Example 1 (15.2)**.** *Set up double integrals with both orders of integration for* $\iint\limits_R f(x,y)\,dA$*, where* R *is the plane region bounded by* $y = x^2$*,* $x = 9$ *and* $y = 0$*.*

Example 2 (15.2)**.** *Evaluate the integral* $\int_0^1 \int_{x^2}^1 \sqrt{y}\, e^{y^2}\, dy\, dx$ *by reversing the order of integration.*

Example 3 (15.3)**.** *(a) Set up (but do NOT evaluate) a double integral using polar coordinates to compute the volume of the solid E that lies below the paraboloid $z = 1 + x^2 + y^2$ and above the region R between the circles $x^2+y^2 = 1$ and $x^2+y^2 = 4$ in the half-space $y \geq 0$.*

(b) Use polar coordinates to compute the volume of the solid E that lies below the paraboloids $z = 5 - x^2 - y^2$ and $z = 4x^2 + 4y^2$.

Example 4 (15.3). *Rewrite (but do NOT evaluate) the integrals using polar coordinates.*

(a) $\displaystyle\int_{-5}^{5}\int_{0}^{\sqrt{25-x^2}} e^{x^2+y^2}\,dy\,dx.$

(b) $\displaystyle\int_{0}^{6}\int_{0}^{\sqrt{6x-x^2}} (x^2+y^2)\,dy\,dx$

Example 5 (15.6). *Evaluate* $\iiint\limits_E 6x\,dV$, *where* E *lies under the plane* $x+y-z+2=0$ *and above the region in the* xy*-plane bounded by the curves* $y=\sqrt{x}$, $y=0$ *and* $x=4$.

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Example 6 (15.6). *Set up the triple integral for the volume of the solid E in the order $dz\,dx\,dy$, where E is the solid bounded by $y = x^2$, $z = 0$, and $y + z = 4$.*

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Example 7 (15.6). *Set up the triple integral in the order* $\underline{dx\,dz\,dy}$ *for* $\iiint\limits_E z\,dV$, *where* E *is the solid bounded by* $y^2 + z^2 = 4$, $x = 0$, $y = 2x$ *and* $z = 0$ *in the first octant.*

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Example 8 (15.7). *Set up an integral using cylindrical coordinates to find the volume of the solid E that lies above the paraboloid $z = x^2 + y^2$ and below the plane $z = 4$.*

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Example 9 (15.7). *Use cylindrical coordinates to set up a triple integral for the volume of the solid E*

(a) that lies between the paraboloids $z = 8 - x^2 - y^2$ *and* $z = 3x^2 + 3y^2$.

(b) that lies within both the cylinder $x^2 + y^2 = 1$ *and the sphere* $x^2 + y^2 + z^2 = 4$.

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(c) that lies below the paraboloid $z = 24 - x^2 - y^2$ *and above the cone* $z = 2\sqrt{x^2 + y^2}$.

Example 10 (15.8)**.** *Evaluate* $\displaystyle\iiint_E \frac{x}{1+(x^2+y^2+z^2)^2}\,dV$*, where* E *is the solid between the spheres* $x^2+y^2+z^2=1$ *and* $x^2+y^2+z^2=4$*.*

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Example 11 (15.8)**.** *Use spherical coordinates to set up a triple integral for the volume of the solid E that lies below the sphere $x^2 + y^2 + z^2 = 5$ and above the cone $z = 2\sqrt{x^2 + y^2}$.*

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Example 12 (15.8). *Convert (but do NOT evaluate) the integral*

$$\int_{-3}^{3}\int_{0}^{\sqrt{9-y^2}}\int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,dz\,dx\,dy$$

into an iterated integral with spherical coordinates.

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Example 13 (15.8)**.** *Consider the density function* $\rho(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ *on the solid* $E:\ x^2 + y^2 + z^2 \leq 4,\ z \geq 0$*. Find the mass.*

Example 14 (15.9)**.** *Use the transformation* $u = x - y$ *and* $v = x + y$ *to evaluate* $\displaystyle\iint_R \frac{x-y}{x+y}\, dA$, *where* R *is the trapezoidal region with vertices* $(1, 0), (3, 0), (0, 3),$ *and* $(0, 1)$.

Example 15 (15.9)**.** *Evaluate the integral* $\iint_R (x-2y)\,e^{3x-y}\,dA$ *by using an appropriate change of variables, where* R *is the region bounded by the lines*

$$x - 2y = 0,\ x - 2y = 4,\ 3x - y = 1,\ \text{and } 3x - y = 8.$$

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