Math 151
Week-In-Review 13
5.1, 5.2, 5.3
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Problem Statements

1. Suppose the area under the curve of a function is estimated using a Riemann Sum with n equal-width rectangles.

   (a) Determine a general formula for the sample points, x_i^*, using right endpoints.

   (b) Determine a general formula for the sample points, x_i^*, using left endpoints.

   (c) Determine a general formula for the sample points, x_i^*, using midpoints.

   (d) Which of these formulas seems to be the most simple to work with?

2. Use this information to answer part (b) of the following.

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} n = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} n^2 = \frac{n(n+1)(2n+1)}{6}$$

(a) Set up a limit to evaluate the exact area under the curve of $f(x) = -2 + 2x$ on the interval from $x = 1$ to $x = 4$.

(b) Use the above information to evaluate the limit.

(c) Write the limit as an equivalent definite integral.

(d) Evaluate the integral using geometry.

3. Use this information to answer part (b) of the following.

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} n = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} n^2 = \frac{n(n+1)(2n+1)}{6}$$

(a) Set up a limit to evaluate the exact area under the curve of $f(x) = x^2$ on the interval $[0, 4]$.

(b) Use the above information to evaluate the limit.

(c) Write the limit as an equivalent definite integral.

(d) Can you evaluate the integral using geometry?

4. Evaluate the following integrals geometrically.

(a) $\int_{-2}^{4} 3x \, dx$

(b) $\int_{0}^{4} \sqrt{16 - x^2} \, dx$

(c) $\int_{0}^{2} 5 - \sqrt{4 - x^2} \, dx$

5. Suppose $\int_{-1}^{3} f(x)\ dx = 5$, $\int_{-1}^{3} g(x)\ dx = 3$ and $\int_{-1}^{3} h(x)\ dx = -7$. Evaluate the following.

(a) $\int_{-1}^{3} 4f(x)\ dx + \int_{3}^{-1} (g(x) - h(x))\ dx$

(b) $\int_{-4}^{0} f(x)\ dx + \int_{-1}^{-4} f(x)\ dx - \int_{3}^{0} f(x)\ dx + \int_{9}^{9} f(x)\ dx + \int_{2}^{5} 3\ dx$

6. Evaluate the integrals, if possible.

(a) $\int_0^4 x^2 \, dx$

(b) $\int_1^9 \sqrt{x} \, dx$

(c) $\int_{-3}^5 |9 - x^2| \, dx$

(d) $\int_0^1 (x^e + e^x + 3^x)\ dx$

(e) $\int_0^{\pi/3} \sec(x)\tan(x)\ dx$

(f) $\int_0^{\pi} \sec^2(x)\ dx$

7. Find the derivative of the following functions.

   (a) $g(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right)\, dt$

   (b) $h(w) = \int_w^3 e^{x^2}\, dx$

   (c) $F(y) = \int_{-1}^y \frac{1}{1+t^3}\, dt$

8. Find the derivative of the following functions.

(a) $g(x) = \int_0^{x^2} t\cos(t)\ dt$

(b) $F(r) = \int_r^{3r} \arctan(t)\ dt$

(c) $g(x) = \int_{\sin(x)}^{\cos(x)} \ln(1+5\theta)\ d\theta$