Math 151 - Week-In-Review 10

Topics for the week:

4.2 The Mean Value Theorem
4.3 How Derivatives Affect the Shape of a Graph

4.2 The Mean Value Theorem

1. Verify that the function, $f(x) = x^3 + 4x^2 - 5$, satisfies the three hypotheses of Rolle's theorem on the interval $[-4, 0]$. Then determine all numbers c that satisfy the conclusion of Rolle's theorem.

2. Does the function, $g(x) = \sqrt[3]{6 - x}$, satisfy the hypotheses of the Mean Value Theorem on the interval $[0, 14]$. If it satisfies the hypotheses, compute all numbers c that satisfy the conclusion of the Mean Value Theorem.

3. Does the function, $h(x) = \frac{1}{2} \cdot 3^{x+1}$, satisfy the hypotheses of the Mean Value Theorem on the interval $[-1, 1]$. If it satisfies the hypotheses, compute all numbers c that satisfy the conclusion of the Mean Value Theorem.

4. A car moves in a straight line. At time, t, (measured in seconds), its position (measured in feet) is $s(t) = t^3 - 3t + 2$, for $t \in [0, 4]$. At what time is the instantaneous velocity of the car equal to its average velocity?

4.3 How Derivatives Affect the Shape of a Graph

5. Determine the intervals on which $f(x) = x^3 - 5x^2 + 8x + 2$ is increasing or decreasing.

6. Determine the intervals on which $f(x) = xe^{x^2-3x}$ is increasing or decreasing.

7. Determine the points in the interval $[0, \pi]$ where $f(x) = \sec^2(3x)$ has any local extrema of $f(x)$.

8. Determine the points where $f(x) = 5x\ln(3x - 6)$ has any local extrema of $f(x)$.

9. Determine the intervals where $f(x) = \dfrac{x^2+1}{x}$ is concave up and concave down. Then determine any inflection points of $f(x)$.

10. Determine the intervals on which $f(x) = (x^2 - 16)^{2/3}$ is concave up and concave down. Then determine any inflection points of $f(x)$.

11. Suppose the function $f(x)$ has a domain of $(-\infty, 4) \cup (4, \infty)$, critical numbers of $x = 0$ and $x = 8$, and a second derivatives of $\dfrac{d^2 f(x)}{dx^2} = \dfrac{160}{(x-4)^3}$.

   (a) State the value(s) of x where $f(x)$ has a local maximum: ______________

   (b) State the value(s) of x where $f(x)$ has a local minimum: ______________

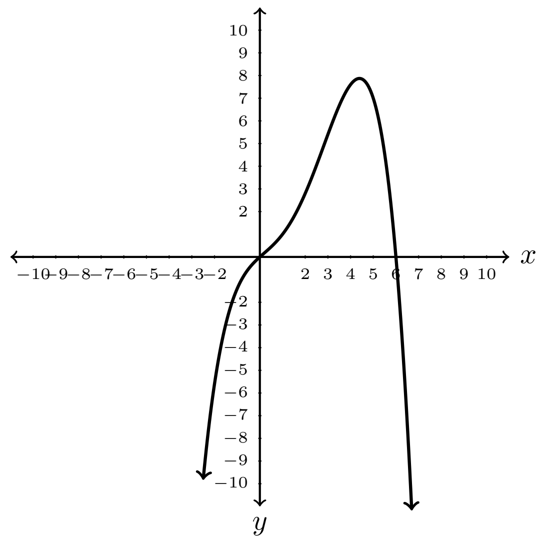
   (c) State the "critical numbers" of the second derivative: ______________

   (d) State the interval(s) where $f(x)$ is concave upward: ______________

   (e) State the interval(s) where $f(x)$ is concave downward: ______________

   (f) State the value(s) of x where $f(x)$ has an inflection point: ______________

12. Given the graph of $f'(x)$ below, determine the interval(s) where $f(x)$ is increasing and decreasing.



13. Using the graph above, determine if there are any intervals where $f(x)$ is decreasing and concave up.

14. Given $f(x)$ is a differentiable function on the intervals $(-\infty, -1) \cup (-1, \infty)$, use the sign charts below to answer each question.

Sign of $f'(x)$ $-\ -\ -$ $+\ +\ +\ +\ +\ +\ +$

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Sign of $f''(x)$ $-\ -\ -$ $-\ -\ -$ $+\ +\ +$

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(a) State the critical numbers of the first derivative: ____________________

(b) State the interval(s) where $f(x)$ is increasing: ____________________

(c) State the interval(s) where $f(x)$ is decreasing: ____________________

(d) State the value(s) of x where $f(x)$ has a local maximum: ____________________

(e) State the value(s) of x where $f(x)$ has a local minimum: ____________________

(f) State the "critical numbers" of the second derivative: ____________________

(g) State the interval(s) where $f(x)$ is concave upward: ____________________

(h) State the interval(s) where $f(x)$ is concave downward: ____________________

(i) State the value(s) of x where $f(x)$ has an inflection point: ____________________

15. Sketch the graph of a function that meets these conditions.

$f(x)$ has a domain of all real numbers

$f(4) = 3$

$\lim_{x\to\infty} f(x) = -\frac{1}{2}$

$f'(4) = 0$

$f'(x)$ does not exist when $x = 0$ and $x = 6$

$f'(x) > 0$ on $(0, 4)$

$f'(x) < 0$ on $(-\infty, 0)$, $(4, 6)$, and $(6, \infty)$

$f''(x) < 0$ on $(-\infty, 6)$

$f''(x) > 0$ on $(6, \infty)$