1. (a) Use the Integral test to determine whether or not the series $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ is convergent.

   (b) Approximate the sum of the series $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ by using the sum of first 4 terms. Estimate the error involved in this approximation.

2. Explain why the Integral Test cannot be used for the series $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n^2+1}$.

3. The tenth partial sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is $s_{10} \approx 1.64522$.

   (a) Find the error when using the tenth partial sum to approximate the sum of the series.

   (b) How many terms n would be required so that the error $s \approx s_n$ is less than 0.001?

4. Evaluate the integral

   (a) $\int \sin^3 x \cos^4 x \; dx$

   (b) $\int \sin^2 x \cos^4 x \; dx$

   (c) $\int_0^{\pi/4} \tan^4 x \sec^4 x \; dx$

   (d) $\int \tan^3 x \sec^3 x \; dx$

   (e) $\int (4x^2 - 25)^{-3/2} \, dx$

   (f) $\int \frac{(x-1)^2 \, dx}{5\sqrt{24 - x^2 + 2x}}$

   (g) $\int \frac{5x^2 + x + 12}{x^3 + 4x} dx$

5. Determine whether the given integral is convergent or divergent.

   (a) $\int_1^{\infty} \frac{4 + \cos^4 x}{x} dx$

   (b) $\int_1^{\infty} \frac{3 + \sin x}{x^2} dx$

   (c) $\int_0^{\infty} \frac{1}{\sqrt{x} + e^{4x}} dx$

6. Compute the following integrals or show that they diverge.

   (a) $\int_e^{\infty} \frac{dx}{x \; \ln^5 x}$

(b) $\int_{-\infty}^{0} (1+x)e^x \; dx$

(c) $\int_{-\infty}^{\infty} \frac{6x^5}{(x^6+3)^3} dx$

(d) $\int_{0}^{2025} \frac{1}{\sqrt{2025-x}} dx$

7. Find the following limits

(a) $\lim_{n\to\infty} \frac{(-1)^n}{n^3}$

(b) $\lim_{n\to\infty} \frac{\sqrt{n}}{\ln n}$

(c) $\lim_{n\to\infty} \frac{1-2n^2}{\sqrt[3]{n^6+1}+2n^2}$

(d) $\lim_{n\to\infty} \left(\frac{1}{3}\ln(n^3+5n-2) - \ln(n-2)\right)$

8. Show that the sequence defined by $a_1 = 3$ and $a_{n+1} = 6 - \frac{8}{a_n}$ is increasing and bounded above. Find its limit.

9. If the series $\sum_{n=1}^{\infty} a_n$ has a partial sum of $s_n = \frac{n}{2n+1}$, find a_4 and the sum of the series.

10. Find the sum of the series

(a) $\sum_{n=1}^{\infty} \frac{2^{2n+1}}{3^{3n-1}}$

(b) $\sum_{n=3}^{\infty} \frac{1}{n^2-4}$

11. Which of the following series is convergent?

(a) $\sum_{n=1}^{\infty} \frac{n^2}{n^{5/7}+1}$

(b) $\sum_{n=1}^{\infty} \frac{\pi^n}{3^n}$

(c) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$