Note: This review is based only on sections 14.7 & 14.8 and 16.7 - 16.9. Students are encouraged to review the previous week's WIR sessions that cover the final exam topics and all other resources provided by their professor.

Example 1 (14.7). *Find the local maximum and minimum values and saddle points of the function*

$$f(x,y) = x^3 + y^3 - 3x^2 - 3y^2 - 9y + 2.$$

Critical points:

$f_x = 3x^2 - 6x = 0 \Rightarrow 3x(x-2) = 0 \Rightarrow x = 0, 2$

$f_y = 3y^2 - 6y - 9 = 0 \Rightarrow 3(y-3)(y+1) = 0 \Rightarrow y = 3, -1$

The critical points are $(0,3), (0,-1), (2,3), (2,-1)$.

$f_{xx} = 6x - 6$, $f_{yy} = 6y - 6$, $f_{xy} = f_{yx} = 0$

$D = f_{xx} f_{yy} - (f_{xy})^2 = 36(x-1)(y-1)$

At $(0,3)$, $D < 0$. So, $(0,3)$ is a saddle point.

At $(0,-1)$, $D > 0$, $f_{xx}(0,-1) < 0$. f has maximum at $(0,-1)$, which is $f(0,-1) = -1 - 3 + 9 = 5$.

At $(2,3)$, $D > 0$ and $f_{xx}(2,3) > 0$. So f has minimum at $(0,-1)$, which is $f(2,3) = 8 + \cancel{27} - 12 - \cancel{27} - 27 + 2 = -29$

At $(2,-1)$, $D < 0$. So, $(2,-1)$ is a saddle point.

Example 2 (14.7)**.** *Find three positive numbers whose sum is 50 and the sum of whose squares is the minimum.*

Let the numbers be x, y, and z. We want to

minimize $f(x,y,z) = x^2+y^2+z^2$

subject to $x+y+z = 50$. — (1)

$x+y+z=50 \Rightarrow z = 50-x-y$. So,

$f(x,y,z) = f(x,y) = x^2+y^2+(50-x-y)^2$ — (2)

$\left.\begin{array}{l} f_x = 0 \\ f_y = 0 \end{array}\right\} \Rightarrow$ $2x - 2(50-x-y) = 0$ — (3)

$2y - 2(50-x-y) = 0$ — (4)

$-\quad +$

$2x - 2y = 0 \Rightarrow \boxed{x = y}$

$x=y$ into (3); $2x - 100 + 4x = 0 \Rightarrow x = \frac{50}{3}$

$y = \frac{50}{3}$

From (1), $z = \frac{50}{3}$.

Verifying that $(x,y,z) = \left(\frac{50}{3}, \frac{50}{3}, \frac{50}{3}\right)$ attains minimum:

$f_{xx} = 6$, $f_{yy} = 6$, $f_{xy} = 2$

$D = 6 \cdot 6 - 4 > 0$ and $f_{xx} > 0$ at the critical point $\left(\frac{50}{3}, \frac{50}{3}, \frac{50}{3}\right)$. So, f attains minimum at this critical point.

Example 3 (14.7)**.** *Find the absolute maximum and minimum values of the function*

$$f(x, y) = x + y - xy$$

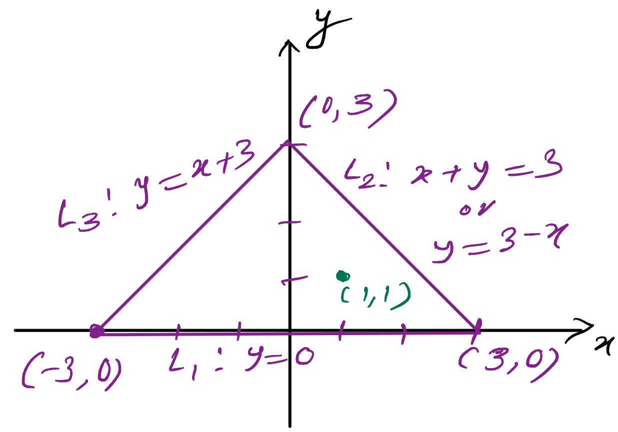
on D, where D is a triangular region with vertices $(-3, 0), (3, 0)$, *and* $(0, 3)$.

Critical points:

$f_x = 0 \Rightarrow 1 - y = 0 \Rightarrow y = 1$

$f_y = 0 \Rightarrow 1 - x = 0 \Rightarrow x = 1.$

$(1, 1)$ is a critical points which lies in D.



$\boxed{f(1,1) = 1}$

Along L_1: $y = 0 \Rightarrow f(x, 0) = x$; $-3 \le x \le 3$

a line

$\boxed{f(-3,0) = -3}$, $\boxed{f(3,0) = 3}$

Along L_2: $y = 3 - x \Rightarrow f(x, 3-x) = \cancel{x} + 3 - \cancel{x} - 3x + x^2 = g(x)$; $0 \le x \le 3$.

$g'(x) = 0 \Rightarrow 2x - 3 = 0 \Rightarrow x = 3/2$

$g(3/2) = \boxed{f(3/2, 3/2) = 3/4}$

$g(0) = \boxed{f(0, 3) = 3}$

Along L_3: $y = x + 3 \Rightarrow f(x, x+3) = g(x) = x + x + 3 - x^2 - 3x$

$= 3 - x - x^2$; $-3 \le x \le 0$.

$g'(x) = 0 \Rightarrow -1 - 2x = 0 \Rightarrow x = -1/2$

$g(-1/2) = \boxed{f(-\frac{1}{2}, \frac{5}{2}) = -\frac{1}{2} + \frac{5}{2} + \frac{1}{2} \cdot \frac{5}{2} = \frac{13}{4}}$

$f(-\frac{1}{2}, \frac{5}{2}) = \frac{13}{4}$ is the maximum.

$f(-3, 0) = -3$ is the minimum.

Example 4 (14.8). *Use Lagrange multipliers' method to find the extreme values of the function*

$$f(x,y) = x^2y$$

subject to the constraint $x^2+y^2=9$. $g(x,y) = x^2+y^2 = 9$

$\nabla f = \lambda \nabla g \Rightarrow \langle 2xy, x^2 \rangle = \lambda \langle 2x, 2y \rangle$

$2xy = 2\lambda x$ — ① $\Rightarrow 2x(y-\lambda) = 0 \Rightarrow x=0$ or $y = \lambda$

$x^2 = 2\lambda y$ — ②

$x^2+y^2 = 9$ — ③

$x=0$ into ③ gives $y = \pm 3 \rightsquigarrow (0, \pm 3)$

$y = \lambda$ into ② $\Rightarrow x^2 = 2y^2$. sub. this into ③,

$2y^2 + y^2 = 9 \Rightarrow y^2 = 3 \Rightarrow y = \pm\sqrt{3}$

And $x^2 = 2(3) \Rightarrow x = \pm\sqrt{6}$

$\rightsquigarrow (\pm\sqrt{3}, \pm\sqrt{6})$.

$f(0, \pm 3) = 0$

$f(\pm\sqrt{3}, \sqrt{6}) = 3\sqrt{6} \longrightarrow$ Max

$f(\pm\sqrt{3}, -\sqrt{6}) = -3\sqrt{6} \longrightarrow$ Min

Suppose f is a continuous function defined on a surface S that is given by a vector valued function

$$\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$$

defined over a region D in the uv-plane. Then the surface integral of f over S is

$$\iint_S f(x,y,z)\,dS = \iint_D f(\mathbf{r}(u,v))|\mathbf{r}_u \times \mathbf{r}_v|\,dA$$

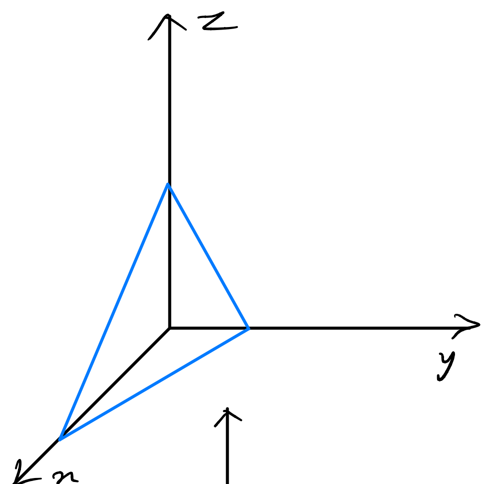
In particular, if the surface S is given by a function $z = g(x,y)$ defined over a region D in the xy-plane, then

$\sqrt{1+g_x^2+g_y^2}$

$$\iint_S f(x,y,z)\,dS = \iint_D f(x,y,g(x,y))|\mathbf{r}_x \times \mathbf{r}_y|\,dA,$$

where $\mathbf{r}(x,y) = \langle x,\ y,\ g(x,y)\rangle$ is a parametric representation of S defined over D.

Example 5 (16.7). *Evaluate $\iint_S xz\,dS$, where S is the part of the plane $x+2y+z=6$ that lies in the first octant.*



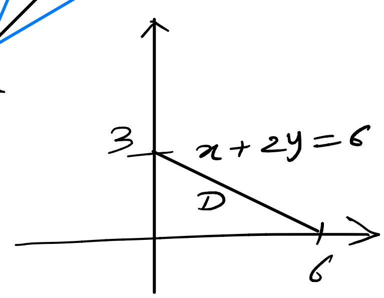
$x+2y+z=6 \Rightarrow z = g(x,y) = 6-x-2y$;

$S: r(x,y) = \langle x, y, 6-x-2y \rangle$; over

$D = \{(x,y): 0 \le x \le 6,\ 0 \le y \le 3-\frac{x}{2}\}$

$r_x \times r_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & -2 \end{vmatrix} = \langle 1, 2, 1 \rangle$

$\Rightarrow |r_x \times r_y| = \sqrt{6}$

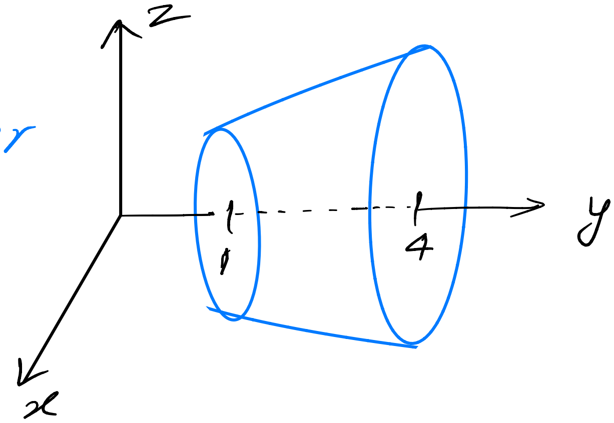


$\iint_S xz\,dS = \iint_D x(6-x-2y)\cdot dA$

$= \int_0^6 \int_0^{3-\frac{x}{2}} 6x - x^2 - 2xy \; dy\,dx$

$= \int_0^6 \left[6xy - x^2y - xy^2 \right]_0^{3-\frac{x}{2}} dx = \cdots = 27$

Example 6 (16.7). *Evaluate* $\displaystyle\iint_S 2z^2\,dS$, *where* S *is the part of the surface* $y = x^2 + z^2$ *between the planes* $y = 1$ *and* $y = 4$.



S : $r(x,z) = \langle x, x^2+z^2, z\rangle$ over

D : $1 \le x^2+z^2 \le 4$.

That is, the projection of S onto xz-plane is the region between the circles $x^2+z^2=1$ and $x^2+z^2=4$.

So, $D = \{(r,\theta) : 1 \le r \le 2,\ 0 \le \theta \le 2\pi\}$

$x = r\cos\theta$, $z = r\sin\theta$

$$|r_x \times r_z| = \sqrt{1 + y_x^2 + y_z^2} = \sqrt{1+4x^2+4z^2} = \sqrt{1+4r^2}$$

$$\iint_S 2z^2\,dS = \iint_D 2z^2 \cdot \sqrt{1+4(x^2+z^2)}\,dA$$

$$= \int_0^{2\pi}\int_1^2 2(r\sin\theta)^2\sqrt{1+4r^2}\,r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_1^2 (1-\cos 2\theta)\cdot r^3\sqrt{1+4r^2}\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_1^2 r^3\sqrt{1+4r^2}\,dr\,d\theta - 0$$

$u = 1+4r^2 \Rightarrow du = 8r\,dr$

$$= \frac{2\pi}{8}\int \sqrt{u}\cdot\left(\frac{u-1}{4}\right)du = \cdots$$

Surface integral of a vector field on parametric surface: If a surface S is given by the vector function $\mathbf{r}(u, v)$ defined on the parameter domain D, then the surface integral of a continuous vector field $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$ is

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v)\, dA \qquad (= \textit{Flux})$$

Here the orientation of S is induced by $\mathbf{r}_u \times \mathbf{r}_v$.

Surface integral of a vector field on a surface that is a graph of a function:. If a surface S is given by a graph of a function $z = g(x, y)$, then a parametrization of the surface is $\mathbf{r}(x, y) = x\,\mathbf{i} + y\,\mathbf{j} + g(x, y)$ then

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_D \mathbf{F} \cdot (\mathbf{r}_x \times \mathbf{r}_y)\, dA \qquad (= \textit{Flux})$$

where D is the projection of the surface S onto the xy-plane.

Example 7 (16.7). *Compute* $\iint_S \mathbf{F}\cdot d\mathbf{S}$, *where* $\mathbf{F}(x,y,z) = y\,\mathbf{i} - x\,\mathbf{j} + 2z\,\mathbf{k}$ *and* S *is the part of the sphere* $x^2+y^2+z^2=1$, $z\geq 0$, *oriented downward.*

$x^2+y^2+z^2=1 \Rightarrow z = \pm\sqrt{1-x^2-y^2} \rightsquigarrow z=\sqrt{1-x^2-y^2} = g(x,y)$

$S: r(x,y) = \langle x, y, \sqrt{1-x^2-y^2}\rangle$ over $\boxed{D: x^2+y^2\leq 1}$

$$r_x \times r_y = \begin{vmatrix} i & j & k \\ 1 & 0 & \frac{-x}{\sqrt{1-x^2-y^2}} \\ 0 & 1 & \frac{-y}{\sqrt{1-x^2-y^2}} \end{vmatrix} = \Big\langle \frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{1-x^2-y^2}}, \underset{\text{upward}}{1}\Big\rangle$$

So, $-r_x\times r_y = -\Big\langle \frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{1-x^2-y^2}}, 1\Big\rangle$ is oriented downward.

$$F\cdot dS = F\cdot(-r_x\times r_y)\,dA = \cancel{\frac{-xy}{\sqrt{1-x^2-y^2}}} + \cancel{\frac{xy}{\sqrt{1-x^2-y^2}}} - 2z = -2\sqrt{1-x^2-y^2}\,dA$$

$$\iint_S F\cdot dS = -2\iint_D \sqrt{1-x^2-y^2}\,dA$$

$$= -2\int_0^{2\pi}\int_0^1 \sqrt{1-r^2}\cdot r\,dr\,d\theta \qquad u = 1-r^2 \Rightarrow du = -2r\,dr$$

$$= -2\cdot 2\pi\cdot \frac{-1}{2}\int \sqrt{u}\,du$$

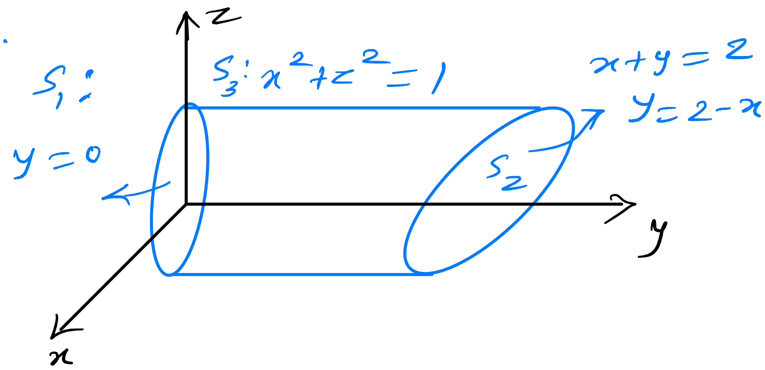
$$= 2\pi\cdot\frac{2}{3}u^{3/2}\Big| = \frac{4\pi}{3}\Big[(1-r^2)^{3/2}\Big]_0^1$$

$$= \frac{4\pi}{3}[0-1] = -\frac{4\pi}{3}$$

Example 8 (16.7). *Compute* $\iint_S \mathbf{F}\cdot d\mathbf{S}$, *where* $\mathbf{F}(x,y,z) = x\,\mathbf{i} + y\,\mathbf{j} + 5\,\mathbf{k}$ *and* S *is the boundary of the cylinder* $x^2+z^2=1$ *and planes* $y=0$ *and* $x+y=2$ *with outward orientation.*

S_1 : $r(x,z) = \langle x, 0, z\rangle$; $x^2+z^2 \le 1$.

$$r_x \times r_z = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \langle 0,-1,0\rangle$$



$$F\cdot(r_x\times r_z) = 0 - y + 0 = -y = 0$$

$$\iint_{S_1} F\cdot dS = 0.$$

S_2 : $r(x,z) = \langle x, 2-x, z\rangle$, $D: x^2+z^2 \le 1$.

$$r_x \times r_z = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \langle -1, -1, 0\rangle$$ → pointed towards $-ve$ y-axis.

$$F\cdot -(r_x\times r_z) = -(-x-y) = x+y = x+(2-x) = 2.$$

$$\iint_{S_2} F\cdot dS = \iint_D 2\, dA = 2A(D) = 2(\pi\cdot 1^2) = 2\pi$$

S_3 : $r(u,v) = \langle \underset{x}{\cos u}, \underset{y}{v}, \underset{z}{\sin u}\rangle$; $\boxed{0\le u\le 2\pi,\ 0\le v\le 2-\cos u}$: D

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ -\sin u & 0 & \cos u \\ 0 & 1 & 0 \end{vmatrix} = \langle -\cos u, 0, -\sin u\rangle$$

$-ve$ → pointed inward.
$0\le u\le \pi \Rightarrow \sin u \ge 0$
$\pi \le u \le 2\pi \Rightarrow \sin u \le 0$.

oriented outward

$$F\cdot(-r_u\times r_v) = \langle \cos u, v, 5\rangle\cdot\langle \cos u, 0, \sin u\rangle = \cos^2 u + 5\sin u$$

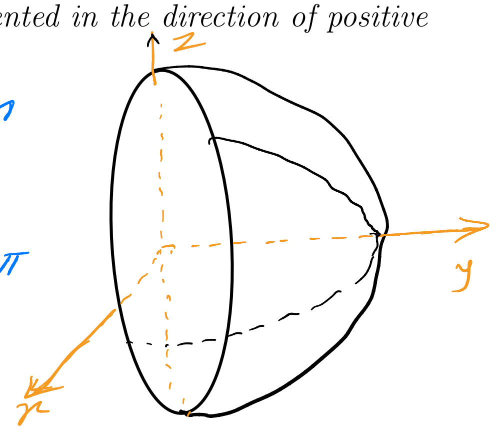
$$\iint_{S_3} F\cdot dS = \int_0^{2\pi}\int_0^{2-\cos u} (\cos^2 u + 5\sin u)\, dv\, du = \cdots = 2\pi$$

Hence, $\iint_S F\cdot dS = \int_{S_1}\cdots + \int_{S_2}\cdots + \int_{S_3}\cdots = 0 + 2\pi + 2\pi = 4\pi$

Example 9 (16.8)**.** *Use the Stokes' Theorem to evaluate* $\iint_S \text{curl}\mathbf{F}\cdot d\mathbf{S}$, *where*

$$\mathbf{F}(x,y,z) = \langle z\,e^{xy},\ -x^2\cos(yz),\ xz\sin^2 y\rangle$$

and S is the part of the sphere $x^2+y^2+z^2=9$, $y\geq 0$, oriented in the direction of positive y-axis.



Intersection of $x^2+y^2+z^2=9$ and $y=0$ is

$x^2+z^2=9$, $y=0$.

C: $x=3\cos t$, $z=3\sin t$, $y=0$; $0\leq t\leq 2\pi$

$dx=-3\sin t\,dt$, $dz=3\cos t\,dt$, $dy=0$.

Note that if we travel along the curve from +ve x-axis to +ve z-axis with head pointed in the +ve y-axis, the surface lies on the right side. So, $\iint_S \text{curl}\,F\cdot dS = \oint_{-C} F\cdot dr = -\oint_C F\cdot dr$

$$= -\oint_C ze^{xy}dx - x^2\cos(yz)dy + xz\sin^2 y\,dz$$

$$= -\int_0^{2\pi} 3\sin t\cdot(-3\sin t\,dt) - 0 + 0$$

$$= 9\int_0^{2\pi}\sin^2 t\,dt = 9\int_0^{2\pi}\frac{1}{2}-\frac{\cos 2t}{2}\,dt$$

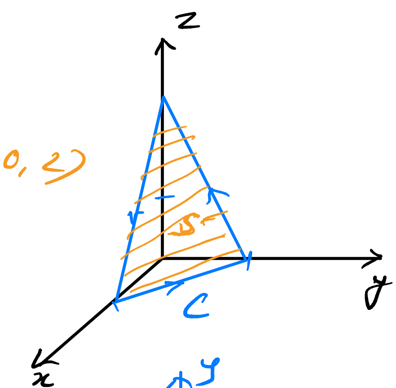
$$= 9\cdot\frac{1}{2}\cdot 2\pi = 9\pi$$

Example 10 (16.8)**.** *Use the Stokes' Theorem to evaluate* $\oint_C \mathbf{F}\cdot d\mathbf{r}$, *where*

$$\mathbf{F}(x,y,z) = \langle 2y + e^x\,, xy,\; 2+2z\rangle$$

and C is the triangle with vertices $(1,0,0)$, $(0,1,0)$, $(0,0,2)$ *with positive orientation.*

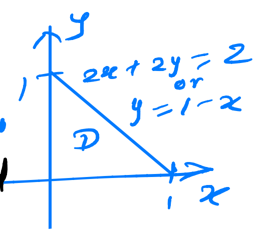
$$\text{curl}\,F = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ 2y+e^x & xy & 2+2z \end{vmatrix} = \langle 0,\ 0,\ y-2\rangle$$



Equation of the plane passing through $(1,0,0)$, $(0,1,0)$, $(0,0,2)$
is $2x+2y+z=2 \rightsquigarrow z = g(x,y) = 2-2x-2y$

The surface S bounded by C in the part of
the plane $z = g(x,y) = 2-2x-2y$.

A parametrization of S is
$r(x,y) = \langle x, y, 2-2x-2y\rangle$ over $D: 0\le x\le 1,\ 0\le y\le 1-x$.



+ve pointed upward

$$r_x \times r_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -2 \\ 0 & 1 & -2 \end{vmatrix} = \langle 2,\ 2,\ 1\rangle$$

$$\text{Curl}\,F\cdot(r_x\times r_y) = 0+0+y-2 = y-2.$$

$$\oint_C F\cdot dr \overset{\text{Stokes' Thm}}{=} \iint_S \text{curl}\,F\cdot dS = \iint_D \text{curl}\,F\cdot(r_x\times r_y)\,dA$$

$$= \int_0^1\int_0^{1-x} (y-2)\,dy\,dx = \int_0^1 \frac{y^2}{2} - 2y\Big|_0^{1-x} dx$$

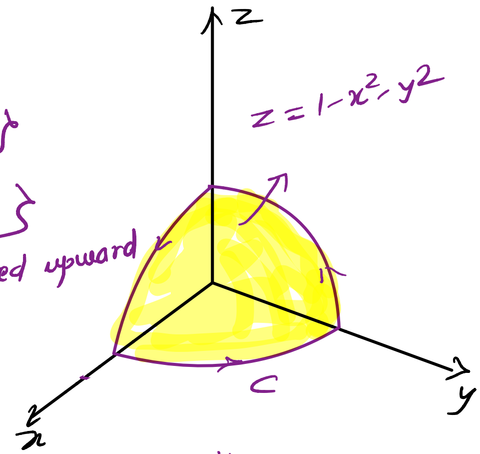
$$= \int_0^1 \frac{(1-x)^2}{2} - 2(1-x)\,dx = \int_0^1 \frac{1}{2} - x + \frac{x^2}{2} - 2 + 2x\,dx$$

$$= \int_0^1 \frac{x^2}{2} + x - \frac{3}{2}\,dx = \frac{x^3}{6} + \frac{x^2}{2} - \frac{3}{2}x\Big|_0^1 = \frac{-5}{6}$$

Example 11 (16.8)**.** *Use the Stokes' Theorem to evaluate* $\oint_C \mathbf{F}\cdot d\mathbf{r}$, *where*

$$\mathbf{F}(x,y,z) = \langle 2y\,,\, x,\, xz\rangle$$

and C *is the boundary of the paraboloid* $z = 1 - x^2 - y^2$ *in the first octant with positive orientation from eagle's view.*

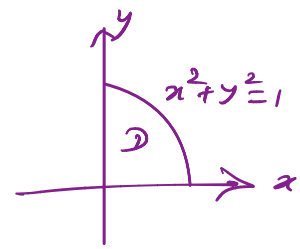


$S: r(x,y) = \langle x, y, 1-x^2-y^2\rangle$ over D

where $D = \{(x,y) : x^2+y^2 \le 1,\ x \ge 0,\ y \ge 0\}$

$= \{(r,\theta)\ ;\ 0 \le r \le 1,\ 0 \le \theta \le \frac{\pi}{2}\}$

$$r_x \times r_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -2x \\ 0 & 1 & -2y \end{vmatrix} = \langle 2x, 2y, 1\rangle$$ → pointed upward



$$\text{curl}F = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ 2y & x & xz \end{vmatrix} = \langle 0, -z, -1\rangle$$

$$\text{curl}F\cdot(r_x\times r_y) = 0 - 2yz - 1 = -2y(1-x^2-y^2) - 1$$

$$\oint_C F\cdot dr \overset{\text{Stokes'}}{=} \iint_D \text{curl}F\cdot(r_x\times r_y)\,dA$$

$$= \int_0^{\pi/2}\int_0^1 [-2r\sin\theta(1-r^2) - 1]\, r\,dr\,d\theta$$

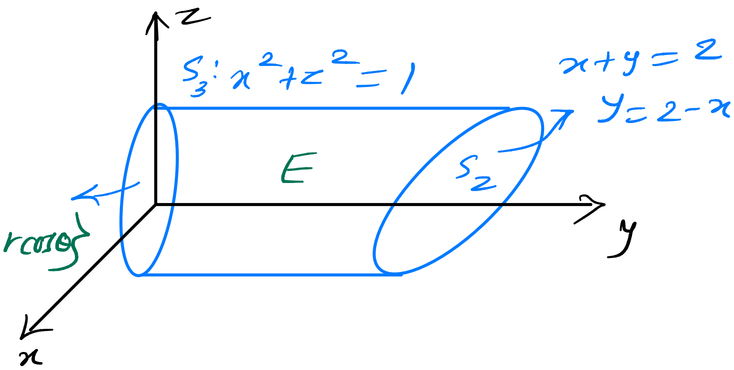
$$= -2\left(\int_0^{\pi/2}\sin\theta\,d\theta\right)\left(\int_0^1 r^2 - r^4\,dr\right) - \left(\int_0^{\pi/2} d\theta\right)\left(\int_0^1 r\,dr\right)$$

$$= -2\cdot(1)\cdot\left[\frac{1}{3} - \frac{1}{5}\right] - \frac{\pi}{2}\cdot\frac{1}{2}$$

$$= -\frac{4}{15} - \frac{\pi}{4}$$

Example 12 (16.9)**.** *(Example 8 above) Use the Divergence Theorem to compute* $\iint_S \mathbf{F}\cdot d\mathbf{S}$, *where* $\mathbf{F}(x,y,z) = x\,\mathbf{i}+y\,\mathbf{j}+5\,\mathbf{k}$ *and* S *is the boundary of the cylinder* $x^2+z^2=1$ *and planes* $y=0$ *and* $x+y=2$ *with outward orientation.*

Let E be the solid bounded by the surface S. Then



$E = \{(r, y, \theta) : 0 \le r \le 1, 0 \le \theta \le 2\pi, 0 \le y \le 2 - r\cos\theta\}$

$x = r\cos\theta$
$z = r\sin\theta$

$\operatorname{div} F = P_x + Q_y + R_z = 1 + 1 + 0 = 2$

$$\iint_S F\cdot dS \overset{\text{Divergence Thm}}{=} \iiint_E \operatorname{div} F\, dV = \iiint_E 2\, dV$$

$$= 2\int_0^{2\pi}\int_0^1\int_0^{2-r\cos\theta} r\, dy\, dr\, d\theta$$

$$= 2\int_0^{2\pi}\int_0^1 (2r - r^2\cos\theta)\, dr\, d\theta$$

$$= 2\int_0^{2\pi}\left[r^2 - \frac{r^3}{3}\cos\theta\right]_{r=0}^{1} d\theta$$

$$= 2\int_0^{2\pi}\left(1 - \tfrac{1}{3}\cos\theta\right) d\theta = 2\int_0^{2\pi} d\theta - \frac{2}{3}\underbrace{\int_0^{2\pi}\cos\theta\, d\theta}_{0}$$

$$= 4\pi$$

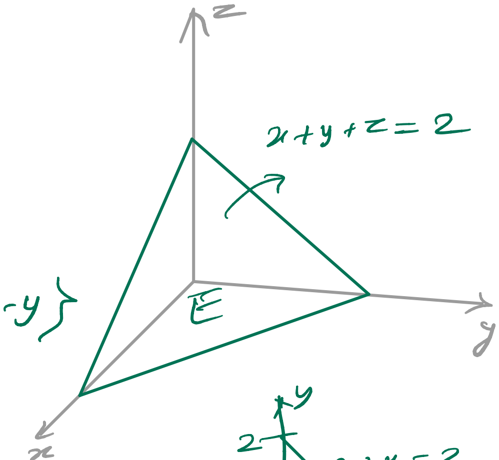
Example 13 (16.9). *Use the Divergence Theorem to compute* $\iint_S \mathbf{F}\cdot d\mathbf{S}$, *where*

$$\mathbf{F}(x,y,z) = \left\langle x^2+yz,\ \sin x - 2yz,\ z^2+3\right\rangle$$

and S *is the boundary of the tetrahedron with vertices* $(0,0,0)$, $(2,0,0)$, $(0,2,0)$, $(0,0,2)$, *with outward orientation.*

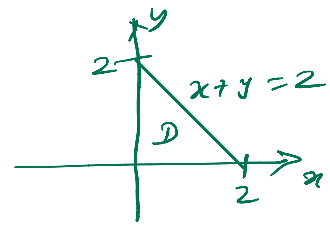
An equation of the plane passing through $(2,0,0)$, $(0,2,0)$ and $(0,0,2)$ is

$x+y+z=2$.



The tetrahedron (solid bounded by S) is

$E=\{(x,y,z):\ 0\le x\le 2,\ 0\le y\le 2-x,\ 0\le z\le 2-x-y\}$



$\operatorname{div} F = 2x - 2z + 2z = 2x$

$$\iint_S F\cdot dS \overset{\text{Divergence Thm}}{=} \iiint_E \operatorname{div} F\, dV$$

$$= 2\int_0^2\int_0^{2-x}\int_0^{2-x-y} x \, dz\, dy\, dx$$

$$= 2\int_0^2\int_0^{2-x} (2x - x^2 - xy)\, dy\, dx$$

$$= 2\int_0^2 \left[2xy - x^2y - \frac{xy^2}{2}\right]_0^{2-x} dx$$

$$= \int_0^2 \left[xy(4-2x-y)\right]_0^{2-x} dx$$

$$= \int_0^2 x(2-x)(2-x)\, dx$$

$$= \int_0^2 4x - 4x^2 + x^3\, dx$$

$$= 2x^2 - \frac{4}{3}x^3 + \frac{x^4}{4}\Big|_0^2 = \frac{4}{3}$$

Good luck!