1.2: SOLUTIONS TO DIFFERENTIAL EQUATIONS

Review

- A **solution to a differential equation** is
- An **initial value problem** is
- A **solution to an initial value problem** is

Exercise 1

Is e^{2x} a solution to the differential equation $y'' - 4y' + 4y = 0$?

Exercise 2

Is $\cos(x)$ a solution to the differential equation $f^{(4)}(x) - f''(x) = 4\cos(x)$?

Exercise 3

Is $\sin(2t)$ a solution to the following initial value problem?

$$\frac{\mathrm{d}^2 g}{\mathrm{d}t^2} - \frac{\mathrm{d}g}{\mathrm{d}t} + 4g = -2\cos(2t), \qquad g(0) = 1.$$

Exercise 4

Find the values of a for which e^{at} is a solution to $y'' - 3y' + y = 0$.

Exercise 5

Find the values of b such that $\sin(bx)$ solves the differential equation $y + 6y'' = 0$.

1.3: CLASSIFICATION OF DIFFERENTIAL EQUATIONS

Review

- An **ordinary differential equation** (ODE) is a differential equation that has derivatives with respect to

- A **partial differential equation** (PDE) is a differential equation that has derivatives with respect to

- The **order** of a differential equation is

- An ODE is **linear** if it can be written in the form

  i.e., in a **linear** ODE,

  - y and its derivatives are all in separate terms.
  - y and its derivatives are not inside any functions or to any powers.
  - Each term can be multiplied by a function of x.
  - There can also be another function of x by itself.

Exercise 6

For each of the following, determine whether it is an ODE or a PDE. Additionally, state the order of the differential equation.

(a) $\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} = f$

(b) $\dfrac{\mathrm{d}^2 g}{\mathrm{d}x^2} - 2\left(\dfrac{\mathrm{d}g}{\mathrm{d}x}\right)^5 = xg$

(c) $r'''(z)r'(z) - z^2 + \tan(z)r(z) = 0$

(d) $u_{xx} + u_{yy}u = 0$

Exercise 7

For each of the following ODEs, determine if it is linear or nonlinear.

(a) $w' - w''w + t^2 w = 7t$

(b) $\dfrac{1}{g'(t)} + g(t) = g''(t)$

(c) $(x^2 + \cos(x))Q(x) - \tan(x)Q'(x) = Q'''(x)$

(d) $y^{(5)} - x^3 y^2 + y''' = 7x^3 - \csc(x)$

(e) $t^2 + z^{(6)} + \cos(t)z''' = \cos(t)$

1.1: DIRECTION FIELDS

Review

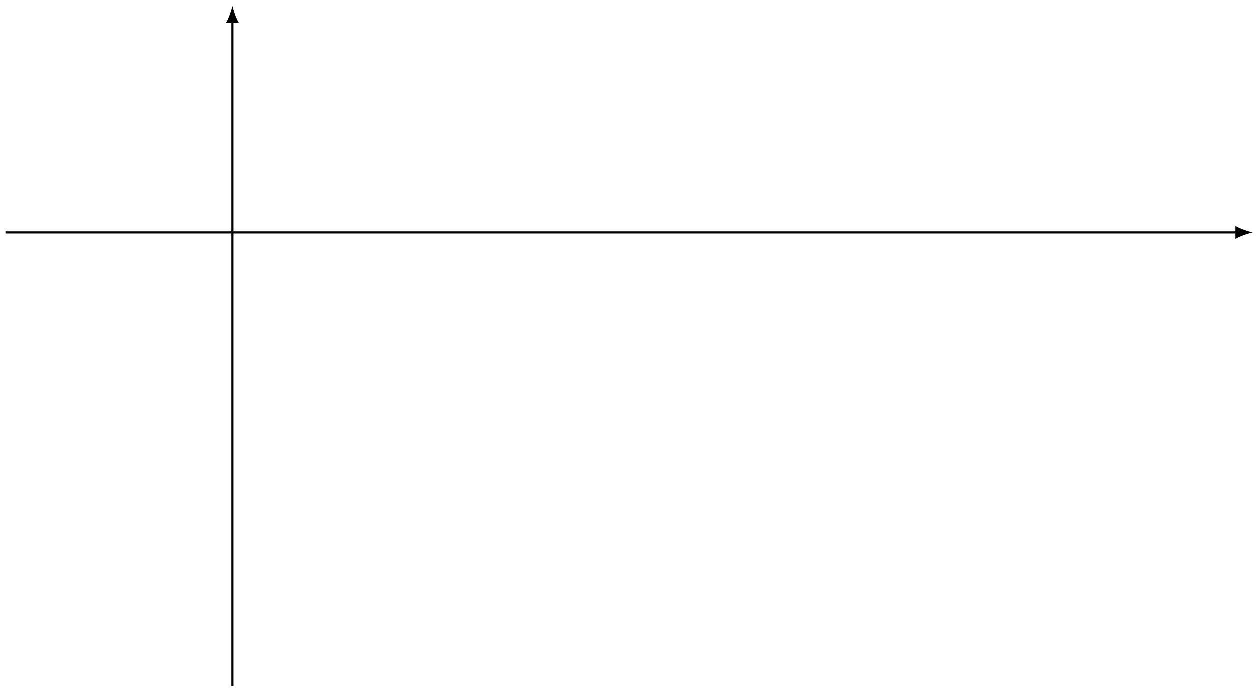
- A **direction field** (or **slope field**)

Exercise 8

Sketch the slope field of for the differential equation $y' = y^2 - 2y$. Draw some example solutions to the ODE. If the initial condition is $y(0) = a$, how does the long-time behavior of $y(t)$ depend on a?

Exercise 9

Sketch the slope field of for the differential equation $y' = \frac{1}{4}y(y+3)^2$. Draw some example solutions to the ODE. If the initial condition is $y(0) = a$, how does the long-time behavior of $y(t)$ depend on a?



2.2: SEPARABLE ODES – SEPARATION OF VARIABLES

Review

- A **separable** ODE is an ODE that has the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)g(y).$$

- Steps for solving a separable ODE:
  1. Treat $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a fraction.
  2. Move all the y's to one side and all the x's to the other.
  3. Integrate both sides.
  4. (If possible) solve for y.

- The **general solution** to a differential equation is the form of the solution that contains all possible solutions inside it. It is the solution you get *before* you plug in the initial condition to solve for c.

- The solution to an initial value problem is **defined** on an *interval* that contains the initial condition. On that interval, the solution must be
  - a **function** that is
  - **defined** and
  - **differentiable**.

Exercise 10

Solve the differential equation $f' = \frac{x^3 + 1}{f^2}$.

Exercise 11

Solve the initial value problem

$$f' = e^{-f}(4 - 2x), \qquad f(2) = 0.$$

Where is the solution defined?

Exercise 12

Solve the initial value problem

$$(e^y - y)x^2y' = 1, \qquad y(1) = 2.$$

Exercise 13

(a) Find the general solution to the differential equation $y' + y^2 \sin(x) = 0$.

(b) Find the solution that satisfies the initial condition $y(\pi) = 3$. Where is the solution defined?

(c) Find the solution that satisfies the initial condition $y(\pi) = 0$. Where is the solution defined?

Exercise 14

Solve the differential equation $\frac{\mathrm{d}g}{\mathrm{d}t} = (g^2 - 9)\cos(t)$.

2.1: LINEAR ODES – METHOD OF INTEGRATING FACTORS

Review

Whenever you have a **linear equation**, you can always solve it using the **method of integrating factors**.

Steps for the **method of integrating factors**:

1. Put in standard form: $y' + p(t)y = g(t)$.
2. Multiply by μ.
3. Find μ to match the product rule.
4. Reverse the product rule.
5. Integrate both sides and solve for y.

Exercise 15

Determine if each of the following are separable or linear.

(a) $u'(t) = \dfrac{\sin(t)}{\cos(u)}$

(b) $\dfrac{\mathrm{d}w}{\mathrm{d}r} = \sin(wr)$

(c) $xz^2\dfrac{\mathrm{d}z}{\mathrm{d}x} = 1$

(d) $y' = 3y + 4$

(e) $\dfrac{\mathrm{d}g}{\mathrm{d}t} = 4g - 3t$

(f) $t^2y - y' = 2$

(g) $f' = 1 + t + f + tf$

Exercise 16

Solve the differential equation $y' = 3y + 4$. (Note that this could also be solved using separation of variables.)

Exercise 17

Solve the initial value problem

$$tf' - (1+t)f = 2t^2, \qquad f(0) = 2.$$