



## MATH 308: WEEK-IN-REVIEW 12 (7.5 - 7.6)

## 7.5: Homogeneous Linear Systems with Constant Coefficients

### Review

- How to solve a homogeneous linear system with constant coefficients (when you have distinct real eigenvalues)
  1. Assume your solution has the form  $\mathbf{x}(t) = \boldsymbol{\xi}e^{rt}$ .
  2. Plug this in to get an eigenvalue problem.
  3. Solve for the eigenvalues  $r_1$  and  $r_2$  and the corresponding eigenvectors  $\boldsymbol{\xi}^{(1)}$  and  $\boldsymbol{\xi}^{(2)}$ .
  4. The general solution is  $c_1\boldsymbol{\xi}^{(1)}e^{r_1t} + c_2\boldsymbol{\xi}^{(2)}e^{r_2t}$ .
- Phase plane/portrait: A phase plane/portrait is essentially a 2D version of the phase line. It shows you where the solution moves as time passes.
- An equilibrium point is a point where if you start there, you will remain there forever. The origin is always an equilibrium point of the differential equation system  $\mathbf{x}' = A\mathbf{x}$ .
- Stability of equilibrium points
  - Asymptotically stable: If you start near the equilibrium point, you will be sucked into it as  $t \rightarrow \infty$ .
  - Stable: If you start near the equilibrium point, you will stay near it.
  - Unstable: There is at least one point near the equilibrium point that goes away from the equilibrium point.

## 7.6: Complex Eigenvalues

### Review

- To solve the system  $\mathbf{x}' = A\mathbf{x}$  when you have complex eigenvectors:
  - Solve for just one of the eigenvectors.
  - Separate  $\boldsymbol{\xi}e^{rt}$  into its real and imaginary parts.
  - The real and imaginary parts form a fundamental set of solutions.
  - (Assuming that  $A$  is  $2 \times 2$ . If  $A$  is larger, then there are also more solutions.)



1. Find the general solution, sketch the phase plane, and determine the stability of the equilibrium point at the origin.

$$\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \mathbf{x}$$



2. Solve the initial value problem when  $\mathbf{x}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ . Draw this solution on the phase plane and sketch the graph of  $x_1(t)$  and  $x_2(t)$ .



3. Find the general solution, sketch the phase plane, and determine the stability of the equilibrium point at the origin.

$$x_1' = -5x_1 + 4x_2$$

$$x_2' = \frac{3}{2}x_1 - 4x_2$$



4. Find the general solution, sketch the phase plane, and determine the stability of the equilibrium point at the origin.

$$x' = 2x + 2y$$

$$y' = x + 3y$$



5. Find the general solution, sketch the phase plane, and determine the stability of the equilibrium point at the origin.

$$x' = 3x + y$$

$$y' = -2x + y$$



6. Find the general solution, sketch the phase plane, and determine the stability of the equilibrium point at the origin.

$$\mathbf{x}' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \mathbf{x}$$



7. Find the general solution, sketch the phase plane, and determine the stability of the equilibrium point at the origin. Solve the initial value problem with  $\mathbf{x}(0) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ .

$$\mathbf{x}' = \begin{bmatrix} 1 & -8 \\ 1 & -3 \end{bmatrix} \mathbf{x}$$





8. Classify the types and stability of the equilibrium point(s) of the system

$$x' = \begin{pmatrix} \alpha - 1 & \alpha + 1 \\ -2/3 & 1/3 \end{pmatrix} x$$

for different values of the parameter  $\alpha$ .