2024_Fall_WeekInReview_Math140_Week2

Sunday, September 1, 2024 3:47 PM

2024_Fall_WeekInRe...

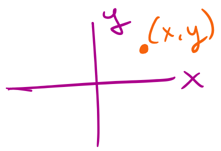
2024 Fall Math 140 Week-In-Review

Week 2: Sections 2.1-2.2

Section 2.1: Review of Lines

Some Key Words and Terms: Point, Quadrant, Line, Linear, Slope, Vertical Line, Horizontal Line, Slope-Intercept Form, Point-Slope Form, Standard Form

Point:

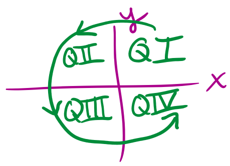


(independent, dependent)

i.e. (time, \$), (quantity, cost), etc....

Quadrant:

the 4 regions of the graph



QI : top right (where both variables are positive)

QII : top left (negative indp values & positive dpnd values)

QIII : bottom left (where both variables are negative)

QIV : bottom right (positive indp values & negative dpnd values)

Line and Linear:

follows a particular relationship for x & y : constant rate of change (slope)

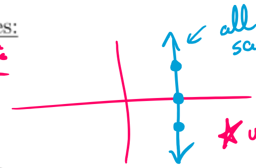
★ ultimately, if something is "linear" we can express it as "$y = mx + b$"

Slope:

constant rate of change $= m = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\Delta y}{\Delta x} = \dfrac{\text{change in } y}{\text{change in } x}$

★ Vertical and Horizontal Lines:

vertical line:



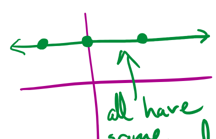
all have same x-value

so: $x = \#$

★ undefined slope

★ no y-intercept unless $x = 0$

horizontal line:



all have same y-value

so: $y = \#$

★ slope $m = 0$

★ no x-intercept unless $y = 0$

Forms of Lines: 3 Forms:

1) Slope-Intercept Form: $y = m \cdot x + b$ (y-intercept)

2) Point-Slope Form: $y - y_1 = m \cdot (x - x_1)$ (x_1, y_1)

3) $Ax + By = C$

★ all variables on left
★ constant on right
} good enough for "augmented matrices" (2.4)

★ no fractions or decimals (whole #s)

★ The skill set we use here is the same skill set in 2.2

Examples:

1. The points $\overset{(x_1,y_1)}{(-6,2)}$ and $\overset{(x_2,y_2)}{(-2,-1)}$ form a line. Determine the following:

   (a) The slope of the line.

   ★ slope $= m = \dfrac{y_2 - y_1}{x_2 - x_1}$ b/c we were given (or could find) two points

   $m = \dfrac{-1-2}{-2-(-6)} = \boxed{\dfrac{-3}{4}}$ can't reduce

   (b) The equation of the line in Point-$\boxed{\text{Slope}}$ Form.

   ↖ it doesn't matter which we choose

   Answer #1: $y - 2 = -\frac{3}{4}(x - (-6))$

   $y - 2 = -\frac{3}{4}(x+6)$

   Answer #2: $y - (-1) = -\frac{3}{4}(x - (-2))$

   $y + 1 = -\frac{3}{4}(x+2)$

   (c) The x and y intercepts, if they exist.

   fastest: ① start w/ $y = mx+b$ & plug-in the m

   ② plug in either point & solve for the b-value (y-value of y-intercept)

   ③ plug in b-value & $y = 0$, to solve for x-value of x-intercept

   ★ALL intercepts are written as points!

   $y = mx + b \Rightarrow y = -\frac{3}{4}x + b$

   $(x,y) = (-2,-1)$

   $-1 = -\frac{3}{\cancel{4}_2}\cdot(\cancel{-2}) + b$

   $-1 = \frac{3}{2} + b$

   $-\frac{3}{2} \quad -\frac{3}{2}$

   $-1 - \frac{3}{2} = -\frac{2}{2} - \frac{3}{2} = -\frac{5}{2} = b$

   $\boxed{\text{y-intercept } (0, -5/2)}$

   $(0, -2.5)$

   $y = -\frac{3}{4}x - \frac{5}{2}$ → $0 = -\frac{3}{4}x - \frac{5}{2}$

   "Slope-Intercept"

   $+\frac{5}{2} \quad +\frac{5}{2}$

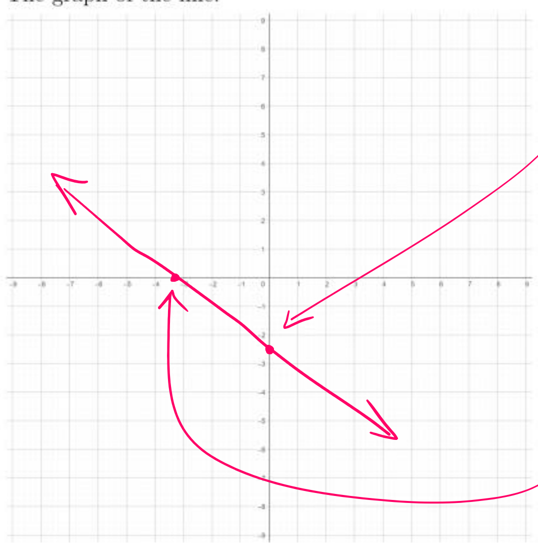
   $-\frac{\cancel{4}^{2}}{3} \cdot \frac{5}{\cancel{2}} = \left(-\frac{3}{4}x\right)\cdot -\frac{4}{3}$

   $-\frac{10}{3} = x$

   $\boxed{\text{x-intercept } \left(-\frac{10}{3}, 0\right)}$

   $(\approx -3\frac{1}{3}, 0)$

   (d) The graph of the line.



standard form ⇒ slope-intercept:

$-3y = -5x + 11$

$y = \frac{5}{3}x - \frac{11}{3}$ $\quad m = \frac{5}{3}$

2. For a point (x, y) on the line given by $5x - 3y = 11$, determine the following:

$m = \dfrac{\Delta y}{\Delta x} = \dfrac{\text{change in } y}{\text{change in } x}$ ⟷ "the x/y increases/decreases by ___ units"

need this #

   (a) The corresponding change in the y-coordinate of the point if the x-coordinate increases by 5 units.

   $m = \dfrac{\Delta y}{\Delta x}$ } need one of these

   $m = \Delta y / \Delta x$

   $\Delta x = +5$

   $5 \cdot \left(\frac{5}{3}\right) = \left(\frac{\Delta y}{\Delta x}\right) \cdot 5$

$m = \frac{\Delta y}{\Delta x}$ need one of these

$m = \Delta y / \Delta x$

$\Delta x = +5$

$5 \cdot \left(\frac{5}{3}\right) = \left(\frac{\Delta y}{5}\right) \cdot 5$

$\frac{25}{3} = \Delta y \rightarrow$ the y increases by 25/3 units

(b) The corresponding change in the x-coordinate of the point if the y-coordinate decreases by 3 units.

so $\Delta y = -3$

then $\frac{5}{3} = \frac{-3}{\Delta x}$ (one fraction = one fraction) then cross-multiply

$-\frac{1}{3} \cdot \left(\frac{5}{3}\right) = \left(-3 \cdot \frac{1}{\Delta x}\right) \cdot -\frac{1}{3}$

reciprocal $-\frac{5}{9} = \frac{1}{\Delta x}$ reciprical

$-\frac{9}{5} = \frac{\Delta x}{1}$

$-\frac{9}{5} = \Delta x$

$\frac{5 \cdot \Delta x}{5} = \frac{-9}{5}$

$\Delta x = -\frac{9}{5} \rightarrow$ the x decreases by 9/5 units

Section 2.2: Modeling with Linear Functions ← taking real things (cost/rev/...) and using $y=mx+b$ to represent them

Some Key Words and Terms: Linear Functions, Linear Depreciation, Rate of Depreciation, Scrap Value, Cost, Variable Cost, Quantity, Fixed Cost, Revenue, Price, Profit, Supply, Demand

Linear Functions:

In this class:

I. Linear Depreciation (time = x, value = y)

II. Total Production Cost (quantity = x, cost = y)
Revenue (quantity = x, revenue = y)
Profit = Revenue − Cost (quantity = x, profit = y)

III. Supply (quantity = x, price per item = y)
Demand (quantity = x, price per item = y)

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☆ All 3 are still just versions of $y=mx+b$ ☆

Linear Depreciation: $V(t) = m \cdot t + b$

- $V(t)$ is the value in \$ of the item after t years
- m is amount of \$ lost each year (always negative)
- t is time in years (x-variable)
- b is the purchase price $(0, b)$ (time is zero)

☆ "The depreciation rate" is $|m|$ dollars per year ☆

Cost, Revenue, and Profit:

Cost = (cost per item) • (quantity) + (fixed costs)

$C(x) = m \cdot x + F$ m = cost per item (cost to produce not buy)

Revenue = (selling price per item) • (quantity)

m = selling price per item & no y-intercept

Revenue = (selling price per item) • (quantity)

$R(x) = m \cdot x$ m = selling price per item & <u>no</u> y-intercept

Profit = (Revenue) − (Cost) ★ will <u>distribute</u> through the cost function

★ $C(x)$, $R(x)$, & $P(x)$ all have <u>positive</u> slopes ★

Supply and Demand:

★ both represent a <u>price</u> per item: $S(x) = p(x)$ OR $D(x) = p(x)$

supply is a price function

demand is a price function

$$p = mx + b$$

- Supply will always have a <u>positive</u> slope
- Demand will always have a <u>negative</u> slope

when $S(x) = D(x)$ "Supply equals Demand" we have "market <u>equilibrium</u>"

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Examples:

1. Before going to college, a student decides to invest in a scooter to get around campus. After one year, the scooter they purchased would still be worth \$563.45. When the student graduates (assume 4 years after starting college), the scooter they purchased would still be worth \$303.80. The scooter uses special electronics that will always be worth at least \$100. Let $V(t)$ be a linear function modeling the current value of the scooter, in dollars, after t years. Determine the following:

$y=mx+b$

$V(t)$ is depreciation, then $(x,y) \to$ (time, value)

$(1, 563.45)$

$(4, 303.8)$

   a. The yearly depreciation rate for the students' scooter.

   $\hookrightarrow |m|$

   $$m = \frac{303.8-563.45}{4-1} = -86.55$$

   the yearly depreciation rate is \$86.55 per year

   b. The purchase price of the scooter.

   $\hookrightarrow$ b-value / y-intercept

   $V(t) = mt + b$

   $V(t) = -86.55t + b$

   (t, V): $(1, 563.45)$

   $563.45 = -86.55(1) + b$

   $650 = b$

   the purchase price is \$650

   c. The linear function, $V(t)$, representing the value of the scooter after t years.

   $$\boxed{V(t) = -86.55t + 650}$$

   d. How many years it will take for the scooter to reach its scrap value? (round your answer to the nearest tenth of a year) $\to$ one decimal

   $\hookrightarrow$ lowest value/minimum value an object can be

   ⋆ if we are not given a scrap-value it is assumed to be \$0

   ⋆ for this question, scrap value is \$100

   "How long until scrap-value?"

   set $V(t)$ = scrap-value and solve for t

   $V(t) = -86.55t + 650 = 100$

   $$\frac{-86.55t}{-86.55} = \frac{-550}{-86.55}$$

   $t \approx 6.4$

   It will take about 6.4 years

2. A company produces a laptop/tablet combo targeted towards college students. When the company produces 50 combos, they incur a total cost of $18,750 and when they produce 100 combos, they incur a total cost of $25,200. Determine the linear cost function, $C(x)$, for the company when they produce x laptop/tablet combos.

$\hookrightarrow C(x) = mx + F$

★ we were not given m = cost per item or F = fixed cost, so we make two points & a line ★

$(x,y) = (\text{quantity}, \text{cost})$

$$\left.\begin{array}{l}(50, 18750)\\(100, 25200)\end{array}\right\} m = \frac{25200 - 18750}{100 - 50} = 129 \quad \text{"cost per item"}$$

then $C(x) = 129x + F$ & plug-in a point

$$25200 = 129(100) + F$$
$$25200 = 12900 + F$$
$$12300 = F$$

$$\boxed{C(x) = 129x + 12{,}300}$$

"total production cost"

3. When the company from the previous example sells 75 combos, they have a total revenue of $18,675. Determine the linear revenue function, $R(x)$, for the company when they sell x laptop/tablet combos.

★ Do not over-complicate revenue ★

Revenue = (selling price per item) • (quantity)

$R = p \cdot x$ & $(75, 18675) = (x, \text{revenue})$

$$18675 = p \cdot 75$$

$$p = m = \frac{18675}{75} = 249 \quad \text{"selling price per item"}$$

$$\boxed{R(x) = 249x}$$

★ For Revenue:

$$m = \frac{\text{revenue}}{\text{quantity}}$$

4. A company produces and sells Red-Eyes: a super-duper alarm clock for college students. The total production cost for the company when they produce x Red-Eyes is given by the function $C(x) = 17x + 1,250$. The total revenue for the company when they sell x Red-Eyes is given by the function $R(x) = 39.99x$. Determine the following:

   a. The profit function, $P(x)$, for the company when they produce and sell x Red-Eyes.

$P(x) = R(x) - C(x)$

$P(x) = 39.99x - (17x + 1250)$

$P(x) = 39.99x - 17x - 1250$

$\boxed{P(x) = 22.99x - 1250}$

★ MUST distribute the negative

   b. How many Red-Eyes the company needs to sell in order to actually turn a profit.

3 scenarios w/ Profit:

① Company losing more than making → negative profit $(P<0)$
cost > revenue

② "Break-Even" → no profit $(P=0)$
cost = revenue

③ Company making more than losing → positive profit $(P>0)$
revenue > cost

profit > 0

$R(x) > C(x)$

$39.99x > 17x + 1250$

$-17x \qquad -17x$

$22.99x > 1250$

$x > \frac{1250}{22.99} \approx 54.4$

x is a # of alarm clocks

"they must sell at least 55 alarm clocks"

5. A company produces and sells a specialized fan designed to be used in dorm rooms called the 'Wind-Mate'. When the selling price of Wind-Mates is set at \$20, the company will supply 5,000 fans. If the selling price increases to \$25, the company is willing to supply an additional 2,000 fans. Determine the linear price function for supplying x Wind-Mates.

this is a supply problem: $S(x) = ?$

↳ $(x, y) = (\text{quantity}, \text{price})$

2 pts so now we make a line
$(5000, 20)$
$(7000, 25)$
5000 + 2000 "additional"

$m = \frac{25-20}{7000-5000} = \frac{5}{2000}$

$m = \frac{1}{400}$ (supply has (+) slope) ✓

$S(x) = \frac{1}{400}x + b$ then plug-in a point

$20 = \frac{1}{400}(5000) + b$ OR $25 = \frac{1}{400}(7000 + b)$

$b = 7.5$

$\boxed{S(x) = \frac{1}{400}x + 7.5 \quad \text{OR} \quad p(x) = \frac{1}{400}x + 7.5}$

6. A T-shirt supplier is planning to capitalize on the reintroduction of the Aggie/Longhorn game this semester. According to their market research, if they set the price of t-shirts at \$30 per t-shirt, then consumers will demand 22,400 t-shirts. If they increase the price by \$9 per t-shirt, then consumers will only demand 19,900 t-shirts. Determine the linear price function for demanding x t-shirts.

$D(x) = ?$

$(x, y) = $ (quantity, price)

$\begin{cases} (22400, 30) \\ (19900, 39) \end{cases}$

$$m = \frac{39 - 30}{19900 - 22400}$$

$$m = -\frac{9}{2500} = -.0036$$ (demand has (-) slope) ✓

$$D(x) = \frac{-9}{2500}x + b$$

$$30 = \frac{-9}{2500}(22400) + b \quad \text{OR} \quad 39 = \frac{-9}{2500}(19900) + b$$

$$b = \frac{2766}{25} = 110.64$$

$$D(x) = p(x) = \frac{-9}{2500}x + \frac{2766}{25}$$

OR

$$D(x) = p(x) = -.0036x + 110.64$$

☆ it is ok to use decimals here b/c the are exact (aka unrounded)

☆ When in doubt, default to fraction, not decimal ☆

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