Math 140: Week-in-Review 12 (Chapters 6.1 & 6.2)

P r = 0.0525 t = 3 I = ?

1. If you invest $2,000 at a simple interest rate of 5.25% per year for 3 years, how much interest will you earn? How much money will be in the account after 3 years? (assuming no additional deposits or withdrawals are made) A = ?

(simple interest) $I = Prt$

$= (2000)(0.0525)(3)$

$= \$315$

$A = P + I$

$= 2000 + 315$

$= \$2315$

t = 4 I = 800 r = 0.04

2. After 4 years, a loan has accumulated $800 in simple interest. If the interest rate is 4% per year, what was the original principal amount? P = ?

(simple interest) $I = Prt \Rightarrow P = \frac{I}{rt}$

$= \frac{800}{(0.04)(4)}$

$= \$5000$

P I t=5

3. If you borrow $2,500 and need to pay $600 in simple interest after 5 years, what is the annual interest rate?

r = ?

(simple interest) $I = Prt$

$$r = \frac{I}{Pt}$$

$$= \frac{600}{(2500)(5)}$$

$$= 0.048$$

Annual interest rate = 4.8%

P r = 0.045 I = 900 t = ?

4. If you invest $2,000 at a simple interest rate of 4.5% per year and earn $900 in interest, how long did the investment last?

(simple interest) $I = Prt$

$$t = \frac{I}{Pr}$$

$$= \frac{900}{(2000)(0.045)}$$

$$= 10$$

t = 10 years

P r = 0.085 t = 5

5. You invest $3,000 at an annual interest rate of 8.5% with continuous compounding for 5 years. How much money will be in the account at the end of 5 years? How much interest will you earn at the end of the investment? A = ? I = ?

(continuous compounding)

$$A = Pe^{rt}$$

$$= 3000e^{(0.085)(5)}$$

$$= 4588.771259 \quad \text{(round to nearest cent)}$$

$A = \$4588.77$ → amount after 5 years

$$I = A - P = 4588.77 - 3000$$

$= \$1588.77$ → interest earned after 5 years

P = ? A t = 5 r = 0.074

6. An initial deposit grows to $15,000 after 5 years at a 7.4% annual interest rate with continuous compounding. How much was the initial deposit, and how much interest did you earn over the 5 years?

(continuous compounding)

$$A = Pe^{rt} \Rightarrow P = \frac{A}{e^{rt}} = \frac{15000}{e^{(0.074)(5)}}$$

$$= 10361.01496$$

$P = \$10361.01$ → initial deposit

$$I = A - P$$

$$= 15000 - 10361.01$$

$I = \$4638.99$ → interest earned after 5 years

PV I%

7. A loan of $5,000 is taken at an annual interest rate of 6.8%, compounded semi-annually. What is the total amount owed after 3 years?

FV=? t=3 m=2

$N = m*t = (2)(3)$ # of compounding periods
$I\% = 6.8$ (annual interest rate, %)
$PV = 5000$ (positive sign, receiving money)
$PMT = 0$ (no periodic payments)
$FV = ? \Rightarrow -6110.731996 \Rightarrow \boxed{\$6110.73}$ → amnt owed after 3years
$P/y = C/y = 2$ (m=2, semi-annual)
PMT: (END) / BEGIN (payment at the end)

PV I% m=4

8. You take out a loan of $12,000 at a 4.5% annual interest rate, compounded quarterly, and plan to pay it off in 5 years. How much total interest will you pay over the 5 years?

FV=0 t=5 I

$N = m*t = (4)(5)$
$I\% = 4.5$
$PV = 12000$
$PMT = 0$
$FV = ? \Rightarrow -15009.00625 \Rightarrow \boxed{\$15009.01}$ ← amnt owed after 5 years
$P/y = C/y = 4$
PMT: (END) / BEGIN

$I = FV - PV$ (total interest)
$= 15009.01 - 12000$
$= \boxed{\$3009.01}$

PV I% m=12

9. Thomas invests $15,000 at a 5.25% annual interest rate, compounded monthly. How long will it take for the investment to reach $18,000? t=?

FV

N = ? ⇒ 41.7646 (round up)

I% = 5.25

PV = −15000 (money out ⇒ negative)

PMT = 0

FV = 18000

P/y = C/y = 12

PMT: (END) / BEGIN

N = 42 months

N = m * t

$t = \frac{N}{m} = \frac{42 \text{ months}}{12 \text{ months/year}}$

t = 3.5 years

PV m = 52 t = 2

10. Amy invests $8,000 in a savings account that pays weekly interest. After 2 years, the investment grows to $9,600. What is the annual interest rate?

FV I% = ?

N = m * t = (52)(2)

I% = ? ⇒ 9.124

PV = −8000 (investment)

PMT = 0

FV = 9600

P/y = C/y = 52

PMT: (END) / BEGIN

The annual interest rate is 9.12%

11. Brian's credit card company charges 18% annual interest rate compounded monthly on any outstanding balance. Brian charged \$2,000 on his credit card and did not pay the bill for 6 months. How much will he owe after the 6 months?

N = 6
I% = 18
PV = 2000 (loan, positive)
PMT = 0
FV = ? ⇒ −2186.8865 (\$2186.89) ← amnt owed in 6 months
P/Y = C/Y = 12
PMT: END / BEGIN

12. A savings account offers an annual interest rate of 6%, compounded monthly. What is the effective annual interest rate for this account?

I% m=12
r_{eff}

$$r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1 \qquad r = 0.06,\ m = 12$$

use EFF! *EFF(I%, m)*

EFF(6, 12) = 6.16778

Effective annual interest rate = 6.17%

13. You invest in a bond that offers a 7% annual interest rate, compounded quarterly. What is the effective annual interest rate for this bond?

I% m=4
r_{eff}

EFF(7, 4) = 7.1859

Effective annual interest rate = 7.19%

14. Sofia is considering three different investment accounts, each offering a different annual interest rate and compounding frequency. Which account would be the best for her investment?

    - Account A: 6.0% annual interest, compounded quarterly. $m=4$
    - Account B: 5.8% annual interest, compounded monthly. $m=12$
    - Account C: 6.1% annual interest, compounded annually. $m=1$

investment → maximize r_{eff}

* The best account maximizes the annual percentage yield (APY) / effective annual interest rate, r_{eff} for an investment

Account A: $EFF(6.0, 4) = 6.1364\%$ → max

Account B: $EFF(5.8, 12) = 5.9567\%$

Account C: $EFF(6.1, 1) = 6.1\%$

* Account A is best since it maximizes annual percentage yield for her investment.

15. You currently have a balance of $3,000 on your credit card and are considering transferring the balance to a different card. Which card would be the better option for you, based on the interest rate and compounding frequency?

    ↳ minimizes r_{eff}

    - Card A: 18.1% annual interest, compounded monthly. $m=12$
    - Card B: 18.0% annual interest, compounded daily. $m=365$
    - Card C: 18.2% annual interest, compounded quarterly. $m=4$

    Which card will result in the lowest total amount of interest paid after 1 year?

* For a loan, the best account minimizes the annual percentage yield / effective annual interest rate

Card A: $EFF(18.1, 12) = 19.67966 \simeq 19.68\%$

Card B: $EFF(18.0, 365) = 19.7164 \simeq 19.72\%$

Card C: $EFF(18.2, 4) = 19.48\%$ minimum

Card C: is best since it minimizes the annual percentage yield

Interest paid after one year

Card A: $I = Pr_{eff} = 3000 * 0.1968 = \$590.40$

Card B: $I = Pr_{eff} = 3000 * 0.1972 = \$591.60$

Card C: $I = Pr_{eff} = 3000 * 0.1948 = \$584.40$ ✓ min interest owed