2024_Fall_WeekInReview_Math140_Week5

Sunday, September 22, 2024 3:24 PM

2024_Fall_WeekInRe...

2024 Fall Math 140 Week-In-Review

Week 5: Sections 3.1 - 3.3

Sections 3.1 - 3.3: The Method of Corners → I) Setup II) Construction III) Solving

Some Key Words and Terms: Variables, Objective Function, Constraints, Graphing a Linear Inequality, True/False Shading, Solution Set, Bounded/Unbounded, Corner Points, Solution to a Linear Programming Problem, Leftovers

Variables: Since these are business applications, our variables are generally what we're making/selling. Possibly also how much we invest.

☆ You must provide units when defining variables ☆
"in dollars" "the number made & sold"

☆ Don't forget a variable for the min/max ☆

Objective Function: the thing we're trying minimize or maximize

- always an equation, not an inequality
- do indicate whether we are maximizing or minimizing

☆ Be careful: something $R = 2x+3y$ is a good objective but

$R(x) = 2x+3y$ is not

one variable two variables

Constraints: restrictions that come from the context of the problem

- will be inequalities, not equation
- look for "totals" as a good indicator of a constraint
- also, look for "relations" between variables
  "at least twice as many ..."
- does the problem require "non-negativity" constraints
  $x \geq 0$, $y \geq 0$, etc...

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Graphing a Linear Inequality:

① Graph the equality if $2x+3y \le 6 \rightarrow 2x+3y=6$

* usually easiest to do by finding the x & y intercept

② Test a point not on the line

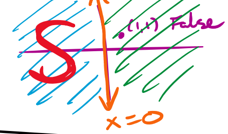
test $(0,0)$ in $2x+3y \le 6$ $(0,15)$ in $2x+3y \le 6$

③ Shade based on true or false result of the test

True/False Shading:

ultimately a matter of preference, BUT unless you use multiple colors, it is easier to "see" the solution set for false shading when there are lots of lines

Look at $x \le 0 \rightarrow$ draw $x=0$ as solid line (b/c "equal to")



test $(1,1)$: $1 \le 0$ false statement

/// true shading

/// false shading

Solution Set:

the set of all points that satisfy all the inequalities (constraints) simultaneously

Is bordered by the inequalities & the corner points give us the possible solutions

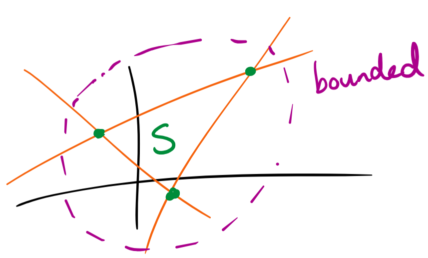
look @ $y \ge 0 \rightarrow y=0$

test $(1,1)$: $1 \ge 0$ true

Bounded/Unbounded:

Bounded solution set is completely enclosed

Unbounded solution set is not completely enclosed



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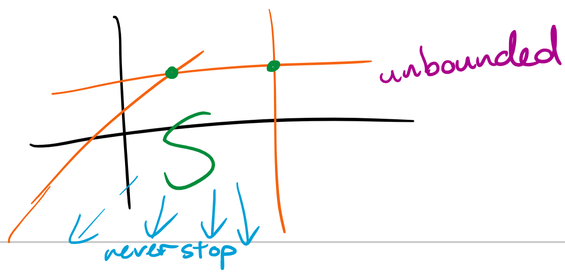


Corner Points: Are the $\boxed{\text{intersections}}$ of two boundary lines.

we need the equations to equal OR we can think of it $\boxed{\text{as solving a system}}$ of equations

generally, this is the way to go if both lines have x & y

Solution to a Linear Programming Problem:

Test all corner points in the objective function

- If only 1 point gives min or max, then that gives the solution
- If two points give min or max, then we need the line segment connecting them as the solution

Leftovers: only come into play with "real-world" scenarios, we plug in the (x,y) point from the solution into all the contraints that came from "totals"

Examples:

1. Setup the following linear programming problem. Do not solve.

   A company makes and sells two types of mini-fridges designed especially for dorm rooms: Space-Savers and Deep-Chills. In order to make the mini-fridges, the company requires coolant, metal, and plastic. Each Space-Saver requires 2 units of coolant, 3 units of metal, and 3 units of plastic. Each Deep-Chill requires 4 units of coolant, 5 units of metal, and 4 units of plastic. After receiving shipments of materials this week, the company has 270 units of coolant, 355 units of metal, and 314 units of plastic. If the company sells each Space-Saver for \$50 and each Deep-Chill for \$85, how many of each mini-fridge should the company make and sell in order to maximize their revenue for this week?

   Variables:

   x = the number of Space-Savers made & sold
   y = the number of Deep-Chills made & sold
   R = the revenue for this week in dollars

   Revenue = (selling price) $\cdot$ (quantity product)

   Objective: Maximize $R = 50x + 85y$

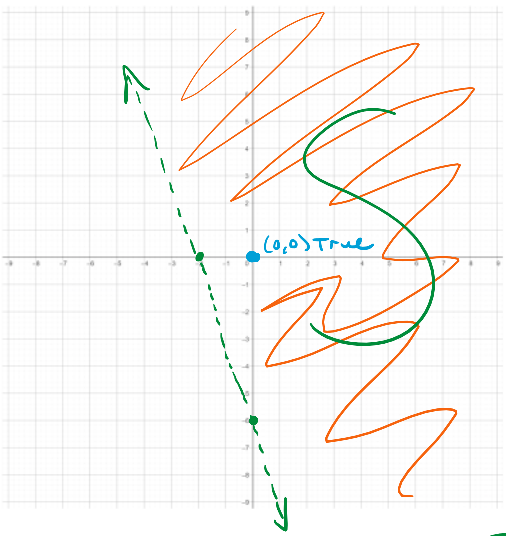
   Constraints:

   (coolant) $2x + 4y \leq 270$
   (metal) $3x + 5y \leq 355$
   (plastic) $3x + 4y \leq 314$
   ☆ $x \geq 0, y \geq 0$ (b/c can't have negative fridges)

2. Graph the following inequality and use $\boxed{\text{True Shading}}$ to determine the solution set.

$$6x + 2y > -12$$

no "equal to" → dashed



$6x + 2y = -12$

$\underline{x=0}$: $2y = -12$
$y = -6$
$(0,-6)$

$\underline{y=0}$: $6x = -12$
$x = -2$
$(-2,0)$

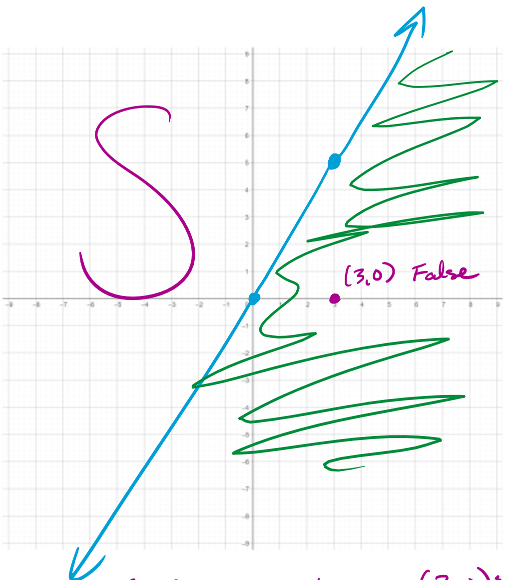
$\underline{\text{test } (0,0)}$: (if I can)

$6(0) + 2(0) > -12$
$0 > -12$ True

3. Graph the following inequality and use $\boxed{\text{False Shading}}$ to determine the solution set.

$$5x - 3y \le 0$$

→ solid



$5x - 3y = 0$

$\underline{x=0}$: $-3y = 0$
$y = 0$
$(0,0)$

$\underline{y=0}$: $5x = 0$
$x = 0$
$(0,0)$

hmm.... choose either

$x = 3$ or $y = 5$
coefficients of the other variable

$\underline{x=3}$: $5(3) - 3y = 0$
$15 - 3y = 0$
$15 = 3y$
$5 = y$
$(3,5)$

$\underline{y=5}$: $5x - 3(5) = 0$
$5x - 15 = 0$
$5x = 15$
$x = 3$
$(3,5)$

CAN'T test (0,0), so choose (3,0):
$5(3) - 3(0) \le 0$
$15 \le 0$ False

4. Use the Method of Corners to determine the solution, if it exists, for the following linear programming problem.

Minimize: $C = 5x + 6y$

Subject to: $2x + 2y \le 14$
$-3x + 2y \le 9$
$3x + 7y \ge 21$
$x \ge 0,\ y \ge 0$

5 lines

★ False Shading

Line 1: $2x+2y=14$
$x=0 \to 2y=14$
$(0,7)$ $y=7$
$y=0 \to 2x=14$
$x=7$
$(7,0)$
test $(0,0)$: $2(0)+2(0)\le 14$
True

Line 2: $-3x+2y=9$
$x=0 \to 2y=9$
$(0,9/2)$ $y=\frac{9}{2}=4.5$
$y=0 \to -3x=9$
$(-3,0)$ $x=-3$
test $(0,0)$: $-3(0)+2(0)\le 9$
True

Solution:

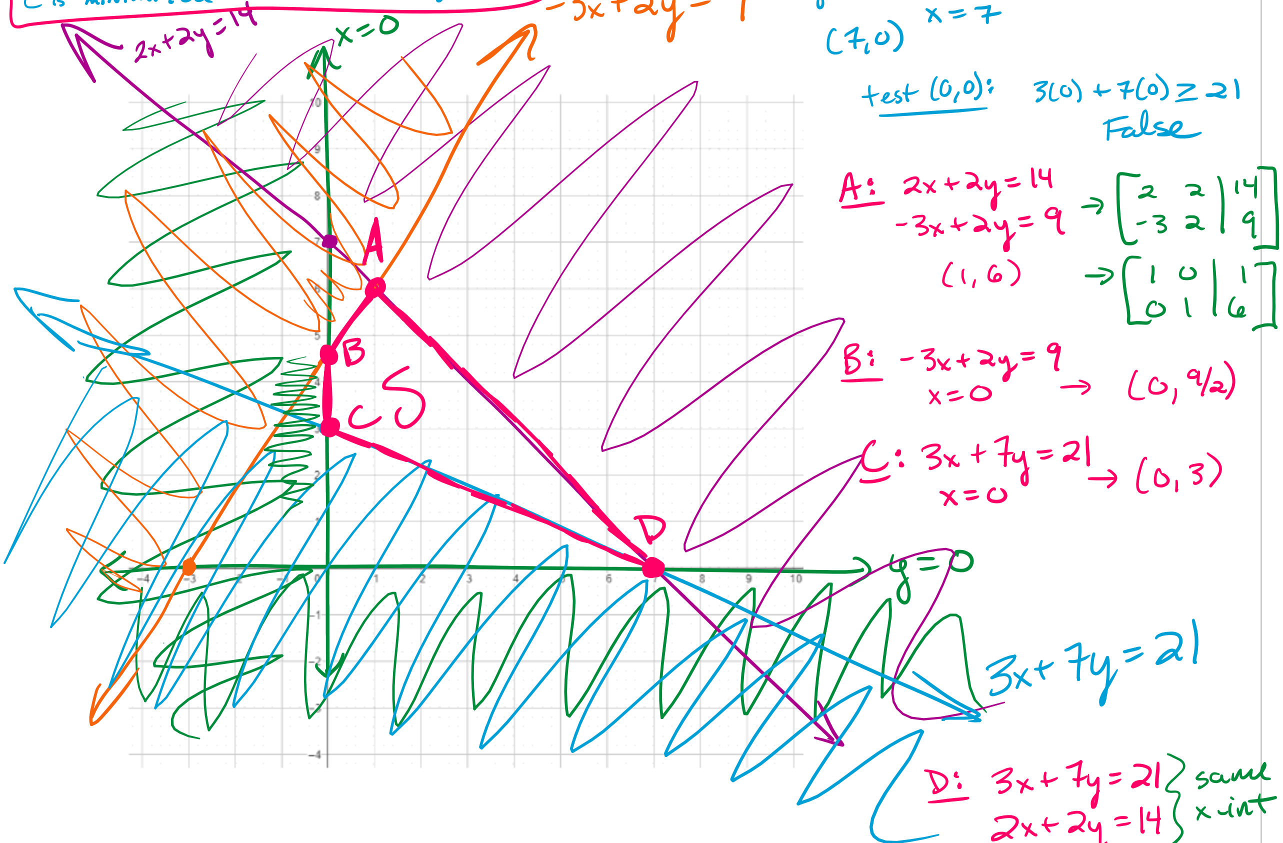
(x,y)	$C=5x+6y$ (min)
$(1,6)$	$C=5(1)+6(6)=41$
$(0,9/2)$	$C=5(0)+6(9/2)=27$
$(0,3)$	$C=5(0)+6(3)=18$
$(7,0)$	$C=5(7)+6(0)=35$

$\boxed{C \text{ is minimized at } 18 \text{ when } (x,y)=(0,3)}$

Line 3: $3x+7y=21$
$x=0 \to 7y=21$
$(0,3)$ $y=3$
$y=0 \to 3x=21$
$(7,0)$ $x=7$
test $(0,0)$: $3(0)+7(0)\ge 21$
False

Line 4: $x=0$

Line 5: $y=0$



A: $2x+2y=14$
$-3x+2y=9$
$\to \left[\begin{array}{cc|c} 2 & 2 & 14 \\ -3 & 2 & 9 \end{array}\right]$
$(1,6)$ $\to \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 6 \end{array}\right]$

B: $-3x+2y=9$
$x=0$ $\to (0,9/2)$

C: $3x+7y=21$
$x=0$ $\to (0,3)$

D: $3x+7y=21$
$2x+2y=14$ } same x-int
$(7,0)$

5. Use the Method of Corners to determine the solution, if it exists, for the following linear programming problem.

Maximize: $Z = \frac{2}{3}x + \frac{5}{4}y$

Subject to: $4x + 5y \geq 20$

$x - 2y \leq 5$

$0 \leq x \leq 7$

$0 \leq x$, $x \leq 7$

Line 3: $x = 0$

Line 4: $x = 7$

Line 1: $4x + 5y = 20$

$x = 0 \rightarrow 5y = 20$

$(0,4)$ $y = 4$

$y = 0 \rightarrow 4x = 20$

$x = 5$

$(5,0)$

test $(0,0)$:

$4(0) + 5(0) \geq 20$

False

Line 2: $x - 2y = 5$

$x = 0 \rightarrow -2y = 5$

$(0, -5/2)$ $y = -5/2$

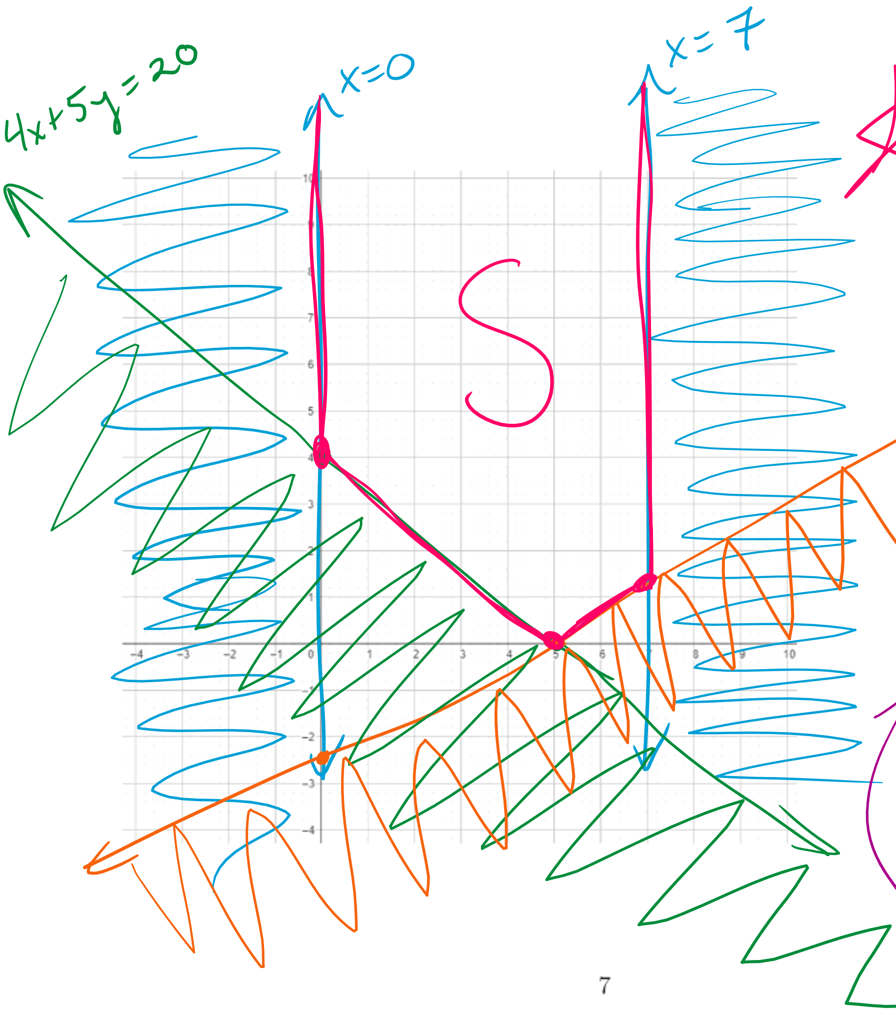
$y = 0 \rightarrow x = 5$

$(5,0)$

test $(0,0)$:

$(0) - 2(0) \leq 5$

True



S is unbounded in quadrant 1

and we want to maximize ax + by, then "no solution"

both positive

corner points

6. ~~For the situation given in Example 1,~~ the corner points of the feasible region are (0, 0), (0, 68), (40, 48), (60, 36), and (108, 0). Determine the solution and any leftovers. Be sure to express your answers in the context of the situation.

Maximize $R = 50x + 85y$

(x,y)	$R = 50x + 85y$
$(0,0)$	$R = 0$
$(0,68)$	$R = 5780$
$(40,48)$	$R = 6080$

Maximize $R = 50x + 85y$

(0,0)	R = 0
(0,68)	R = 5780
$\boxed{(40,48)}$	$\boxed{R = 6080}$
(60,36)	R = 6060
(108,0)	R = 5400

(coolant) $2x + 4y \le 270$

(metal) $3x + 5y \le 355$

(plastic) $3x + 4y \le 314$

☆ $x \ge 0, y \ge 0$ (b/c can't have negative fridges)

Answer Part 1

the revenue for the week is maximized @ \$6,080 when 40 Space-Savers & 48 Deep-Chills are made and sold

leftovers:

$2(40) + 4(48) \le 270$

$272 \le 270$ (shouldn't happen, I must have a typo)

assume: $2x + 4y \le 272$

$272 \le 272 \rightarrow$ no leftovers of coolant

$272 - 272 = 0$

$3(40) + 5(48) \le 355$

$360 \le 355$ ← again, I must have a typo

$355 - 312 = 43$ leftover units of metal

$3(40) + 4(48) \le 314$

$312 \le 314 \rightarrow 314 - 312 =$ 2 leftover units of plastic

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