



MATH 140: WEEK-IN-REVIEW 5 (3.4 & 4.1)

1. Determine if the following linear programming problems are standard maximization problems. If they are, transform the constraints of the linear programming problems to linear equations with slack variables and write down the corresponding simplex tableau.

(a)

$$\begin{aligned} \text{Objective: Maximize } P &= 2x + 5y \\ \text{Subject to: } 5y &\leq -2x + 20 \\ 3 - 4y &\leq x \\ x \geq 0, y &\geq 0. \end{aligned}$$

(b)

$$\begin{aligned} \text{Objective: Maximize } P &= 7x + 3y + 5z \\ \text{Subject to: } x + y + z &\leq 20 \\ -3x + 4y &\geq -18 + 5z \\ 2x &\leq -3z + 30 \\ x \geq 0, y \geq 0, z &\geq 0. \end{aligned}$$



2. For the following simplex tableau, identify the pivot column, pivot row, and the pivot element to be used in the next iteration of the Simplex Method. Show supporting work.

$$(a) \left[\begin{array}{cccccc|c} x & y & s_1 & s_2 & s_3 & P & \text{constant} \\ 3 & 1 & 1 & 0 & 0 & 0 & 24 \\ 2 & 0 & 0 & 1 & 0 & 0 & 18 \\ 1 & 3 & 0 & 0 & 1 & 0 & 24 \\ \hline -4 & -6 & 0 & 0 & 0 & 1 & 24 \end{array} \right]$$

$$(b) \left[\begin{array}{cccccc|c} x & y & z & s_1 & s_2 & s_3 & P & \text{constant} \\ -1/2 & 0 & 1/4 & 1 & -1/4 & 0 & 0 & 17/2 \\ 1/2 & 1 & 3/4 & 0 & 1/4 & 0 & 0 & 19/2 \\ 2 & 0 & 3 & 0 & 0 & 1 & 0 & 30 \\ \hline -1 & 0 & -1/2 & 6 & 3/2 & 0 & 1 & 63 \end{array} \right]$$



3. Read the values of each variable from the following tableau below and classify each variable as basic or non-basic. What corner point does each tableau correspond to, if the problem had been solved using the Method of Corners? Determine if the solution is optimal and explain why.

$$(a) \left[\begin{array}{cccccc|c} x & y & s_1 & s_2 & s_3 & P & \text{constant} \\ 3 & 0 & 1 & 0 & -1 & 0 & 3 \\ -7 & 0 & 0 & 1 & 3 & 0 & 5 \\ -2 & 1 & 0 & 0 & 1 & 0 & 2 \\ \hline -4 & 0 & 0 & 0 & 3 & 1 & 6 \end{array} \right]$$

$$(b) \left[\begin{array}{cccccc|c} x & y & s_1 & s_2 & s_3 & P & \text{constant} \\ 0 & 0 & 7/3 & 1 & 2/3 & 0 & 4 \\ 1 & 0 & 0 & -1/7 & -3/7 & 0 & 3/7 \\ 0 & 1 & 4/7 & -2/7 & 1/7 & 0 & 20/7 \\ \hline 0 & 0 & 0 & -4/7 & 9/7 & 1 & 54/7 \end{array} \right]$$

$$(c) \left[\begin{array}{cccccc|c} x & y & s_1 & s_2 & s_3 & P & \text{constant} \\ 0 & 0 & 7/3 & 1 & 2/3 & 0 & 4 \\ 1 & 0 & 1/3 & 0 & -1/3 & 0 & 1 \\ 0 & 1 & 2/3 & 0 & 1/3 & 0 & 4 \\ \hline 0 & 0 & 4/3 & 0 & 5/3 & 1 & 10 \end{array} \right]$$



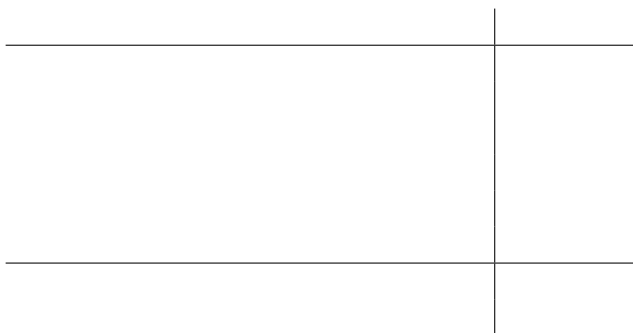
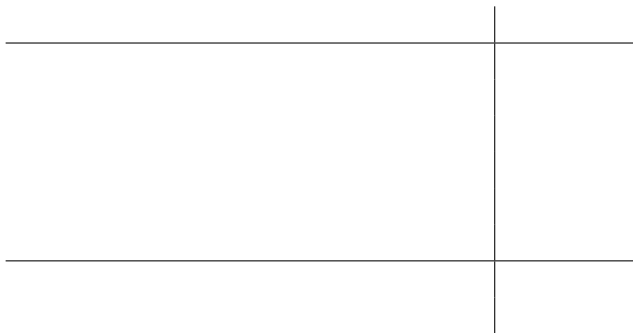
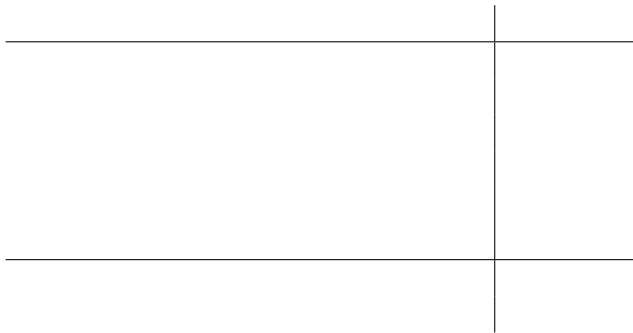
4. Solve the following linear programming problem using the Simplex Method.

Objective: Maximize $P = 2x + 3y$

Subject to: $5x + 4y \leq 32$

$x + 2 \leq 10$

$x \geq 0, y \geq 0.$



**5. Solve the following linear programming problem using the Simplex Method.**

A farmer owns a 12 acre piece of land and cultivates greens, carrots, and cucumbers. Each acre of crop that is planted has certain requirements for labor and capital given in the table below.

	Capital (\$)	Labor (hrs)	Profit (\$)
Greens (per acre)	36	6	40
Carrots (per acre)	24	6	30
Cucumbers (per acre)	18	2	20

If the farmer has \$360 available for capital and 48 hours of labor for cultivating these crops, how many acres of each crop should be planted to maximize profit? What is the maximum profit? Is there any leftover land, capital, or labor-hours?





6. Consider an experiment where you select a letter at random from the word “chance”.

(a) State the sample space for this experiment.

(b) State all simple events for the experiment.

(c) Give an example of an impossible event for the experiment.

(d) State the total number of possible events.

(e) Write the event, V , “a vowel is selected”.



7. You have two bowls containing pingpong balls. One bowl contains 4 identical blue balls labeled 1 to 4. The other bowl contains 5 identical green balls labeled 1 to 5. A blue ball and a green ball are drawn at random, one from each bowl, noting the numbers.

(a) What is the sample space for this experiment?

(b) Write the event, E , that a ball labeled 2 is drawn.

(c) Write the event, F , that a sum of 4 or a sum of 5 is drawn.

(d) Write the event, G , that a sum of 7 is drawn.



(e) Write the event, H , that the number on the green ball is a 3.

(f) Verbally describe the event H^C

(g) Are the events G and H mutually exclusive? Explain why or why not.



8. A fair coin is tossed three times, noting the side landing up on each toss.

(a) State the sample space for this experiment.

(b) State the event, E , that the second toss is “tails”.

(c) State the event, F , that all tosses land on “heads”.

(d) State the event $E \cup F$.

(e) State the event $E \cap F$.

(f) Are events E and F mutually exclusive? Explain why or why not.



9. A card is drawn at random from a standard 52-card deck, noting the suit drawn, and a ball is drawn at random from a bowl containing three balls labeled 1 to 3.

(a) State the number of outcomes in the sample space, S , of this experiment.

(b) State the number of simple events of the experiment. Write one.

(c) State the event, E , that a ball labeled 2 is drawn from the bowl.

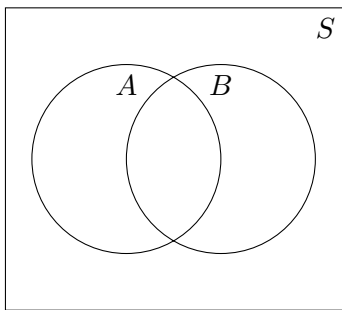
(d) State the event, F , that a red card is drawn and a number less than 3 is drawn.

(e) State the event, G , that a Spade is drawn or a number greater than 1 is drawn from the bowl.

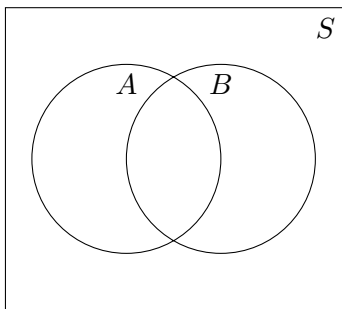


10. Shade the region on the Venn diagram representing the events given below.

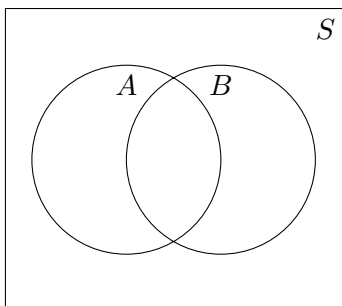
a. $B^C \cap A$



b. $(A \cap B)^C$



c. $(A^C \cup B)^C$





11. A letter is selected at random from the word “chance,” noting the letter, and then a spinner divided into three equal regions (maroon, white, and gray) is spun, noting the color. If the spinner lands on a dividing line, it is re-spun. Consider the following events.

W := the event “a consonant is drawn”.

X := the event, “an h is drawn”.

Y := the event “the color spun is maroon”.

- (a) Verbally describe the outcomes in each of the following events.

(i) W^C

(ii) $W \cap Y^C$

(iii) $Y^C \cup (W \cap X)$

- (b) Write each of the following events using symbolic notation.

(i) The event “a vowel is drawn and the color spun is maroon”.

(ii) The event “a consonant other than “h” is drawn, and a color other than maroon is spun”.