1 Math 251/221 WEEK in REVIEW 5. Fall 2024

1.1 Sections 14.5 - 14.8.

1. If $z = \dfrac{y}{y + x^2}$, $x = \sqrt{t}$ and $y = \ln(t)$, find $\dfrac{dz}{dt}$.

2. Let
$$w = \cos xy + y \cos x,$$
where
$$x = e^{-t} + 3s,\ y = 5e^{2t} - \sqrt{s}$$
Find $\dfrac{\partial w}{\partial t}$ and $\dfrac{\partial w}{\partial s}$.

3. If
$$yz^4 + xz^3 = e^{xyz}$$
find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

4. Let
$$f(x, y, z) = x^2 y + x\sqrt{1 + z}$$
   (a) Find $\nabla f(1, 2, 3)$, the gradient of the function at $(1, 2, 3)$.
   (b) Find $D_{\mathbf{v}} f(1, 2, 3)$, the directional derivative of f at $(1, 2, 3)$ in the direction of $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

5. (a) Find the direction in which $f(x, y, z) = ze^{xy}$ increases most rapidly at the point $(0, 1, 2)$.
   (b) What is the maximum rate of increase?
   (c) What is the largest rate of decrease of f at this point? In which direction does this change occur?
   (d) When is the directional derivative at this point is half of its maximum value?

6. (a) Find parametric equations of the normal line and an equation of the tangent plane to the surface
$$x^3 + y^3 + z^3 = 5xyz$$
at the point $(2, 1, 1)$.

7. Find the local maximum and local minimum values, and saddle points if any, of the function $f(x, y) = x^2 - y^2 + xy$

8. Find the absolute maximum and minimum values of $f(x, y) = 3x^2 y - x^3 - y^4$ on the closed triangular region in the xy -plane with the vertices $(0, 0)$, $(1, 1)$, and $(1, 0)$.

9. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = xy$ subject to the constraint $9x^2 + y^2 = 4$.

1.2 Review for Exam 2.

1. Find the domain of the function
$$f(x, y) = \sqrt{x^2 + y^2 - 1} + \ln(4 - x^2 - y^2)$$

2. Identify level curves for the function $z = x^2 - y^2$.

3. Find f_{xyz} if $f(x, y, z) = x \sin(yz)$.

4. The dimensions of a closed rectangular box are 80 cm, 60 cm, and 50 cm with a possible error of 0.2 cm in each dimension. Use differential to estimate the maximum error in surface area of the box.

5. Approximate the number $\sqrt{8.94 + (9.99)^2 - (1.01)^3}$