Math 308: Week-in-Review 1 (1.1, 1.2, 1.3 & 2.1)

1. Verify that each of the given functions is a solution of the corresponding differential equation.

   (a) $\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt} - 12y = 0, \qquad y(t) = e^{4t},\ y(t) = e^{-3t}$

   (b) $\dfrac{dy}{dt} + y = \sin(t), \qquad y(t) = Ae^{-t} - \dfrac{1}{2}\cos(t) + \dfrac{1}{2}\sin(t).$

(c) $y''' - 2y'' = 0, \qquad y(x) = A + Bx + Ce^{2x}$ (where A, B and C are constants).

2. (a) Find values of A and B so that the function $y(x) = Ae^{4x} + Be^{-3x}$ is a solution of the initial value problem $y'' - y' - 12y = 0$, $y(0) = 1$, $y'(0) = -1$.

(b) Find the values of A, B and C so that $y(x) = A + Bx + Ce^{2x}$ is a solution of the initial value problem $y''' - 2y'' = 0,\ \ y(0) = 0,\ y'(0) = 1,\ y''(0) = 3$.

3. Use direct integration to find the general solution of each of the differential equations

   (a) $\dfrac{dy}{dx} = \dfrac{x}{\sqrt{x^2 - 16}}$

   (b) $\dfrac{dy}{dx} = xe^{-x}$

   (c) $\dfrac{dy}{dx} = \dfrac{1}{x^2 - 16}$

4. For each of the following, determine whether it is an ODE or PDE. Additionally, state (a) the order of the differential equation and (b) whether it is linear or nonlinear.

(a) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y = x^3$

(b) $y\dfrac{dy}{dx} + y^4 = \sin(x)$

(c) $\dfrac{1}{c^2}\dfrac{\partial^2 z}{\partial t^2} = \dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2}$, $\quad c$ is constant

(d) $\left(\dfrac{dy}{dx}\right)^2 + y = 0$

(e) $u_t + uu_x = \sigma u_{xx}$, $\quad \sigma$ is a constant

(f) $r^2R''(r) + rR'(r) + (r^2 - \alpha^2)R(r) = 0 \quad \alpha$ is a constant

(g) $u_x + u_y + u^2 = 0$

(h) $t^4 + w^{(7)}(t) + \tan(t)w''(t) = \sin(t)$

5. Consider the following list of differential equations, some of which produced the direction fields shown in the figures. Identify the differential equation that corresponds to the given direction field.

(a) $y' = y(y-2)^2$ (b) $y' = y(2-y)$ (c) $y' = y+2$ (d) $y' = -y(2-y)$ (e) $y' = -2-y$

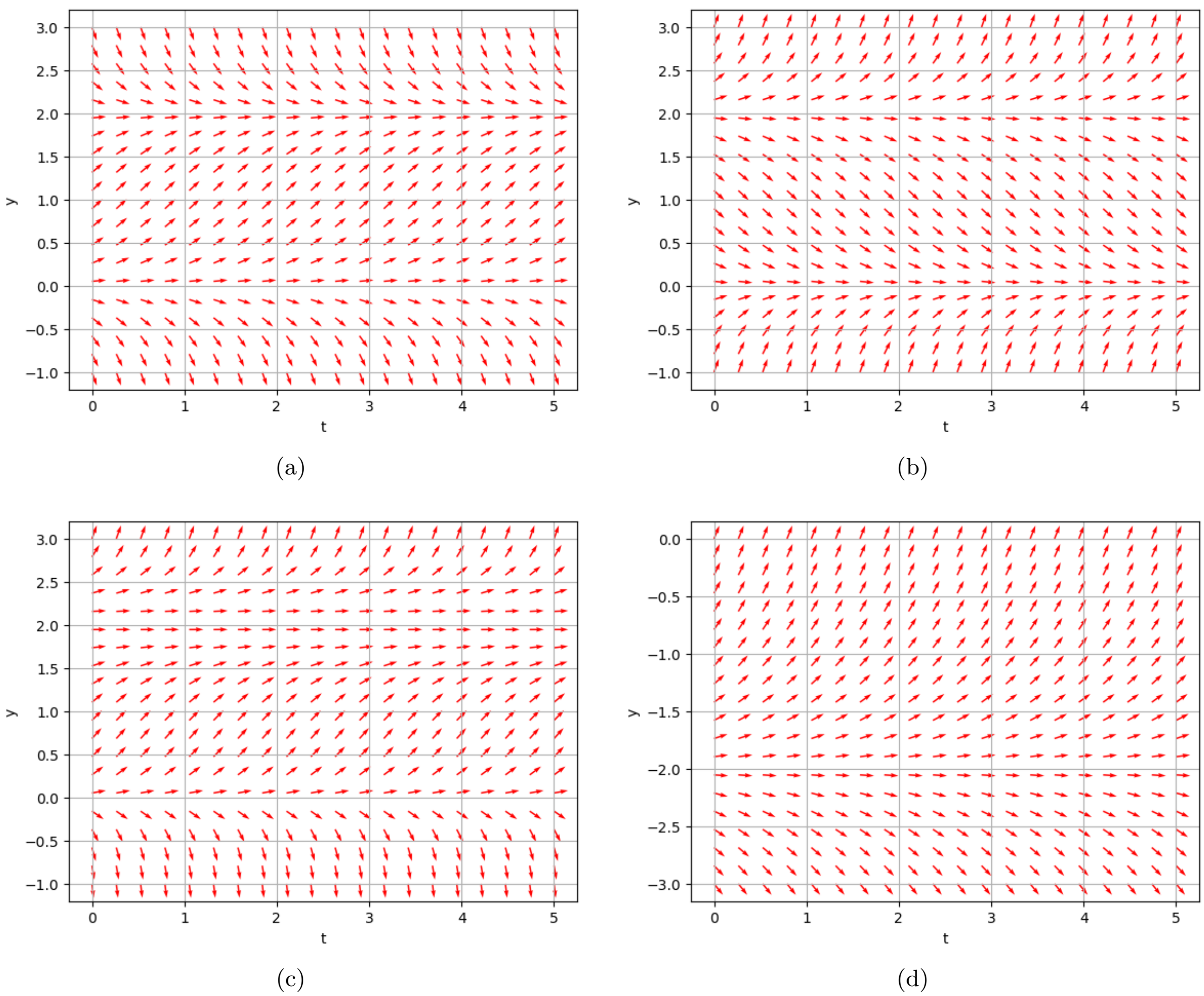

Figure 1: Direction fields.

6. Use the direction fields below to determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe this dependency.

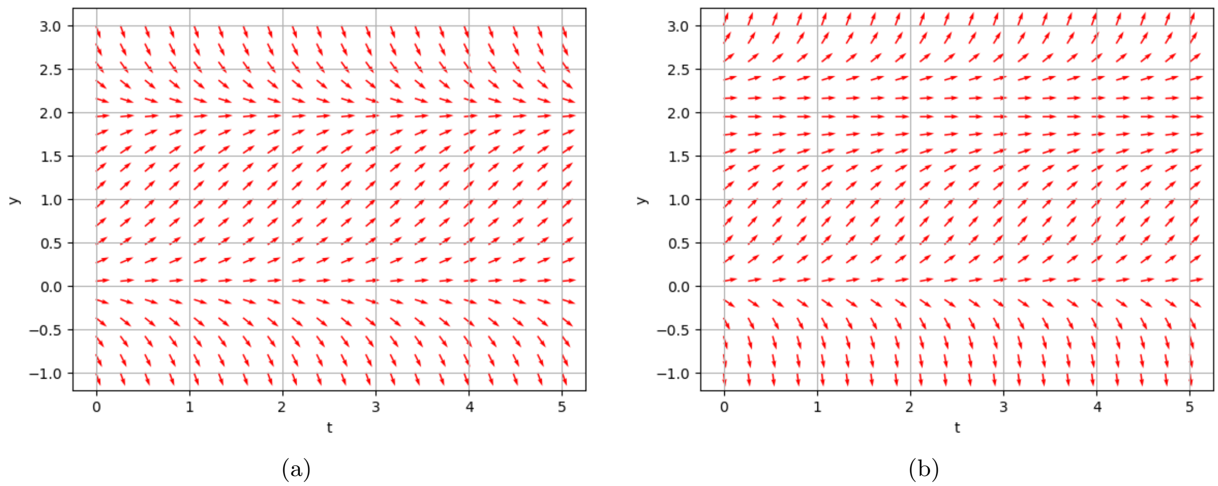

Figure 2: Direction fields.

7. Solve each of the following differential equations.

   (a) $ty' + y = t^2$

   (b) $(e^t + 1)\dfrac{dy}{dt} + e^t y = t, \quad y(0) = -1.$

(c) $\dfrac{dy}{dt} + \dfrac{2t}{1+t^2}y = \dfrac{1}{1+t^2}, \quad y(0) = 4.$

(d) $t\dfrac{dy}{dt} + (t+1)y = t, \quad y(\ln\ 2) = 1,\ t > 0.$

(e) Find the solution of the initial value problem and describe its behavior for large t.

$$y' + \frac{1}{4}y = 3 + 2\cos(2t), \quad y(0) = 0$$