Math 308: Week-in-Review 3

1. Determine (without solving the problem) an interval in which the solution of the following initial value problem is certain to exist.

   (a)
   $$y' + (\sec x)y = x^2, \quad y(0) = 5$$

   (b)
   $$y' + \frac{t}{t^2 - 9}y = \frac{1}{t}, \quad y(-1) = 2$$

   (c)
   $$\sin(t)g' + 2tg = \ln(2 + t), \quad g(\pi/2) = 3.$$

2. (a) Consider the differential equation

$$y' = (2t + y)^{\frac{2}{3}}.$$

If the initial condition is $y(0) = 1$, does the IVP have a unique solution? What if the initial condition is $y(1) = -2$?

(b) Consider the initial value problem $y' = \sin(2t)y^{\frac{1}{3}}$, $y(0) = 0$. One solution is $y(t) = 0$. Find two other solutions to the initial value problem. Why does the Existence and Uniqueness Theorem not apply to this case?

3. Solve the following initial value problems and determine how the interval in which the solution exists depends on y_0.

   (a) $y' = y^2, \quad y(0) = y_0$

   (b) $y' = -\dfrac{4t}{y}, \quad y(0) = y_0$

4. Determine if the following equations are autonomous or not.

(a) $f''(x) - 3f(x)f'(x) + 4 = 0$

(b) $\dfrac{q''(x)}{x^2+1} - q(x)^{3/2} = 4\cos(x)$

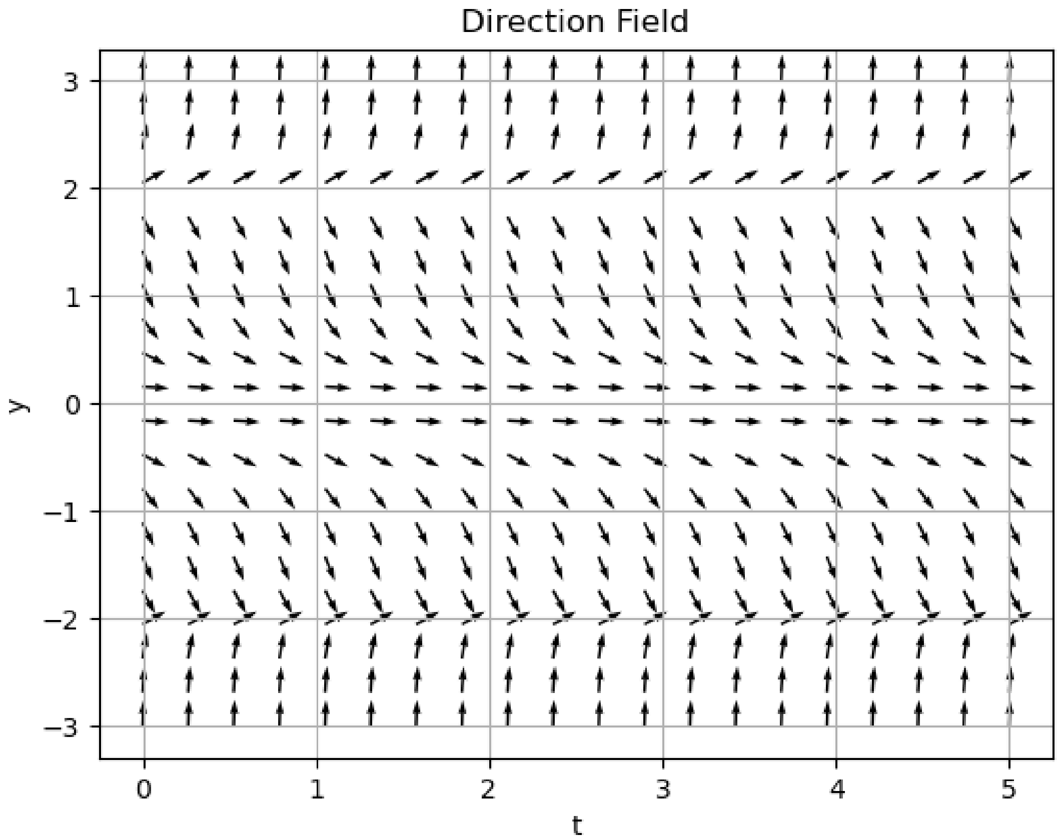
(c) $y'' + y' + y = 0$

(d) $\dfrac{g''}{g^2} + g = \sqrt{g}$

(e) $\dfrac{d^2y}{dx^2} + 3(x^2-1)y - x = 5\sin(2x)$

(f) $\sin(u^3) + \dfrac{d^3u}{dx^3} = 0$

5. Given the following slopefield, determine the equilibrium solutions and their stability. Also, draw the phaseline diagram.



6. Given the differential equation

$$y' = (1+y)(y-2)^2$$

   (a) Find the equilibrium solutions.
   (b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, or semistable
   (c) Graph some solutions
   (d) If $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$ for some $y_0 \in (-\infty, \infty)$, find the limit of $y(t)$ as $t \to \infty$

7. Suppose that the population of rabbits obeys the logistic equation

$$\frac{dP}{dt} = 0.1P\left(1 - \frac{P}{1000}\right).$$

Initially there are $P(0) = 100$ rabbits.

(a) Solve the differential equation to find the population $P(t)$ as a function of time.

(b) Find the time it takes for the population to reach 90% of the carrying capacity.

8. (a) Show that the equation is exact and find the general solution

$$(2x \sin y + y^3 e^x) + (x^2 \cos y + 3y^2 e^x)\frac{dy}{dx} = 0.$$

(b) Show that the equation is exact and find the solution to the initial value problem

$$(2t \cos y + 3t^2 y) + (t^3 - t^2 \sin y - y)\frac{dy}{dt} = 0, \quad y(0) = 2.$$