

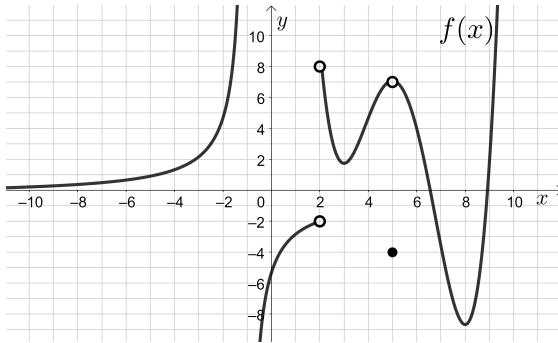


Section 3.4

- **Definition of Absolute Extrema:**
 - $f(c)$ is an **absolute maximum** of f if $f(c) \geq f(x)$ for all x in the domain of f .
 - $f(c)$ is an **absolute minimum** of f if $f(c) \leq f(x)$ for all x in the domain of f .
 - $f(c)$ is an **absolute extremum** of f if $f(c)$ is an absolute maximum or absolute minimum.
- **Extreme Value Theorem:** If a function is continuous on a closed interval $[a, b]$, then it must have both an absolute maximum and absolute minimum on that interval.
- **The Closed Interval Method:** This method is used to find the absolute extrema of a continuous function on a closed interval, $[a, b]$.
 - Find the critical values on the interval (a, b) .
 - Evaluate f at the endpoints, a and b , and at the critical values that are on the interval.
 - The largest value obtained from the previous step is the absolute maximum of $f(x)$ on $[a, b]$ and the smallest value obtained is the absolute minimum of $f(x)$ on $[a, b]$.
- **The First Derivative Test for Absolute Extrema:** This method is used to find the absolute extrema of a continuous function, $f(x)$, on an interval (open, closed, half-open) that has only **ONE** critical value, $x = c$.
 - If $f'(x)$ changes from positive to negative at $x = c$, then $f(c)$ is an **absolute maximum** of $f(x)$.
 - If $f'(x)$ changes from negative to positive at $x = c$, then $f(c)$ is an **absolute minimum** of $f(x)$.
- **The Second Derivative Test for Absolute Extrema:** This method is used to find the absolute extrema of a continuous function, $f(x)$, on an interval (open, closed, half-open) that has only **ONE** critical value, $x = c$, for which $f'(c) = 0$.
 - If $f''(c) < 0$, then $f(c)$ is an **absolute maximum** of $f(x)$.
 - If $f''(c) > 0$, then $f(c)$ is an **absolute minimum** of $f(x)$.
- Note: For the **First Derivative Test for Absolute Extrema** and the **Second Derivative Test for Absolute Extrema**, if you are dealing with an **open** interval and you make a conclusion about the absolute extrema at the critical value, you can conclude that the other absolute extrema does not exist. For instance, if you are dealing with an open interval and conclude that there is an absolute minimum at $x = c$ (the only critical value on the interval), you can conclude that that function does not have an absolute maximum on the interval. The same idea holds in the opposite direction.



1. Find the absolute extrema of the function graphed below on each of the following intervals.



(a) $(-\infty, \infty)$

(c) $(2, 5)$

(d) $[0, 6)$

(e) $[6, 9]$

(b) $[-5, -2]$

2. Find the absolute extrema of $f(x) = \frac{3}{4}x^4 + \frac{19}{3}x^3 - 7x^2 + 30$ on each of the intervals given below:

(a) $[-5, 1]$



(b) $[3, 4]$

(c) $(-\infty, -6)$

(d) $(5, 8)$

3. Find the absolute extrema of $g(x) = \frac{1}{3}x - \sqrt{x+2}$ on each of the intervals given below:

(a) $[-1, 1]$



(b) $(0, 2)$

Section 3.5

Strategy for Solving Optimization Problems

1. Translate the word problem into mathematical form.
 2. State the objective function in terms of one variable.
 3. Find the interval on which the objective function must be optimized.
 4. Use Calculus (the Closed Interval Method, First Derivative Test, or Second Derivative Test from Section 3.4) to find the optimal solution.
 5. Answer the original question.
4. Find two positive numbers x and y whose product is 250 and for which $4x + 10y$ is minimized.



5. A company that makes pencil sharpeners has found that it can sell 400 pencil sharpeners each month when the price of each pencil sharpener is \$50. For every one dollar decrease in price, the company can sell 8 additional pencil sharpeners each month.

- (a) Find the price that the company must charge per pencil sharpener to maximize its revenue.
- (b) What is the maximum revenue?

6. You are building a right-angled triangular garden along a river. The fencing of the left border costs \$4 per foot, while the fencing of the lower border costs \$1 per foot. (No fencing is required along the river.) You want to spend \$200 on the fence. What are the dimensions of the garden that will maximize the amount of area enclosed?



7. A closed rectangular box of volume 324 cubic inches is to be made with a square base. If the material for the bottom costs \$4 per square inch and the material for the sides and top costs \$2 per square inch, find the dimensions of the box that minimize the cost of materials.