Week in Review #8

Tuesday, March 25, 2025 7:25 PM

Section 5.2 Part a: Polynomial Functions

- General Notation of a Polynomial
  - Degree -> highest power
  - Leading Coefficient -> coefficient for highest power
  - Constant Term -> coefficient of x^0
- End Behavior
- Domain
- Intercepts
- Parent Polynomial Functions
  - Zero $f(x) = 0$
  - Constant $f(x) = b$, where $b \neq 0$
  - Linear $f(x) = x$
  - Quadratic $f(x) = x^2$
  - Cubic $f(x) = x^3$

$5x^3 - \pi x^2 + x^1$

Pr 1. Determine if the given function is a polynomial function. If the answer is yes, state the degree, leading coefficient, and constant term.

(a) $f(x) = -42x^{-1} + 3x^{\pi} - 6x^{3.1}$ -> not a positive integer

Not a polynomial.

(b) $g(w) = \sqrt{3}w^2 - w^3 + \frac{1}{7}w - 21 \cdot w^0$

$-w^3$

This is a polynomial 2, 3, 1, 0

degree = 3

leading coefficient = −1

$g(w) = -w^3 + \sqrt{3}w^2 + \frac{1}{7}w - 21$

constant term = −21

Pr 2. Describe the end behavior of each polynomial function, both symbolically and with a quick sketch of the end behavior.

(a) $f(x) = -x^4 + x^3 - 6x - 2048$

as $x \to \infty$, $f(x) \to -\infty$

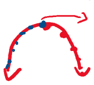
as $x \to -\infty$, $f(x) \to -\infty$

Shortcute

highest degree term

$-1x^4 \to -x^2$

$-x^2$



$+x^{2k} \to x^2$

$-x^{2k} \to -x^2$

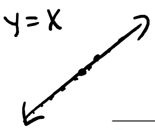
(b) $g(x) = 12x^4 - 9 + 9x^7 - x^2$

$9x^7$ $9 > 0$

odd ~> $1 \cdot x$

$g(x)$ and $y = x$ have same behavior

$y = x$

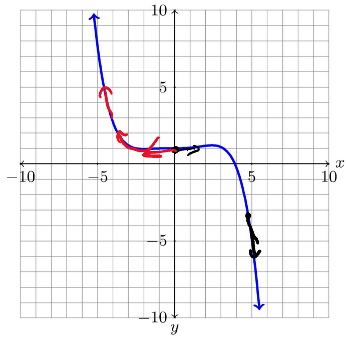


as $x \to \infty$, $g(x) \to \infty$

as $x \to -\infty$, $g(x) \to -\infty$

checks: if degree is even then answers should match

Pr 3. Describe the end behavior symbolically for the polynomial function, $f(x)$, graphed below.



as $x \to \infty$, $f(x) \to -\infty$

as $x \to -\infty$, $f(x) \to \infty$

Pr 4. State the domain of each polynomial function.

domain of a polynomial is $(-\infty, \infty)$

(a) $f(x) = 2x^{13} - 6x^2 - 40x$

$(-\infty, \infty)$

(b) $g(w) = 15w^2 - w^3 + 5w - 12$

$(-\infty, \infty)$

or roots

Pr 5. Determine all exact real zeros, the x-intercept(s), and y-intercept of each given polynomial function, if possible.

(a) $f(x) = -5(2-3x)(4x+9)$

$-5(2-3x)(4x+9)=0$

$ab=0$
$a=0$ or $b=0$

$-5=0$ or $2-3x=0$ or $4x+9=0$

not possible

$2-3x=0$
$+3x \quad +3x$
$\frac{3x}{3}=\frac{2}{3}$
$\boxed{x=2/3}$

$4x+9=0$
$-9 \quad -9$
$\frac{4x}{4}=\frac{-9}{4}$
$\boxed{x=-9/4}$

$\boxed{\begin{array}{l}\text{x-intercepts:}\\ (2/3,0)\\ (-9/4,0)\end{array}}$

$\boxed{\text{y-intercept: } (0, f(0)) = (0,-90)}$

$f(0) = -5\cdot(2-\cancel{3\cdot 0})(\cancel{4\cdot 0}+9)$

$f(0)=-5\cdot 2\cdot 9$
$=-90$

(b) $g(x) = 6x^3 - 3x^2 - 18x = 3x(2x+3)(x-2)$

constant term: 0

y-intercept : $(0, f(0)) = (0,0)$

x-intercepts: $(0,0)$
$(2,0)$
$(-3/2,0)$

real zeros:
$3x(2x+3)(x-2)=0$

$3x=0$ $\quad$ $2x+3=0$ $\quad$ $x-2=0$
$\boxed{x=0}$ $\quad$ $-3 \quad -3$ $\quad$ $+2 \quad +2$
$\frac{2x}{2}=\frac{-3}{2}$ $\quad$ $\boxed{x=2}$
$\boxed{x=-3/2}$

(c) $h(w) = 5w^2 - w^3 + 4w - 20$ $= -w^3+5w^2+4w-20$

constant term = -20

$x^2-y^2=(x+y)(x-y)$

$\boxed{\text{y-intercept} = (0,-20)}$

$-w^3+5w^2+4w-20$ $\quad$ $w^3=w\cdot w\cdot w$
$=w^2\cdot w$

$w^2(-w+5)+4(w-5)$
$=-w^2(w-5)+4(w-5)$
$=(-w^2+4)(w-5)$
$=-(w^2-4)(w-5)$
$=-(w-2)(w+2)(w-5)=0$

$w-2=0$ $\quad$ $w+2=0$ $\quad$ $w-5=0$
$\boxed{w=2 \quad w=-2 \quad w=5}$

$\boxed{\text{x-intercepts: } (2,0),\ (-2,0),\ (5,0)}$

(d) $k(x) = (x^2+9)(x^2-4)$

$x^2+9=(x+3i)(x-3i)$ not in this course

$x^2+9=0 \rightarrow x^2=-9$
$x=\sqrt{-9}$

$x^2+9=0$
real
no solutions

$x^2-4=0$
$+4 \quad +4$
$x^2=4$
$x=\pm\sqrt{4}$
$=\pm 2$

$\boxed{\text{real zeros } x=-2,\ x=2}$

$\boxed{\begin{array}{l}\text{x-intercepts:}\\ (-2,0),\ (2,0)\end{array}}$

$\boxed{\begin{array}{l}\text{y-intercept:}\\ k(0)=(0^2+9)(0^2-4)\\ =(9)(-4)\\ =-36\\ (0,-36)\end{array}}$

Pr 6. Match each of the following polynomials with the graph of its parent function:

(a) $f(x) = 1 - x^3 + 13x^{1}$

replace $f(x)$ with x^{degree}

parent function : $p(x) = x^3$

graph D

(b) $g(x) = 2028$ $p(x) = x^0 = 1$

degree 0

$x^1 + 5$

graph A

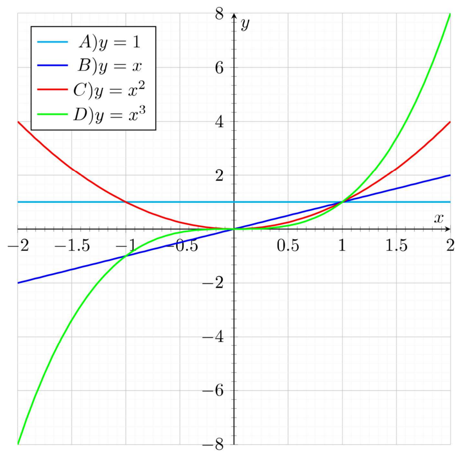
(c) $h(x) = 32x^{1} - 2027$ $p(x) = x$

degree 1

graph B

(d) $k(x) = 3x^2 - 2x + 1$ $p(x) = x^2$

graph C

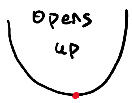


SECTION 5.2 PART B: QUADRATIC FUNCTIONS

- General form of a Quadratic Function - $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers with $a \neq 0$
  - Vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
  - Axis of Symmetry $\boxed{x = -\frac{b}{2a}}$
  - Domain and Range
- Quadratic Formula - used to solve equations of the form $ax^2 + bx + c = 0 \rightarrow \boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$
- Recall: Profit = Revenue - Cost

Pr 1. Determine the vertex, axis of symmetry, domain, range, x-intercept(s), y-intercept, maximum value and minimum value for each quadratic function, if they exist.

(a) $f(x) = 2x^2 + 6x + 0$



k is the minimum value

domain: $(-\infty, \infty)$

axis of symmetry:

$a = 2$, $b = 6$, $c = 0$

$x = -\frac{b}{2a} = -\frac{\cancel{2}}{\cancel{2}\cdot 6} = \frac{-1}{6}$

$\boxed{x = -\frac{1}{6}}$

$\boxed{y\text{-intercept: } (0, 0)}$

$\boxed{x\text{-intercepts: } (0,0) \quad (-3, 0)}$

$2x^2 + 6x = 0$

$2x(x+3) = 0$

$2x = 0 \quad x + 3 = 0$

$x = 0 \quad x = -3$

vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$= \boxed{\left(-1/6, -17/18\right)}$

$f\left(-\frac{b}{2a}\right) = f\left(-\frac{1}{6}\right)$

$= 2\cdot\left(-\frac{1}{6}\right)^2 + 6\cdot\left(-\frac{1}{6}\right)$

$= 2\cdot\frac{1}{6^2} + (-1)$

$= \frac{2}{36} - 1$

$= \frac{1}{18} - 1 = \frac{1}{18} - \frac{18}{18}$

$= -\frac{17}{18}$

minimum value: $-17/18$

Maximum value: DNE

Range: $\left[-\frac{17}{18}, \infty\right)$ ← $[k, \infty)$ when leading coef > 0

(b) $g(x) = 3x^2 - 6x + 3$

lead. coef > 0

$3x^2 - 6x + 3$

domain: $(-\infty, \infty)$

axis of symmetry:

$x = -\frac{b}{2a} = -\frac{(-6)}{2\cdot 3} = \frac{6}{6} = 1$

$x = 1$

$3x^2 - 6x + 3$

$\boxed{y\text{-intercept: } (0,3)}$

$\boxed{x\text{-intercept: } (1,0)}$

$3(x^2 - 2x + 1) = 0$

$3(x-1)(x-1) = 0$

$x - 1 = 0 \rightarrow x = 1$

vertex: $k = g\left(-\frac{b}{2a}\right) = g(1) = 3\cdot(1)^2 - 6(1) + 3$

$= 3 - 6 + 3 = 0$

$\boxed{\text{vertex: } (1, 0)}$ → x-intercept.

- No Max value
- Minimum value is 0

Range: $[0, \infty)$

(c) $h(x) = 36 - 49x^2$ $= 6^2 - (7x)^2 = (6-7x)(6+7x)$

$h(x) = -49x^2 + 0x + 36$

Negative leading coef.

$\boxed{\text{domain: } (-\infty, \infty)}$

$\boxed{\text{axis of symmetry: } x = \frac{-b}{2a} = \frac{-0}{2(-49)} = 0}$

$\boxed{x=0}$

vertex: $h(0) = 36$

$\boxed{\text{vertex: } (0, 36)} \rightarrow \boxed{\text{y-intercept: } (0, 36)}$

No minimum value

Maximum value is 36

Range : $(-\infty, 36] \rightarrow (-\infty, k]$

$\boxed{\text{y-intercept: } (0, 36)}$

x-intercept:

$-49x^2 + 36 = 0$
$\quad -36 \quad -36$

$-49x^2 = -36$

$\frac{-49x^2}{-49} = \frac{-36}{-49}$

$x^2 = \frac{36}{49} \rightarrow x = \pm\sqrt{\frac{36}{49}}$

$= \pm \frac{6}{7}$

$\boxed{\left(-\frac{6}{7}, 0\right), \left(\frac{6}{7}, 0\right)}$

x-intercepts:

(d) $j(x) = \frac{1}{5}x^2 + \frac{49}{500}x - \frac{31}{100}$

why?

$a = \frac{1}{5}$

$b = \frac{49}{500}$

domain: $(-\infty, \infty)$ $\rightarrow$ y-intercept: $\left(0, \frac{-31}{100}\right)$

$\rightarrow$ x-intercept?

axis of symmetry:

$x = \frac{-b}{2a} = \frac{-\left(\frac{49}{500}\right)}{2 \cdot \left(\frac{1}{5}\right)} = \frac{\left(\frac{-49}{500}\right)}{\left(\frac{2}{5}\right)} = \frac{-49}{500} \times \frac{5}{2}$

$= \frac{-49}{100} \times \frac{1}{2}$

$= \frac{-49}{200}$

$\boxed{x = -\frac{49}{200}}$

vertex: $j\left(\frac{-49}{200}\right) = \frac{1}{5}\left(\frac{-49}{200}\right)^2 + \frac{49}{500}\left(\frac{-49}{200}\right) - \frac{31}{100}$

$= \frac{-64401}{200000}$

x-intercept:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{\frac{-49}{500} \pm \sqrt{\left(\frac{-49}{500}\right)^2 - 4\left(\frac{1}{5}\right)\left(\frac{-31}{100}\right)}}{2\left(\frac{1}{5}\right)}$

$\left(-\frac{49}{200}, \frac{-64401}{200000}\right)$

$\boxed{x \approx 1.023, -1.513}$

No maximum

minimum value: $-\frac{64401}{200000}$

$= \frac{-49}{}$

Range: $\left[\frac{-64401}{200000}, \infty\right)$

Pr 2. Graph the quadratic function with the following properties

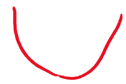
i. As $x \to -\infty$, $h(x) \to \infty$ and as $x \to \infty$, $h(x) \to \infty$

ii. $h(x)$ has a single real zero of -3.

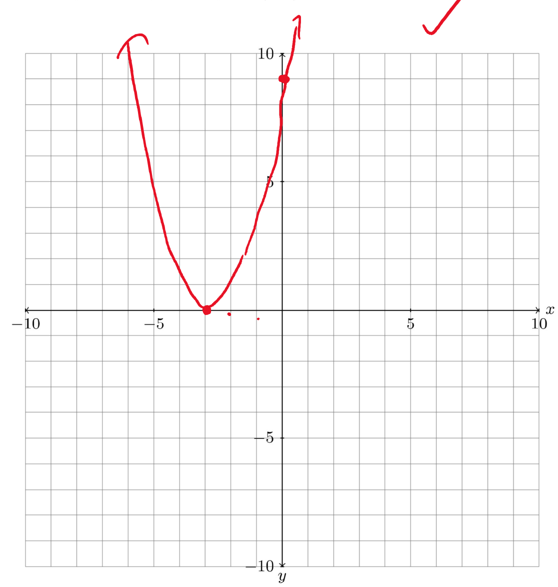
iii. There is a minimum value of 0.

iv. The graph has a y-intercept of $(0, 9)$.

$x = -3$



leading coef > 0

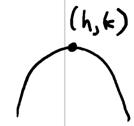


$R(x) = \text{price} \cdot \text{quantity}$ $\qquad$ $P(x) = R(x) - C(x)$

Pr 3. The cost to produce bottled mineral water is given by $C(x) = 18x + 7500$, where x is the number of thousands of bottles produced. The profit from the sale of these bottles is given by the function $P(x) = -x^2 + 300x - 7500$.

(a) What is the revenue function, $R(x)$, in dollars, where x is the number of bottles of mineral water made and sold.

(b) How many bottles must be sold to maximize the revenue?



$$-x^2 + 300x - 7500 = R(x) - (18x + 7500)$$

cancel

$$= R(x) - 18x - 7500$$

$$+18x + 7500 \qquad\qquad + 18x + 7500$$

a) $\boxed{R(x) = -x^2 + 318x} = R(x)$

@ (h,k), k is the maximum revenue

h is the # of bottles

$h = -\frac{b}{2a}$ $\qquad$ $b = 318$, $a = -1$

$$= -\frac{318}{2(-1)} = \frac{318}{2} = \boxed{159 \text{ thousand bottles}}$$

(c) What is the maximum revenue?

$$k = R\left(-\frac{b}{2a}\right) = -(159)^2 + 318 \times 159$$

$$= 159(-159 + 318)$$

$$= \$25,281.$$

Funko Pop

Pr 4. The fixed cost of manufacturing collectible ~~bobble head~~ figurines is \$350, while the production cost is \$30. If we sell figurines at \$350, then none are demanded, while 300 are demanded if we give the figurines away for free.

(a) Determine the company's profit function $P(x)$, in dollars, as a function of x, the number of figurines made and sold.

$P(x) = R(x) - C(x)$

$R(x) = \text{price} \cdot x$

$p(x) = -\frac{7}{6}x + 350$

quantity

$C(x) = 30x + 350$

price - 350 ?

price = p(x) ← demand function

$p(0) = 350$

$p(300) = 0$

$p(x) = mx + b$

$p(0) = b = 350$

$m = \frac{350-0}{0-300} = -\frac{350}{300} = -\frac{35}{30} = -\frac{7}{6}$

$R(x) = \left(-\frac{7}{6}x + 350\right) \cdot x$

$\boxed{R(x) = -\frac{7}{6}x^2 + 350x}$

$\boxed{P(x)} = R(x) - C(x)$

$= -\frac{7}{6}x^2 + 350x - 30x - 350$

$= \boxed{-\frac{7}{6}x^2 + 320x - 350}$

(b) How many figurines must be sold in order to maximize profit?

vertex $h = -\frac{b}{2a}$

$h = \frac{-320}{2\left(-\frac{7}{6}\right)} = \frac{-320}{-2\left(\frac{7}{6}\right)} = \frac{-320}{-7/3}$

$\frac{2 \cdot 7}{1 \cdot 6} = \frac{14}{6} = \frac{7}{3}$

$-320 \cdot \frac{-3}{7} =$ $\boxed{\frac{960}{7} \text{ figurines}}$

(c) At what $\boxed{\text{price per figurine}}$ will the maximum profit be achieved?

want : $p(h)$ demand function

$p\left(\frac{960}{7}\right) = -\frac{7}{6}\left(\frac{960}{7}\right) + 350$

$= -\frac{1}{6} \cdot 960 + 350 = -160 + 350$

$\boxed{= \$190 \text{ per figurine}}$

(d) How many bobble-heads need to be sold in order to break-even?

where $R(x) = C(x)$ (or $P(x) = 0$)

only want x-coordinate

break-even point $(x, R(x))$

solving $P(x) = 0$

$-\frac{7}{6}x^2 + 320x - 350 = 0$

$x = \frac{-320 \pm \sqrt{320^2 - 4\left(-\frac{7}{6}\right)\cdot(-350)}}{2\left(-\frac{7}{6}\right)}$

$= \frac{-320 \pm \sqrt{100766 + 2/3}}{2\left(-\frac{7}{6}\right)}$

prefer this

this situation