



2.3: MODELING WITH FIRST ORDER ODES

Review

- If f is **proportional to** g , then it means

Exercise 1

Suppose we initially have 3 rabbits. After 2 years, we have 14 rabbits. Assuming the population growth of the rabbits is proportional to the number of rabbits, how many rabbits will we have in 3 more years?



Exercise 2

Suppose we invest \$100 per month in a savings account that makes 5% interest compounded continuously. Initially, we have nothing in the savings account. How much money will be in the account after 8 years?



Exercise 3

Suppose we have a 200 L tank filled with water. We start pouring sugar water into the tank at a rate of 3 L/min. The sugar water contains 5 g/L of sugar. At the same time, the well-mixed fluid flows out of the tank at a rate of 3 L/min. How much sugar is in the tank after 1 hr?



Exercise 4

Suppose we leave a bucket of ice cream out on the counter in a 70°F room. The ice cream was initially 15°F . However, after 1 minute of sitting there, its temperature is 20°F . How long until the ice cream starts to melt? (Assume that the ice cream obeys Newton's law of cooling: The rate at which the temperature changes is proportional to the temperature difference of the object and its surroundings.)



2.4: EXISTENCE AND UNIQUENESS OF SOLUTIONS

Review

- A solution **exists** if

- A solution is **unique** if

- **Theorem for linear ODEs:** If _____ on an interval $I = (a, b)$ containing the initial condition t_0 , then the initial value problem

$$y' + p(t)y = g(t), \quad y(t_0) = y_0$$

has a unique solution on I .

- **Theorem for nonlinear ODEs:** Let the functions f and $\frac{\partial f}{\partial y}$ be continuous in some rectangle $(a, b) \times (c, d)$ containing the point (t_0, y_0) . Then, there is a unique solution to the initial value problem

$$y' = f(t, y), \quad y(t_0) = y_0$$

on a sufficiently small interval $I_h = (t_0 - h, t_0 + h)$ around t_0 .

Exercise 5

Without solving the IVP, determine where a unique solution is guaranteed to exist.

$$y' - \tan(t)y = \sqrt{t+1}, \quad y(0) = 2\pi.$$

Exercise 6

Without solving the IVP, determine where a unique solution is guaranteed to exist.

$$\cos(x)f' - 4x^2f = \ln(1+x), \quad f(\pi) = 7.$$

Exercise 7

Consider the differential equation

$$y' = (x + y)^{1/3}$$

If the initial condition is $y(2) = -2$, does the IVP have a unique solution? What if the initial condition is $y(1) = 2$?



Exercise 8

Consider the initial value problem

$$y' = \frac{x}{2y - 1}, \quad y(0) = 1.$$

Why does the existence and uniqueness theorem not apply to this IVP? Show that this IVP has more than one solution.



2.5: AUTONOMOUS EQUATIONS

Review

- A differential equation is **autonomous** if
- An **equilibrium solution** to a differential equation is
- The **stability** of an equilibrium solution can be any of the following:
 - **Unstable**
 - **Semi-stable**
 - **(Asymptotically) stable**

Exercise 9

Are the following autonomous or not?

1. $y'' + y' + y = 7$

2. $x^2 - y' = y^4$

3. $f'(x) - 3f(x) - 12 = 0$

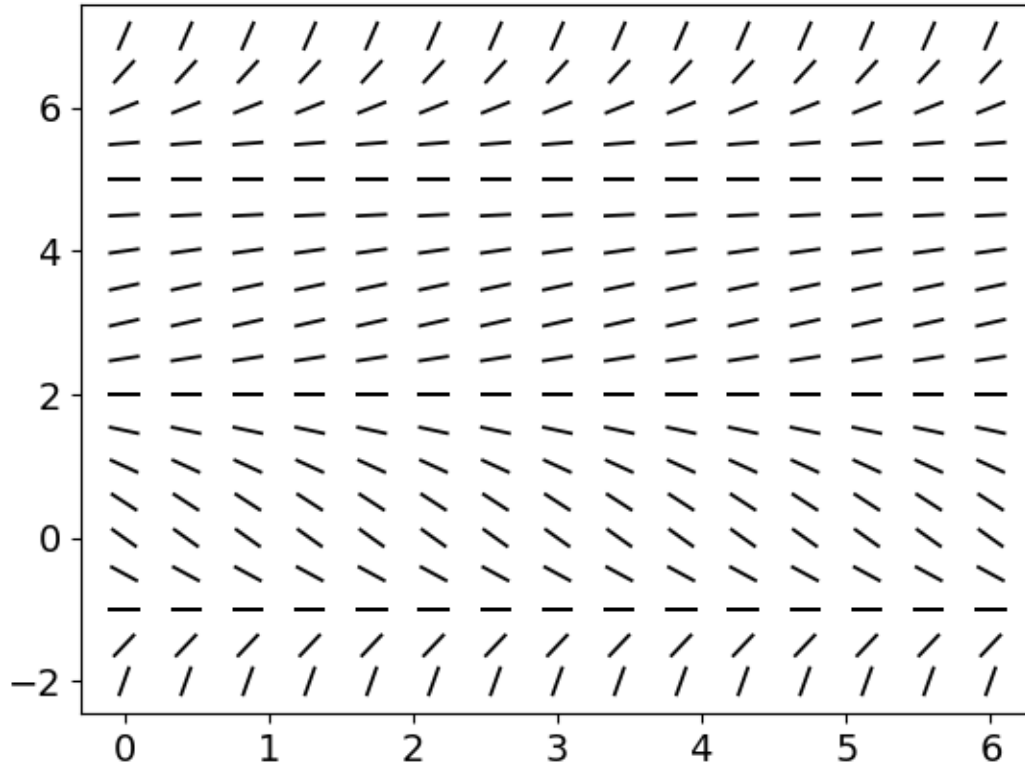
4. $\frac{g''}{g'} + g = \sqrt{g}$

5. $\frac{w''(x)}{x^2 + 1} - (w(x))^{3/2} = 6 \sin(x)$

6. $\cos(u^2) + \frac{du}{dx} = u$

Exercise 10

Given the following slope field, determine the equilibrium solutions and their stability. Also, draw the phase line diagram.





Exercise 11

Suppose the population of armadillos is governed by the equation

$$\frac{dA}{dt} = 1000A - A^2.$$

What are the equilibrium solutions? Interpret them physically. What is the stability of each equilibrium solution? Interpret this physically.

Exercise 12

Given the phase line diagram, sketch the corresponding slope field. Draw some representative solutions on the slope field for different initial conditions. Also, determine the stability of each equilibrium point.