to vector $-3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$? to the vector $\mathbf{k}$?

9. Find the scalar and vector projections of $\mathbf{b} = \langle 4,2,0 \rangle$ onto $\mathbf{a} = \langle 1,2,3 \rangle$.