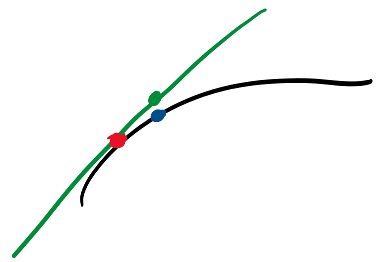
Math 151
Week-In-Review 9
3.10, 4.1

Todd Schrader



Problem Statements

1. (a) Find the linear approximation to $f(x) = x^{-\frac{1}{2}}$ at $a = 9$.

Point: $f(9) = 9^{-1/2} = \frac{1}{\sqrt{9}} = \frac{1}{3}$ $\quad (9, \frac{1}{3})$

Slope: $f'(x) = \frac{-1}{2} x^{-3/2}$

$f'(9) = \frac{-1}{2} \cdot \frac{1}{9^{3/2}} = \frac{-1}{2} \cdot \frac{1}{3^3} = \frac{-1}{2} \cdot \frac{1}{27} = \frac{-1}{54}$

$y - \frac{1}{3} = \frac{-1}{54}(x - 9)$

$y = \frac{1}{3} - \frac{1}{54}(x - 9)$

$f(x) \approx L(x) = \frac{1}{3} - \frac{1}{54}(x - 9)$

Linear Approximation or Linearization

(b) Approximate $10^{-\frac{1}{2}}$. $\quad f(x) = x^{-1/2}$ $\quad x = 10$

$f(10) \approx L(10) = \frac{1}{3} - \frac{1}{54}(10 - 9) = \boxed{\frac{1}{3} - \frac{1}{54}}$

$\frac{18}{54} - \frac{1}{54} = \boxed{\frac{17}{54}}$

2. Find the linear approximation for the following equations when $a = 0$.

(a) $f(x) = \sin(x)$

Point: $f(0) = \sin(0) = 0$ $(0,0)$

Slope: $f'(x) = \cos x$

$f'(0) = \cos(0) = 1$

$y - 0 = 1(x - 0)$

$y = x$

$f(x) \approx L(x) = x$ $\sin(x) \approx x$

For small x-values (or x-values near zero) $\sin(x) \approx x$.

(b) $g(x) = \cos(x)$ $(0,1)$

Point: $g(0) = \cos(0) = 1$

Slope: $g'(x) = -\sin(x)$

$g'(0) = -\sin(0) = 0$

$y - 1 = 0(x - 0)$

$y = 1$

$L(x) = 1$

For small values of x, $\cos(x) \approx 1$

$(5-x^2)^{1/2}$

$x=2$

3. (a) Find the linearization of $f(x) = \sqrt{5 - x^2}$ at $a = 2$.

Point: $f(2) = \sqrt{5-2^2} = \sqrt{1} = 1 \quad (2,1)$

Slope: $f'(x) = \frac{1}{2}(5-x^2)^{-1/2} \cdot (-2x) = \frac{-x}{\sqrt{5-x^2}}$

$f'(2) = \frac{-2}{\sqrt{5-2^2}} = -2$

$\boxed{L(x) = 1 - 2(x-2)}$

(b) Approximate $f(2.1)$ using a linear approximation.

$f(2.1) \approx L(2.1) = 1 - 2(2.1 - 2)$

$= 1 - 2(0.1) = 1 - 0.2 = \boxed{0.8}$

$y = f(x)$

$\frac{d}{dx}(y) = \frac{dy}{dx} \approx \frac{\Delta y}{\Delta x}$

(c) Find the differential dy if $y = \sqrt{5 - x^2}$.

$\frac{dy}{dx} = f'(x)$

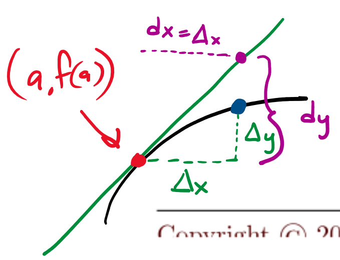
$dy = f'(x)\,dx$

$dy = \frac{1}{2} \cdot (5-x^2)^{-1/2} \cdot (-2x)\,dx$

$\boxed{dy = \frac{-x}{\sqrt{5-x^2}}\,dx}$

(d) Evaluate dy for $x = 2$ and $dx = 0.1$.

$dy = \frac{-2}{\sqrt{5-2^2}}(0.1) = \frac{-2}{1}(0.1) = \boxed{-0.2}$



4. (a) Use a linear approximation to estimate $e^{0.1}$.

$a = 0$

Point: $f(0) = e^0 = 1$ $(0,1)$

$f(x) = e^x$

Slope: $f'(x) = e^x$

$f'(0) = e^0 = 1$

$L(x) = 1 + 1(x-0)$

$L(x) = 1 + x$

$\sqrt[3]{65}$

$e^{0.1} \approx L(0.1) = 1 + 0.1 = \boxed{1.1}$

$f(x) = \sqrt[3]{x}$

$a = 64$

(b) Use differentials to estimate $e^{0.1}$.

$x=0$

$y = e^x$ $a = 0$

$dx = \Delta x = 0.1 - 0 = 0.1$

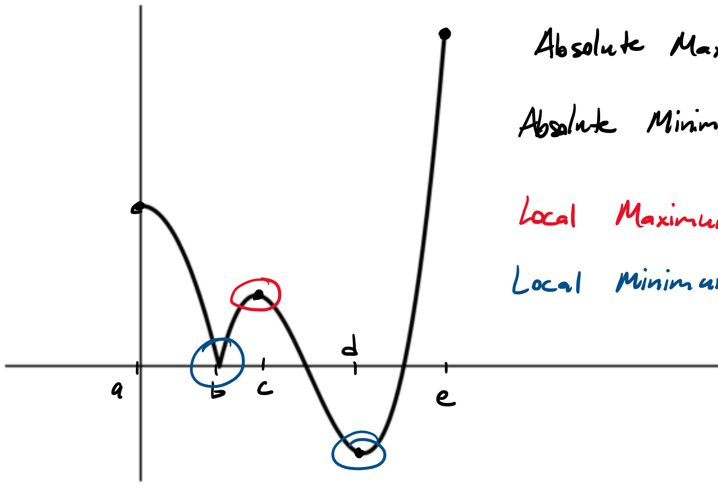
$dy = e^x\, dx$

$e^0 = 1$

$dy = e^0 (0.1) = 0.1$

$e^{0.1} \approx 1 + dy = 1 + 0.1 = \boxed{1.1}$

5. For each graph below,determine the locations of each absolute maximum or minimum. Also determine the location of each local maximum or minimum.



(a)

Absolute Maximum: @ x = e

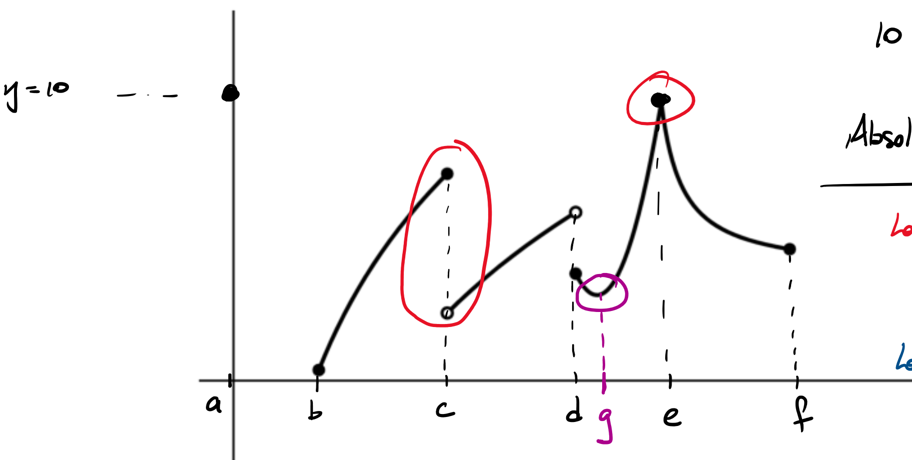
Absolute Minimum: @ x = d

Local Maximum: @ x = c

Local Minimum: @ x = b

and

@ x = d



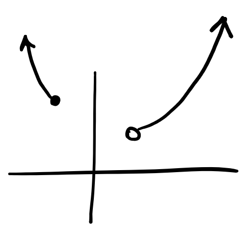
(b)

Absolute Maximum:
10 @ x = a and x = e

Absolute Minimum: @ x = b

Local Maximum: @ x = e
and @ x = c

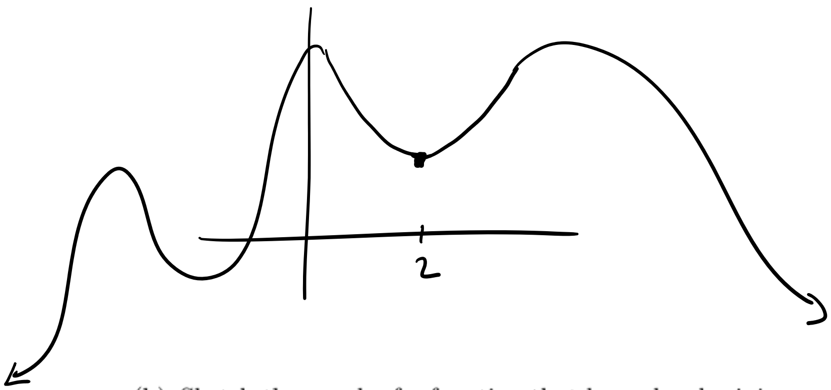
Local Minimum: @ x = g



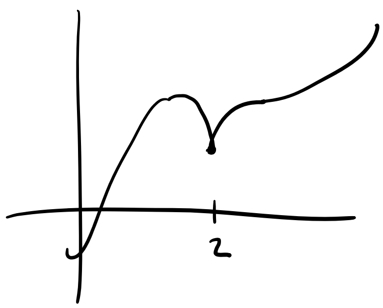
Absolute Maximum: None

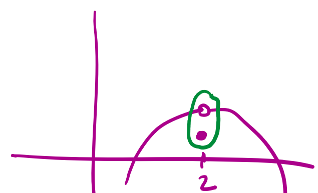
Absolute Minimum: None

6. (a) Sketch the graph of a function that has a local minimum at 2 and is differentiable at 2.

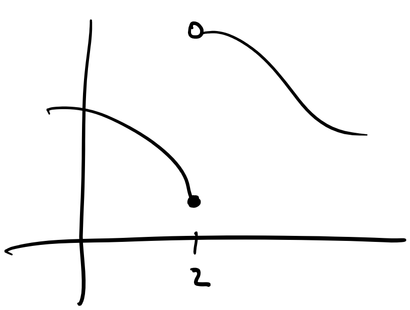


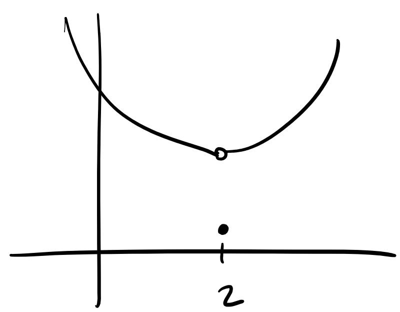
(b) Sketch the graph of a function that has a local minimum at 2 and is continuous but not differentiable at 2.





(c) Sketch the graph of a function that has a local minimum at 2 and is not continuous at 2.





7. Find the critical numbers of the functions.

Critical Numbers/Values

Either $f'(x)$ DNE

or

$f'(x) = 0$

(a) $g(v) = v^3 - 12v + 4$

$g'(v) = 3v^2 - 12$

$g'(v)$ DNE : None N/A

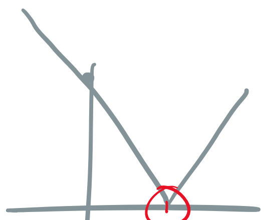
$g'(v) = 0$ $\quad 3v^2 - 12 = 0$

$3v^2 = 12$

$v^2 = 4$

$v = \pm 2$

$$\boxed{\begin{array}{l}\text{Critical \#s of } g(v): \\ v = -2, \quad v = 2\end{array}}$$



$\frac{5}{3}$ (b) $A(x) = |5 - 3x|$

$5 - 3x \geq 0$

$-3x \geq -5$

$x \leq \frac{5}{3}$

if $5 - 3x \geq 0$

$$A(x) = \begin{cases} 5 - 3x & \text{if } x \leq \frac{5}{3} \\ -(5-3x) & \text{if } x > \frac{5}{3} \end{cases}$$

$$A'(x) = \begin{cases} -3 & \text{if } x < \frac{5}{3} \\ 3 & \text{if } x > \frac{5}{3} \end{cases}$$

$A'(x)$ DNE : $x = \frac{5}{3}$

$A'(x) = 0$: N/A

$$\lim_{x \to \frac{5}{3}^-} A'(x) = \lim_{x \to \frac{5}{3}^-} (-3) = -3$$

$$\lim_{x \to \frac{5}{3}^+} A'(x) = \lim_{x \to \frac{5}{3}^+} (3) = 3$$

$A'(x)$ DNE @ $x = \frac{5}{3}$

$$\boxed{\begin{array}{l}\text{Critical \#s of } A(x): \\ x = \frac{5}{3}\end{array}}$$

* Edit

(c) $h(t) = t^{3/4} - 2t^{1/4}$ $\quad t \geq 0$

$$h'(t) = \frac{3}{4}t^{-1/4} - \frac{1}{2}t^{-3/4} = \frac{t^{1/2}}{t^{1/2}} \cdot \frac{3}{4t^{1/4}} - \frac{1}{2t^{3/4}} \cdot \frac{2}{2}$$

$$h'(t) = \frac{3t^{1/2} - 2}{4t^{3/4}}$$

$$\boxed{\begin{array}{c}\text{Critical \#s of } h(t) \\ t=0, \quad t = \frac{4}{9}\end{array}}$$

$h'(t)$ DNE : $\quad 4t^{3/4} = 0$

$$\boxed{t=0}$$

$h'(t) = 0$

$$\frac{3t^{1/2} - 2}{4t^{3/4}} = 0 \qquad 3t^{1/2} = 2$$

$$t^{1/2} = \frac{2}{3} \qquad \boxed{t = \frac{4}{9}}$$

$$\Rightarrow 3t^{1/2} - 2 = 0$$

(d) $f(x) = x^{1/3}(4-x)^{2/3}$

$$f'(x) = x^{1/3} \cdot \frac{2}{3}(4-x)^{-1/3} \cdot (-1) + (4-x)^{2/3} \cdot \frac{1}{3}x^{-2/3}$$

$$= \frac{x^{2/3}}{x^{2/3}} \cdot \frac{-2x^{1/3}}{3(4-x)^{1/3}} + \frac{(4-x)^{2/3}}{3x^{2/3}} \cdot \frac{(4-x)^{1/3}}{(4-x)^{1/3}}$$

$$= \frac{-2x + (4-x)}{3x^{2/3}(4-x)^{1/3}} = \frac{-3x+4}{3x^{2/3}(4-x)^{1/3}}$$

$$\boxed{\begin{array}{c}\text{Critical \#s of } f(x) \\ \hline x=0, \quad x=4, \quad x = \frac{4}{3}\end{array}}$$

$f'(x)$ DNE : $\quad 3x^{2/3}(4-x)^{1/3} = 0$

$x=0 \qquad x=4$

$f'(x) = 0$: $\quad \frac{-3x+4}{3x^{2/3}(4-x)^{1/3}} = 0 \Rightarrow -3x+4 = 0 \qquad -3x = -4 \qquad x = \frac{4}{3}$

8. Determine the absolute maximum and minimum values of the function on the given interval.

(a) $f(x) = \dfrac{x}{x^2 - x + 1}$, $[0, 3]$

$$f'(x) = \frac{(x^2-x+1)(1) - x(2x-1)}{(x^2-x+1)^2} = \frac{x^2-x+1-2x^2+x}{(x^2-x+1)^2}$$

$$= \frac{-x^2+1}{(x^2-x+1)^2}$$

$f'(x) = 0$: $-x^2+1 = 0$

$-(x^2-1) = 0$

$-(x-1)(x+1) = 0$

$x = 1$, $\cancel{x = -1}$ Not in Interval

$f'(x)$ DNE: $(x^2-x+1)^2 = 0$

$x^2 - x + 1 = 0$

$b^2 - 4ac = 1 - 4(1)(1) = -3$

No Solutions

x	$f(x)$
0	0
1	$\frac{1}{1} = 1$
3	$\frac{3}{7}$

Absolute Maximum: 1 @ $x = 1$

Absolute Minimum: 0 @ $x = 0$

(b) $g(x) = 2\cos(x) + \sin(2x)$, $[0, \pi]$

$g'(x) = -2\sin(x) + \cos(2x) \cdot 2$

$g'(x)$ DNE: None

$g'(x) = 0$: $-2\sin(x) + 2\cos(2x) = 0$

$-\sin(x) + \cos(2x) = 0$

$-\sin(x) + 1 - 2\sin^2(x) = 0$

$-(2\sin^2(x) + \sin(x) - 1) = 0$

$-(2\sin(x) - 1)(\sin(x) + 1) = 0$

$-(2x^2 + x - 1) = 0$ $\quad -(2x-1)(x+1) = 0$

$2\sin(x) - 1 = 0$

$2\sin(x) = 1$

$\sin(x) = \frac{1}{2}$

$x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$

$\sin(x) + 1 = 0$

$\sin(x) = -1$

$\cancel{x = \frac{3\pi}{2}}$ Not in Interval

x	$g(x)$
0	2
$\frac{\pi}{6}$	$2\left(\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$
$\frac{5\pi}{6}$	$2\left(-\frac{\sqrt{3}}{2}\right) + \frac{-\sqrt{3}}{2} = \frac{-3\sqrt{3}}{2}$
π	-2

Abs Max: $\frac{3\sqrt{3}}{2}$ @ $x = \frac{\pi}{6}$

Abs. Min: $\frac{-3\sqrt{3}}{2}$ @ $x = \frac{5\pi}{6}$