Math 150 - Week-In-Review 3 Solutions

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Section 3.1 – Simplifying Rational Expressions

1. **Simplify each expression and determine for what values of x, if any, the simplified expression is not equivalent to the original expression.**

   (a) $\dfrac{x^2+4x-12}{3x-6}$

   Solution:

   $$\frac{x^2+4x-12}{3x-6} = \frac{(x+6)\cancel{(x-2)}}{3\cancel{(x-2)}} = \frac{x+6}{3}$$

   Since we cancelled the factor $(x-2)$ in finding the simplified expression, the simplified expression is equivalent to the original expression for $x \neq 2$.

   (b) $\dfrac{\frac{2}{x}-3}{1-\frac{1}{x-1}}$

   Solution:

   $$\frac{\frac{2}{\cancel{x}}-3}{1-\frac{1}{\cancel{x-1}}} \cdot \frac{\cancel{x}(x-1)}{x\cancel{(x-1)}} = \frac{2(x-1)-3x(x-1)}{x(x-1)-x} = \frac{(x-1)(2-3x)}{x(x-2)}$$

   Since we cancelled the factor $(x-1)$ in finding the simplified expression, the simplified expression is equivalent to the original expression for $x \neq 1$. We also cancelled the factor x, but since both expressions are undefined (divide by zero) at $x=0$, both expressions are equivalent at $x=0$.

   (c) $\dfrac{x^2+4x-5}{3x+18} \cdot \dfrac{2x-1}{x+5}$

   Solution:

   $$\frac{x^2+4x-5}{3x+18} \cdot \frac{2x-1}{x+5} = \frac{\cancel{(x+5)}(x-1)}{3(x+6)} \cdot \frac{2x-1}{\cancel{x+5}} = \frac{(x-1)(2x-1)}{3(x+6)}$$

   Since we cancelled the factor $(x+5)$ in finding the simplified expression, the simplified expression is equivalent to the original expression for $x \neq -5$.

   (d) $\dfrac{2x^2+x-6}{x^2-1} \div \dfrac{x^2-4}{x^2+2x+1}$

Solution:

$$\frac{2x^2+x-6}{x^2-1} \div \frac{x^2-4}{x^2+2x+1} = \frac{2x^2+x-6}{x^2-1} \cdot \frac{x^2+2x+1}{x^2-4}$$

$$= \frac{(2x-3)\cancel{(x+2)}}{\cancel{(x+1)}(x-1)} \cdot \frac{\cancel{(x+1)}(x+1)}{\cancel{(x+2)}(x-2)}$$

$$= \frac{(2x-3)(x+1)}{(x-1)(x-2)}$$

Since we cancelled the factors $(x+1)$ and $(x+2)$ in finding the simplified expression, the simplified expression is equivalent to the original expression for $x \neq -1$, and $x \neq -2$.

(e) $\dfrac{6}{x^2+4x+4} + \dfrac{2}{x^2-4}$

Solution:

$$\frac{6}{x^2+4x+4} + \frac{2}{x^2-4} = \frac{6}{(x+2)(x+2)} + \frac{2}{(x+2)(x-2)}$$

$$= \frac{6}{(x+2)(x+2)} \cdot \frac{x-2}{x-2} + \frac{2}{(x+2)(x-2)} \cdot \frac{x+2}{x+2}$$

$$= \frac{6x-12}{(x+2)^2(x-2)} + \frac{2x+4}{(x+2)^2(x-2)}$$

$$= \frac{8x-8}{(x+2)^2(x-2)} = \frac{8(x-1)}{(x+2)^2(x-2)}$$

Since we didn't cancel any factors in finding the simplified expression, the simplified expression is equivalent to the original expression for all x.

(f) $\dfrac{x}{x^2+5x+6} - \dfrac{3}{x^2+7x+12}$

Solution:

$$\frac{x}{x^2+5x+6} - \frac{3}{x^2+7x+12} = \frac{x}{(x+2)(x+3)} - \frac{3}{(x+3)(x+4)}$$

$$= \frac{x}{(x+2)(x+3)} \cdot \frac{x+4}{x+4} - \frac{3}{(x+3)(x+4)} \cdot \frac{x+2}{x+2}$$

$$= \frac{x^2+4x}{(x+2)(x+3)(x+4)} - \frac{3x+6}{(x+2)(x+3)(x+4)}$$

$$= \frac{x^2+4x-3x-6}{(x+2)(x+3)(x+4)} = \frac{x^2+x-6}{(x+2)(x+3)(x+4)}$$

$$= \frac{(x-2)\cancel{(x+3)}}{(x+2)\cancel{(x+3)}(x+4)} = \frac{x-2}{(x+2)(x+4)}$$

Since we cancelled the factor $(x+3)$ in finding the simplified expression, the simplified expression is equivalent to the original expression for $x \neq -3$.

(g) $x\,(1-2x)^{-3} + (1-2x)^{-2}$

Solution:

$$x\,(1-2x)^{-3} + (1-2x)^{-2} = \frac{x}{(1-2x)^3} + \frac{1}{(1-2x)^2}$$

$$= \frac{x}{(1-2x)^3} + \frac{1}{(1-2x)^2} \cdot \frac{1-2x}{1-2x}$$

$$= \frac{x+1-2x}{(1-2x)^3} = \frac{1-x}{(1-2x)^3}$$

Since we didn't cancel any factors in finding the simplified expression, the simplified expression is equivalent to the original expression for all x.

Instead, we can factor out $(1-2x)^{-3}$,

$$x\,(1-2x)^{-3} + (1-2x)^{-2} = (1-2x)^{-3}\left[x + (1-2x)^1\right]$$

$$= (1-2x)^{-3}\,[1-x]$$

$$= \frac{1-x}{(1-2x)^3}.$$

(h) $\dfrac{\left(4-x^2\right)^{1/2}+x^2\left(4-x^2\right)^{-1/2}}{4-x^2}$

Solution:

$$\begin{aligned}\frac{\left(4-x^2\right)^{1/2}+x^2\left(4-x^2\right)^{-1/2}}{4-x^2} &= \frac{\left(4-x^2\right)^{1/2}+x^2\left(4-x^2\right)^{-1/2}}{4-x^2}\cdot\frac{\left(4-x^2\right)^{1/2}}{\left(4-x^2\right)^{1/2}}\\ &= \frac{4-x^2+x^2\left(4-x^2\right)^0}{(4-x^2)^{3/2}} = \frac{4-x^2+x^2}{(4-x^2)^{3/2}}\\ &= \frac{4}{(4-x^2)^{3/2}}\end{aligned}$$

We cancelled the factor $\sqrt{4-x^2}$, but since both expressions are undefined (divide by zero) at $x=\pm 2$, both expressions are equivalent for all x.

For a slightly different method,

$$\begin{aligned}\frac{\left(4-x^2\right)^{1/2}+x^2\left(4-x^2\right)^{-1/2}}{4-x^2} &= \frac{\dfrac{\sqrt{4-x^2}}{1}+\dfrac{x^2}{\sqrt{4-x^2}}}{4-x^2}\\ &= \frac{\dfrac{\sqrt{4-x^2}}{1}\cdot\dfrac{\sqrt{4-x^2}}{\sqrt{4-x^2}}+\dfrac{x^2}{\sqrt{4-x^2}}}{4-x^2}\\ &= \frac{\dfrac{4-x^2}{\sqrt{4-x^2}}+\dfrac{x^2}{\sqrt{4-x^2}}}{4-x^2}\\ &= \frac{\dfrac{4}{\sqrt{4-x^2}}}{4-x^2} = \frac{4}{(4-x^2)^{3/2}}.\end{aligned}$$

2. **Find and simplify the difference quotient** $\dfrac{f(x+h)-f(x)}{h}$, $h\neq 0$**, for each function** f **given.**

   (a) $f(x)=x^2-12x+5$

   Solution: Since

$$\begin{aligned}f(x+h)-f(x) &= (x+h)^2-12(x+h)+5-(x^2-12x+5)\\ &= x^2+2xh+h^2-12x-12h+5-x^2+12x-5\\ &= 2xh+h^2-12h,\end{aligned}$$

then,
$$\frac{f(x+h)-f(x)}{h} = \frac{\cancel{h}(2x+h-12)}{\cancel{h}} = 2x+h-12.$$

(b) $f(x) = \sqrt{x+3}$

Solution: Rationalizing the numerator,
$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\sqrt{x+h+3}-\sqrt{x+3}}{h} \\ &= \frac{\sqrt{x+h+3}-\sqrt{x+3}}{h} \cdot \frac{\sqrt{x+h+3}+\sqrt{x+3}}{\sqrt{x+h+3}+\sqrt{x+3}} \\ &= \frac{x+h+3-(x+3)}{h\left(\sqrt{x+h+3}+\sqrt{x+3}\right)} \\ &= \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h+3}+\sqrt{x+3}\right)} \\ &= \frac{1}{\sqrt{x+h+3}+\sqrt{x+3}}.\end{aligned}$$

(c) $f(x) = \dfrac{1}{2x-1}$

Solution: Since
$$\begin{aligned}f(x+h)-f(x) &= \frac{1}{2(x+h)-1} - \frac{1}{2x-1} = \frac{1}{2x+2h-1} - \frac{1}{2x-1} \\ &= \frac{1}{2x+2h-1}\cdot\frac{2x-1}{2x-1} - \frac{1}{2x-1}\cdot\frac{2x+2h-1}{2x+2h-1} \\ &= \frac{2x-1-(2x+2h-1)}{(2x-1)(2x+2h-1)} \\ &= -\frac{2h}{(2x-1)(2x+2h-1)},\end{aligned}$$

then,
$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{-\dfrac{2h}{(2x-1)(2x+2h-1)}}{h} = -\frac{2\cancel{h}}{\cancel{h}(2x-1)(2x+2h-1)} \\ &= -\frac{2}{(2x-1)(2x+2h-1)}.\end{aligned}$$

3. **The height, s, in feet, of a football kicked off at the start of an A&M game, after t seconds, is given by the function $s(t) = -16t^2 + 80t$. Estimate the instantaneous velocity of the football one second after the kickoff by first finding the average velocity of the football over the interval $[1, t]$.**

Solution: By the definition of average velocity over the interval $[1, t]$,

$$\overline{v} = \frac{s(t) - s(1)}{t - 1} = \frac{-16t^2 + 80t - (-16(1)^2 + 80(1))}{t - 1} = \frac{-16t^2 + 80t - 64}{t - 1}$$

$$= \frac{-16(t^2 - 5t + 4)}{t - 1} = \frac{-16\cancel{(t-1)}(t - 4)}{\cancel{t-1}} = -16(t - 4).$$

Since the instantaneous velocity of the football one second after kickoff is what the average velocity $\overline{v}$ approaches as $t \to 1$,

$$v(1) = \overline{v(1)} = -16(1 - 4) = -16(-3) = 48 \text{ ft/s}.$$

As the graph shows, the football reaches its peak height after 2.5 seconds, so, the football is rising after one second, meaning the velocity is positive.