Math 152 - Exam 3 Review

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Review of Maclaurin series.

1. $\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n$
2. $e^x = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$
3. $\cos(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$
4. $\sin(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$
5. $\arctan(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$
6. $\ln(1+x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$

Evaluate the following integral as Power Series.

1. $f(x) = \displaystyle\int 5x^2 \arctan(7x^3)\, dx$

2. Find a Power series representation of the functions $f(x) = \dfrac{x^2}{(5-3x)^2}$.

3. If $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(3x)^n}{n!}$, find the power series for $f'(x)$ and $\displaystyle\int f(x)dx$. Identify $f(x)$.

4. Find the 25^{th} derivative for the function $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n(n+2)} x^n$ centered at $x = 0$.

Find the Taylor Series Representions for the following functions

5. $f(x) = xe^{3x}$ centered at $x = 5$

6. $f(x) = \ln(1 + x)$ centered at $a = 2$

Find the Maclaurin Series Representation for the following functions.

7. $f(x) = \int_0^x e^{-t^2}\, dt.$

8. $f(x) = x^3 \cos(2x)$

Find the sum of the following series.

9. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (3)^n (\pi^n)}{n!}$

10. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1}}{(2n+1)!}$

11. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1}}{(2n)!}$

12. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1}}{2^{2n+1}(2n)!}$

13. Find the third degree Taylor polynomial for $f(x) = \sqrt{x}$, centered at $x = 4$.

14. Find the second degree Taylor polynomial for $f(x) = \arctan(x)$, centered at $x = 1$.

15. Find the radius of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n(x-5)^n}{n!}$

16. Given that the radius of convergence for the series $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(x-3)^n}{2^n n^4}$ is 2, find the interval of convergence.

17. If the power series $\displaystyle\sum_{n=0}^{\infty} C_n(x-2)^n$ has a radius of convergence of 5. which of the following series will also converge?

   (a) $\displaystyle\sum_{n=0}^{\infty} C_n 7^n$

   (b) $\displaystyle\sum_{n=0}^{\infty} C_n 5^n$

   (c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n C_n 4^n$

18. Which of the following series diverge?

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2 - 2n - 1}{n^3 + 4n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2 + 2n + 4}$

(c) $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

19. Which of these series converge absolutely?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n e^n}{e^{n+1}}$

(c) $\displaystyle\sum_{n=1}^{\infty} (\sqrt{n+2} - \sqrt{n})$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

20. How many terms would be needed to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^5}$ to within 2×10^{-9}?

21. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^5}$ correct to 3 decimal places.

22. Using the 5^{th} partial sum to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{(n+3)!}$, find the upper bound for the error in the estimate of the sum of the series.

