Math 151 - Week-In-Review 9

Topics for the week:

3.10 Linear Approximations and Differentials
4.1 Maximum and Minimum Values

3.10 Linear Approximations and Differentials

1. Write the linearization $L(x)$ of the function $f(x) = \ln(3x+4)$ at $a = -1$.

$f(x) = \ln(3x+4)$
$a = -1$
$f(-1) = \ln(3(-1)+4)$
$= \ln(1)$
$= 0$

$\frac{df(x)}{dx} = \frac{3}{3x+4}$

$\left.\frac{df(x)}{dx}\right|_{x=-1} = \frac{3}{3(-1)+4}$
$= \frac{3}{1}$
$= 3$

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = 0 + 3(x-(-1))$

$\boxed{L(x) = 3(x+1) \text{ or } 3x+3}$

2. Write the linearization $L(x)$ of the function $f(x) = \sqrt[5]{x-32}$ at $a = 0$. Then use $L(x)$ to approximate $\sqrt[5]{-31}$.

$f(x) = \sqrt[5]{x-32} = (x-32)^{1/5}$
$a = 0$
$f(0) = \sqrt[5]{(0)-32}$
$= \sqrt[5]{-32}$
$= -2$

$f'(x) = \frac{1}{5}(x-32)^{-4/5} = \frac{1}{5(x-32)^{4/5}}$

$f'(0) = \frac{1}{5((0)-32)^{4/5}}$
$= \frac{1}{5(-2)^4}$
$= \frac{1}{80}$

$L(x) = f(a) + f'(a)(x-a)$
$L(x) = (-2) + \frac{1}{80}(x-0)$
$L(x) = \frac{1}{80}x - 2$

$\sqrt[5]{-31}$ means $x-32=-31$
$x = 1$

so
$\sqrt[5]{-31} \approx L(1) = \frac{1}{80}(1) - 2$
$\approx \frac{1}{80} - 2$
$\sqrt[5]{-31} \approx -\frac{159}{80}$

3. Approximate $\sqrt[44]{1.3}$.

$\sqrt[44]{1.3}$ means

$f(x) = \sqrt[44]{x}$ and $a = 1$

$f(1) = \sqrt[44]{1} = 1$

$\frac{df(x)}{dx} = \frac{1}{44}(x)^{-43/44}$

$\left.\frac{df(x)}{dx}\right|_{x=1} = \frac{1}{44}(1)^{-43/44}$

$= \frac{1}{44}$

$L(x) = f(a) + f'(a)(x-a)$

$= 1 + \frac{1}{44}(x-1)$

$L(x) = \frac{1}{44}(x-1) + 1$

$\sqrt[44]{1.3}$ means $x = 1.3$

$\sqrt[44]{1.3} \approx L(1.3) = \frac{1}{44}(1.3-1) + 1$

$\approx \frac{1}{44}(0.3) + 1$

$\approx \frac{3}{440} + 1 \approx \frac{443}{440}$

4. Find the differential of the function, $y = e^{2x}\sin(3x)$.

$y = e^{2x}\sin(3x)$

$dy = \left(2e^{2x}\sin(3x) + 3\cos(3x)\,e^{2x}\right)dx$

5. The radius of a circle increases from an initial value of 10 cm by a change of 0.1 cm. Estimate the corresponding change in the circle's area. Then calculate the relative error and the percentage error in the estimate.

$A = \pi r^2$ $\quad r = 10\text{cm}$ $\quad dr = 0.1\text{cm}$

$dA = 2\pi r\,dr$

$dA = 2\pi(10)(0.1)$

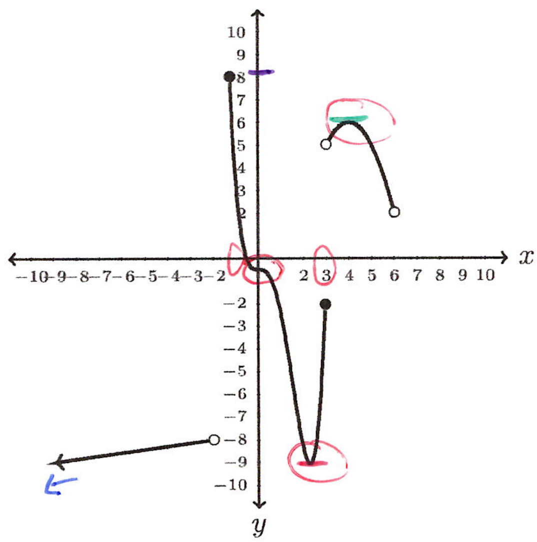
$dA = 2\pi \text{ cm}^2$

relative error $= \frac{dA}{A} = \frac{2\pi r\,dr}{\pi r^2} = \frac{2dr}{r} = \frac{2(0.1)}{10} = \frac{1}{50}$

percentage error = (relative error)(100%) $= \frac{1}{50} \cdot 100\% = 2\%$

4.1 Maximum and Minimum Values

6. Use the graph of $f(x)$ below to identify any values of x for which the function



(a) has an absolute minimum.

none

(b) has an absolute maximum.

8 (when $x = -1$)

(c) has a local minimum.

-9 (when $x = 2$)

(d) has a local maximum.

6 (when $x = 4$)

(e) has a critical number.

when $f(x)$ is defined but not continuous or any horizontal tangents

$x = -1, 0, 2, 3, 4$

7. Compute any absolute and local extrema for $f(x) = 6x^3 + 9x^2 - 36x + 8$ on the interval $[0, 2]$.

$f(x) = 6x^3 + 9x^2 - 36x + 8$ is continuous on $[0,2]$ as $f(x)$ is a polynomial

$f'(x) = 18x^2 + 18x - 36$

Critical numbers: $f'(x) = 0$ or $f'(x)$ does not exist in the domain of $f(x)$

$18x^2 + 18x - 36 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x + 2 = 0$ or $x - 1 = 0$

$x \neq -2$ $\quad x = 1$

$x = -2$ is not in the interval $[0,2]$

	x	f(x)
endpt	0	$f(0) = 6(0)^3 + 9(0)^2 - 36(0) + 18 = 18$
	1	$f(1) = 6(1)^3 + 9(1)^2 - 36(1) + 18 = -3$
endpt	2	$f(2) = 6(2)^3 + 9(2)^2 - 36(2) + 18 = 30$

- Absolute Minimum of -3
- Absolute Maximum of 30
- Local Minimum of -3
- Local Maximum of none

8. Does our answers to the previous question change if we adjust the interval to $(-\infty, \infty)$?

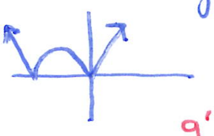
No absolute extrema when the interval is open

We also add the critical number $x = -2$

$f(-2) = 6(-2)^3 + 9(-2)^2 - 36(-2) + 18 = 78$

Now a local maximum of 78.

9. Compute any critical numbers and absolute and local extrema for $g(x) = |3x^2 + 9x|$.



$$g(x) = |3x^2+9x| = \begin{cases} (3x^2+9x) & \text{if } x \le -3 \\ -(3x^2+9x) & \text{if } -3 < x < 0 \\ (3x^2+9x) & \text{if } x \ge 0 \end{cases}$$

$3x^2+9x \ge 0$

$3x(x+3) \ge 0$

$x+3$	$-$	$+$	$+$
$3x$	$-$	$-$	$+$
	$(+)$	-3 $(-)$	0 $(+)$

$$g'(x) = \begin{cases} 6x+9 & \text{if } x < -3 \\ -6x-9 & \text{if } -3 < x < 0 \\ 6x+9 & \text{if } x > 0 \end{cases}$$

Critical Numbers:

$g'(x) = 0$

$6x+9=0$ or $-6x-9=0$

$6x=-9$ $\quad -6x = 9$

$x = -3/2$ $\quad x = -3/2$

$g'(x)$ does not exist

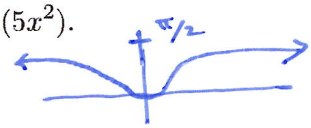
$x = -3, 0$

x	$f(x)$
-3	$\lvert 3(-3)^2+9(-3)\rvert = 0$
$-\frac{3}{2}$	$\lvert 3(\frac{-3}{2})^2+9(-\frac{3}{2})\rvert = \frac{27}{4}$
0	$\lvert 3(0)^2+9(0)\rvert = 0$

- Absolute Minimum of 0
- No Absolute Maximum
- Local Minimum of 0
- Local Maximum of $\frac{27}{4}$

10. Compute any critical numbers and absolute and local extrema for $h(x) = \arctan(5x^2)$.

$h(x) = \arctan(5x^2)$ is continuous on $(-\infty, \infty)$



$$\frac{dh(x)}{dx} = \frac{1}{1+(5x^2)^2} \cdot 10x = \frac{10x}{1+25x^4}$$

Critical Numbers:

$$\frac{10x}{1+25x^4} = 0$$

$10x = 0$

$x = 0$

x	$h(x)$
0	$\arctan(5(0)^2) = 0$

- No Local or Absolute Maximum even though the horizontal asymptote is $y = \pi/2$
- Local and Absolute Minimum of 0

11. Compute any critical numbers and absolute and local extrema for $f(x) = x(x-3)^{-1}$.

$f(x) = x(x-3)^{-1} = \frac{x}{x-3}$ Domain: $x \in (-\infty, 3) \cup (3, \infty)$

$f'(x) = 1(x-3)^{-1} - 1(x-3)^{-2} \cdot x$

$= \frac{(x-3) - x}{(x-3)^2}$

$= \frac{-3}{(x-3)^2}$

Critical Numbers:

$\frac{-3}{(x-3)^2} = 0$ No Critical numbers

- No local extrema
- No absolute extrema

12. Compute any absolute extrema for $g(x) = 4\cos^2(2x) + 4x$ on the interval $\left[0, \frac{\pi}{2}\right]$.

$g(x) = 4\cos^2(2x) + 4x$

$g(x)$ is continuous on the closed interval $[0, \pi/2]$

$\frac{dg(x)}{dx} = 8\cos(2x)(-\sin(2x)) \cdot 2 + 4$

$= -16\cos(2x)\sin(2x) + 4$

$= -8\sin(4x) + 4$

(Note: $2\sin(\theta)\cos(\theta) = \sin(2\theta)$)

Critical Numbers:

$-8\sin(4x) + 4 = 0$

$\sin(4x) = 1/2$

$4x = \frac{\pi}{6} + 2\pi k$ or $4x = \frac{5\pi}{6} + 2\pi k$

$x = \frac{\pi}{24} + \frac{\pi}{2}k$ $x = \frac{5\pi}{24} + \frac{\pi}{2}k$

$x = \frac{\pi}{24}, \frac{5\pi}{24}$

x	$g(x)$
0	$4\cos^2(0) + 4(0) = 4$
$\pi/24$	$4\cos^2(2(\pi/24)) + 4(\pi/24) \approx 4.256$
$\frac{5\pi}{24}$	$4\cos^2(2(\frac{5\pi}{24})) + 4(\frac{5\pi}{24}) \approx 2.886$
$\pi/2$	$4\cos^2(2(\pi/2)) + 4(\pi/2) \approx 10.283$

- Absolute Minimum of 2.886
- Absolute Maximum of 10.283