1. Let $f(x,y) = \sin(xy) + \pi$. Find $f(1, \pi/2)$. $= \sin(1\cdot\frac{\pi}{2}) + \pi = \sin(\frac{\pi}{2}) + \pi = \boxed{1+\pi}$

2. Let $f(x,y,z) = y + xz$. Find $f(-3, 2, 1)$. $= 2 + (-3)(1) = 2 - 3 = -1$

3. Find the domain and range of the functions:

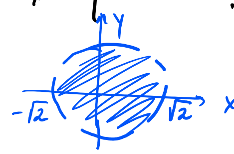
   (a) $f(x,y) = \ln(2 - x^2 - y^2)$;

   The domain of $f(x,y)$ is a subset in $\mathbb{R}^2$

   The range is an interval

   $\ln(2-x^2-y^2) \rightarrow 2-x^2-y^2>0$

   Domain $\{(x,y) \text{ in } \mathbb{R}^2 \mid x^2+y^2<2\}$

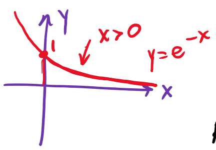
   

   Range $\boxed{(-\infty, \ln 2)}$

   (b) $f(x,y,z) = e^{-\frac{1}{x^2+y^2+z^2}}$

   $\frac{1}{x^2+y^2+z^2}$ is always positive.

   $x^2+y^2+z^2 \neq 0$

   $x^2+y^2+z^2=0$ if and only if $x=y=z=0$

   $D = \{(x,y,z) \text{ in } \mathbb{R}^3 \mid x^2+y^2+z^2 \neq 0\}$ (all the points in $\mathbb{R}^3$ except the origin).

   Range : $\boxed{(0,1)}$

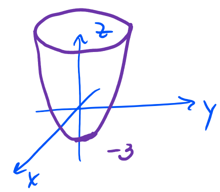
4. Sketch the graphs of the functions:

   (a) $f(x,y) = x^2 + y^2 - 3$

   the graph of $f(x,y)$ is the surface

   $z = x^2+y^2-3$

   $z+3 = x^2+y^2$

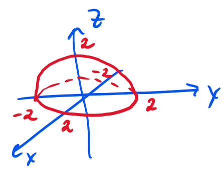
   

   (b) $f(x,y) = \sqrt{4 - x^2 - y^2}$

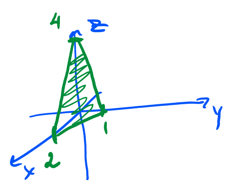
   $(z)^2 = \left(\sqrt{4-x^2-y^2}\right)^2 \Rightarrow$

   $z^2 = 4-x^2-y^2$

   $x^2+y^2+z^2=4$ – sphere. (the top half)

   (c) $f(x,y) = -2x - 4y + 4$

   $z = -2x-4y+4$ or $2x+4y+z=4$

   a plane

   intercepts: x-axis (plug $y=z=0$) $2x=4$, $x=2$ the point $(2,0,0)$

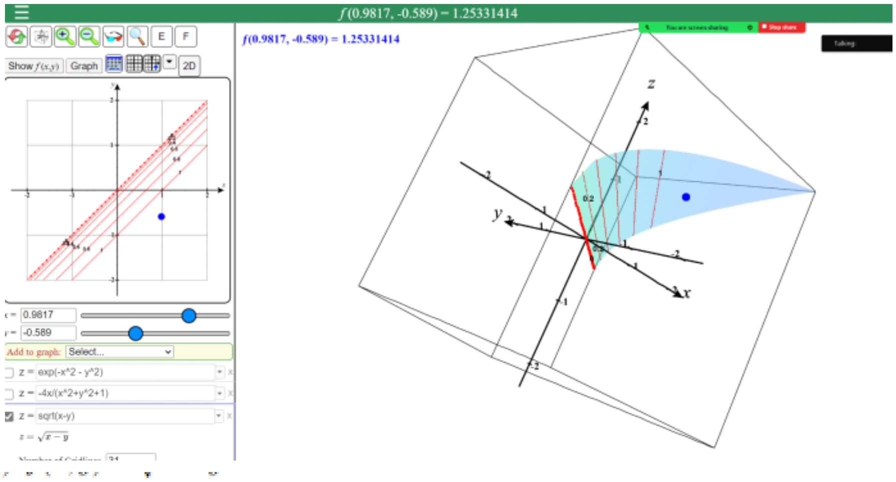
   y-axis (plug $x=z=0$) $4y=4$, $y=1$ the point $(0,1,0)$

   z-axis (plug $x=y=0$) $z=4$ the point $(0,0,4)$

5. Classify all level curves of the functions $k^2 = (\sqrt{x-y})^2 \Rightarrow k^2 = x - y$ lines.

level curves are

(a) $f(x, y) = \sqrt{x - y}$



(b) $f(x, y) = e^{-x^2 - y^2}$

level curves are $\ln(k) = \ln(e^{-x^2-y^2}) \Rightarrow -1(-x^2-y^2) = -(\ln k)$

The equation would make sence only if $\ln k < 0$ or $0 < k < 1$

$\ln k < 0$ then $\ln \frac{1}{k} > 0$

$x^2 + y^2 = \ln \frac{1}{k}$ circles centered at the origin.

6. Describe level surfaces of the function $f(x, y, z) = -x^2 - y^2 - z^2$

level surfaces : $k = -x^2 - y^2 - z^2$

$x^2 + y^2 + z^2 = -k$, and $k < 0$

spheres, centered @ origin of radius $\sqrt{-k}$.

7. Find the first partial derivatives of the functions

(a) $f(x,y) = x^4 + 5xy^3$

$\frac{\partial f}{\partial x} = 4x^3 + 5y^3$

$\frac{\partial f}{\partial y} = 0 + 5x(3y^2) = 15xy^2$

(b) $f(x,y) = y^2\cos(xy)$ ← the derivative of xy with respect to x

$\frac{\partial f}{\partial x} = y^2(-\sin xy)(xy)'_x = -y^2\sin(xy)(y) = -y^3\sin(xy)$

$\frac{\partial f}{\partial y} = 2y\cos(xy) + y^2[-\sin(xy)](xy)'_y = 2y\cos(xy) - xy^2\sin(xy)$

(c) $f(x,y) = x^y$

$[x^n]'$ $\frac{\partial f}{\partial x} = y\,x^{y-1}$

$[a^y]'$ $\frac{\partial f}{\partial y} = x^y \ln x$

(d) $f(x,y,z) = xy^2e^{-xz}$

$\frac{\partial f}{\partial x} = y^2e^{-xz} + xy^2e^{-xz}(-xz)'_x = y^2e^{-xz} - xy^2z\,e^{-xz}$

$\frac{\partial f}{\partial y} = 2xy\,e^{-xz}$

$\frac{\partial f}{\partial z} = xy^2e^{-xz}(-xz)'_z = -x^2y^2e^{-xz}$

(e) $f(u,v,w) = \sqrt{u^4 + v^2\cos w} = (u^4 + v^2\cos w)^{1/2}$

$\frac{\partial f}{\partial u} = \frac{1}{2}(u^4+v^2\cos w)^{-1/2}(u^4+v^2\cos w)'_u = \frac{1}{2}(u^4+v^2\cos w)^{-1/2}(4u^3)$

$\frac{\partial f}{\partial v} = \frac{1}{2}(u^4+v^2\cos w)^{-1/2}(u^4+v^2\cos w)'_v = \frac{1}{2}(u^4+v^2\cos w)^{-1/2}(2v\cos w)$

$\frac{\partial f}{\partial w} = \frac{1}{2}(u^4+v^2\cos w)^{-1/2}(u^4+v^2\cos w)'_w = \frac{1}{2}(u^4+v^2\cos w)^{-1/2}(-v^2\sin w)$

8. Find all second-order derivatives of the function

(a) $z = xe^{-2y}$

First-order.

$$\frac{\partial z}{\partial x} = e^{-2y} \qquad \frac{\partial z}{\partial y} = xe^{-2y}(-2) = -2xe^{-2y}$$

second order.

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(e^{-2y}\right) = 0 \qquad \frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(-2xe^{-2y}\right) = -2xe^{-2y}(-2) = 4xe^{-2y}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y}\left(e^{-2y}\right) = -2e^{-2y} \qquad \frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial x}\left(-2xe^{-2y}\right) = -2e^{-2y}$$

do match!

(b) $v = r\cos(x + 2t)$

First-order

$$\frac{\partial v}{\partial r} = \cos(x+2t) \qquad \frac{\partial v}{\partial x} = r(-\sin(x+2t)) = -r\sin(x+2t) \qquad \frac{\partial v}{\partial t} = -r\sin(x+2t)(2) = -2r\sin(x+2t)$$

2nd order

$$\frac{\partial^2 v}{\partial r^2} = \frac{\partial}{\partial r}\left(\cos(x+2t)\right) = 0$$

$$\frac{\partial^2 v}{\partial r \partial x} = \frac{\partial}{\partial x}\left(\cos(x+2t)\right) = -\sin(x+2t)$$

$$\frac{\partial^2 v}{\partial r \partial t} = \frac{\partial}{\partial t}\left(\cos(x+2t)\right) = -2\sin(x+2t)$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial}{\partial x}\left(-r\sin(x+2t)\right) = -r\cos(x+2t)$$

$$\frac{\partial^2 v}{\partial r \partial x} = -\sin(x+2t)$$

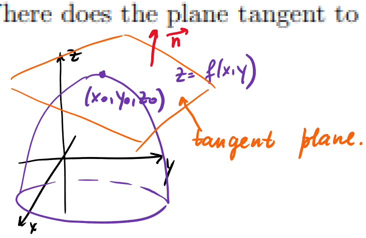
$$\frac{\partial^2 v}{\partial x \partial t} = \frac{\partial}{\partial t}\left(-r\sin(x+2t)\right) = -r\cos(x+2t)(2)$$

$$\frac{\partial^2 v}{\partial t^2} = \frac{\partial}{\partial t}\left(-2r\sin(x+2t)\right) = -2r\cos(x+2t)(2)$$

$$\frac{\partial^2 v}{\partial t \partial r} = -2\sin(x+2t)$$

$$\frac{\partial^2 v}{\partial t \partial x} = -2r\cos(x+2t)$$

9. Where does the plane tangent to the surface $z = e^{x-y}$ at $(1, 1, 1)$ meet the z-axis?



$\vec{n} = \langle z_x, z_y, -1\rangle$

Eqn. $\boxed{z - z_0 = z_x(x_0, y_0)(x - x_0) + z_y(x_0, y_0)(y - y_0)}$

$z = e^{x-y}$ @ $(1,1,1)$ $\Rightarrow x_0 = y_0 = z_0 = 1$

$$z_x = e^{x-y} \qquad z_x(1,1) = e^0 = 1$$

$$z_y = e^{x-y}(-1) = -e^{x-y} \qquad z_y(1,1) = -e^0 = -1$$

Tangent plane: $\boxed{z - 1 = 1(x-1) - 1(y-1)}$

$z - 1 = x - y$ or $\boxed{z - x + y = 1}$

10. Show that the surfaces given by $f(x, y) = x^2 + y^2$ and $g(x, y) = -x^2 - y^2 + xy^3$ have the same tangent plane at $(0, 0)$.

at $(0,0)$

$f(0,0) = 0 \Rightarrow x_0 = y_0 = z_0$

$$f_x = 2x \qquad f_x(0,0) = 0$$

$$f_y = 2y \qquad f_y(0,0) = 0$$

tangent plane $z - 0 = 0(x-0) + 0(y-0)$

$\boxed{z=0}$

$g(0,0) = 0$

$x_0 = y_0 = z_0 = 0$

$$g_x = -2x + y^3 \qquad g_x(0,0) = 0$$

$$g_y = -2y + 3xy^2 \qquad g_y(0,0) = 0$$

tangent plane : $z - 0 = 0(x-0) + 0(y-0)$

$\boxed{z=0}$

same

11. Find the differential of the function

$$f(x,y,z)=\sqrt{x^2+y^2+z^2} = (x^2+y^2+z^2)^{1/2}$$

$df = f_x\,dx + f_y\,dy + f_z\,dz$

$f_x = \frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-1/2}(\cancel{2}x) = x(x^2+y^2+z^2)^{-1/2}$

$f_y = \frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2y) = y(x^2+y^2+z^2)^{-1/2}$

$f_z = \frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2z) = z(x^2+y^2+z^2)^{-1/2}$

$$\boxed{df = x(x^2+y^2+z^2)^{-1/2}dx + y(x^2+y^2+z^2)^{-1/2}dy + z(x^2+y^2+z^2)^{-1/2}dz}$$

$$\boxed{df = (x\,dx + y\,dy + z\,dx)(x^2+y^2+z^2)^{-1/2}}$$

12. Use differentials to estimate

$$\sqrt{(\underset{x}{4.01})^2+(\underset{y}{3.98})^2+(\underset{z}{2.02})^2}$$

$(a,b,c) = (4,4,2)$

$f(x,y,z) = \sqrt{x^2+y^2+z^2}$

$f(a+\Delta x, b+\Delta y, c+\Delta z) \approx f(a,b,c) + f_x(a,b,c)\Delta x + f_y(a,b,c)\Delta y + f_z(a,b,c)\Delta z$

$\Delta x = 4.01-4 = 0.01$

$\Delta y = 3.98-4 = -0.02$

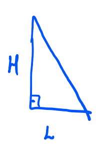
$\Delta z = 2.02-2 = 0.02$

$f(x,y,z)=\sqrt{x^2+y^2+z^2}$	$f(4,4,2)=\sqrt{16+16+4}=\sqrt{36}=6$
$f_x = x(x^2+y^2+z^2)^{-1/2}$	$f_x(4,4,2) = \frac{4}{6} = \frac{2}{3}$
$f_y = y(x^2+y^2+z^2)^{-1/2}$	$f_y(4,4,2) = \frac{4}{6} = \frac{2}{3}$
$f_z = z(x^2+y^2+z^2)^{-1/2}$	$f_z(4,4,2) = \frac{2}{6} = \frac{1}{3}$

$\sqrt{(4.01)^2+(3.98)^2+(2.02)^2} \approx f(4,4,2) + f_x(4,4,2)\Delta x + f_y(4,4,2)\Delta y + f_z(4,4,2)\Delta z$

$= 6 + \frac{2}{3}(0.01) + \frac{2}{3}(-0.02) + \frac{1}{3}(0.02) = 6$

13. The two legs of a right triangle are measured as 5 m and 12 m respectively, with a possible error in measurement of at most 0.2 cm in each. Use differentials to estimate the maximum error in the calculated value of the area of the triangle.



$A = \frac{1}{2}L(H) = \frac{1}{2}LH$

$L = 5\,m = 500\,cm, \quad H = 12\,m = 1200\,cm$

$dA = \frac{\partial A}{\partial L}dL + \frac{\partial A}{\partial H}dH$

$dL = dH = 0.2$

Find $\boxed{dA(500, 1200) = ?}$

$\frac{\partial A}{\partial L} = \frac{1}{2}H, \quad \frac{\partial A}{\partial H} = \frac{1}{2}L$

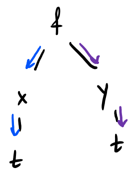
$dA = \frac{1}{2}H\,dL + \frac{1}{2}L\,dH$

$dA(500, 1200) = \frac{1}{2}(500)(0.2) + \frac{1}{2}(1200)(0.2)$

$= 50 + 120 = 170\ (cm^2)$

Find $\frac{df}{dt}$, when $f(x,y) = x\cos(xy)$

and $x = \sqrt{t}$, $y = \ln t$



$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$= \left[\cos xy + x(-\sin xy)\,y\right]\frac{1}{2\sqrt{t}} - x\sin(xy)\,x\,\frac{1}{t}$$

$$= \left[\cos xy - xy\sin(xy)\right]\frac{1}{2\sqrt{t}} - x^2\sin(xy)\,\frac{1}{t}$$

15. Let

$$w = \cos xy + y\cos x,$$

where

$$x = e^{-t} + 3s,\ y = 5e^{2t} - \sqrt{s}$$

Find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$.

∂t ∂s

16. If

$$yz^4 + xz^3 = e^{xyz}$$

find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.