Session 9: Sections 4-1, 4-2, and part of 4-3

1. Determine if $F(t) = 7t + et + C$ is the antiderivative of $f(t) = 7 + e$

$F'(t) = 7 + e = f(t)$ ✓

2. Determine the following indefinite integrals.

(a) $\int x^4 \, dx = \boxed{\frac{1}{5}x^5 + C}$

check: $\frac{d}{dx}\left(\frac{1}{5}x^5 + c\right) = x^4$ ✓

(b) $\int \frac{1}{\sqrt{x}} \, dx = \int x^{-1/2} \, dx = \boxed{2x^{1/2} + c}$

check: $\frac{d}{dx}\left(2x^{1/2} + c\right) = x^{-1/2}$ ✓

(c) $\int 4t^4 - 5t - 6 \, dt = \boxed{\frac{4}{5}t^5 - \frac{5}{2}t^2 - 6t + c}$

check: $\frac{d}{dx}\left(\frac{4}{5}t^5 - \frac{5}{2}t^2 - 6t + c\right)$

$= 4t^4 - 5t - 6$ ✓

(d) $\int \sqrt[3]{x^2} - 3x^{1/4} \, dx = \int x^{2/3} - 3x^{1/4} \, dx = \frac{3}{5}x^{5/3} - 3\left(\frac{4}{5}\right)x^{5/4} + c$

$\boxed{= \frac{3}{5}x^{5/3} - \frac{12}{5}x^{5/4} + c}$

check: $\frac{d}{dx}\left(\frac{3}{5}x^{5/3} - \frac{12}{5}x^{5/4} + c\right)$

$= x^{2/3} - \frac{60}{20}x^{1/4} = x^{2/3} - 3x^{1/4}$ ✓

(e) $\int \frac{4x^4 - 5x^3}{x^2} \, dx = \int \frac{4x^4}{x^2} - \frac{5x^3}{x^2} \, dx = \int 4x^2 - 5x \, dx = \boxed{\frac{4}{3}x^3 - \frac{5}{2}x^2 + c}$

check:

$\frac{d}{dx}\left(\frac{4}{3}x^3 - \frac{5}{2}x^2 + c\right) = 4x^2 - 5x$ ✓

3. Find $F(t)$ such that $F'(t) = \dfrac{1-t^4}{t^3}$ and $F(1) = 4$.

$$F'(t) = \frac{1}{t^3} - \frac{t^4}{t^3} = t^{-3} - t$$

$$\int t^{-3} - t \; dt = -\frac{1}{2}t^{-2} - \frac{1}{2}t^2 + c$$

$$4 = -\frac{1}{2}(1)^{-2} - \frac{1}{2}(1)^2 + c$$

$$4 = -\frac{1}{2} - \frac{1}{2} + c$$

$$4 = -1 + c$$

$$5 = c$$

$$\boxed{F(t) = -\frac{1}{2t^2} - \frac{t^2}{2} + 5}$$

4. The daily marginal revenue function for the BlackDay Sunglass Company is given by

$$MR(x) = 30 - 0.00003x^2, \quad 0 \le x \le 1732$$

where x represents the number of sunglasses produced and sold. Recover the revenue function R and find the price at which the sunglasses should be sold to obtain maximum revenue.

Find $MR(x) = 0$

$\int MR(x)\,dx$

max revenue:

$$0 = 30 - .00003x^2$$

$$\frac{-30}{-.00003} = x^2$$

$$1\,000\,000 = x^2$$

$$1000 = x$$

Sell 1000 sunglasses to get max revenue

$$P(1000) = 30 - .00001(1000)^2$$

$$= 20$$

The sunglasses should be sold at \$20 each to reach max. revenue

$$\int 30 - .00003x^2\,dx$$

$$R(x) = 30x - \frac{.00003}{3}x^3 + c$$

c = 0 because no revenue is made when no sunglasses are made.

$$\boxed{R(x) = 30x - .00001x^3}$$

$$= x\underbrace{(30 - .00001x^2)}_{\substack{p(x)\\ \text{price}}}$$

5. Find the following indefinite integrals.

(a) $\int 2x(\underbrace{x^2+15}_{u})^{14}\,dx$ need $= \int u^{14}\,du = \frac{1}{15}u^{15} + C$

$= \boxed{\frac{1}{15}(x^2+15)^{15} + C}$

$u = x^2 + 15$

$du = 2x\,dx$ need $2x\,dx$ ✓

check:

$\frac{d}{dx}\left(\frac{1}{15}(x^2+15)^{15} + c\right) = (x^2+15)^{14}(2x)$ ✓

(b) $\int x^6 e^{x^7}\,dx$ need, u $= \int \frac{1}{7} e^u\,du = \frac{1}{7}e^u + c$

$\boxed{= \frac{1}{7}e^{x^7} + c}$

$u = x^7$

$du = 7x^6\,dx$ (need $x^6\,dx$....divide by 7)

$\frac{1}{7}du = x^6\,dx$

check:

$\frac{d}{dx}\left(\frac{1}{7}e^{x^7} + c\right) = \cancel{\frac{1}{7}}e^{x^7}(\cancel{7}x^6)$ line

$= x^6 e^{x^7}$ ✓

(c) $\int \frac{16x^7}{(\underbrace{3-x^8}_{u})^2}\,dx$ need $= \int \frac{-2}{u^2}\,du = \int -2u^{-2}\,du = \frac{-2}{-1}u^{-1} + c$

$= \boxed{2(3-x^8)^{-1} + c}$

$u = 3 - x^8$

$du = -8x^7\,dx$ (need $16x^7\,dx$...mult. by -2)

$-2\,du = 16x^7\,dx$

Check:

$\frac{d}{dx}\left(2(3-x^8)^{-1} + c\right) = -2(3-x^8)^{-2}(-8x^7)$

$= 16x^7(3-x^8)^{-2}$

$= \frac{16x^7}{(3-x^8)^2}$ ✓

(d) $\int 3x\sqrt{8-x^2}\,dx = \int 3x(\underbrace{8-x^2}_{u})^{1/2}\,dx$ need $= \int -\frac{3}{2}u^{1/2}\,du = -\frac{3}{2}\cdot\frac{2}{3}u^{3/2} + c$

$= -u^{3/2} + c$

$\boxed{= -(8-x^2)^{3/2} + c}$

$u = 8 - x^2$

$du = -2x\,dx$ (need $3x\,dx$...mult by $-\frac{3}{2}$)

$-\frac{3}{2}du = -\frac{3}{\cancel{2}}(\cancel{-2}x)\,dx$

$-\frac{3}{2}du = 3x\,dx$

check:

$\frac{d}{dx}\left(-(8-x^2)^{3/2} + c\right)$

$= -\frac{3}{\cancel{2}}(8-x^2)^{1/2}(-\cancel{2}x)$

$= 3x(8-x^2)^{1/2}$

(e) $\int \frac{15x^3}{3+5x^4}\,dx$ $= \int 15x^3(3+5x^4)^{-1}\,dx = \int \frac{3}{4}u^{-1}\,du$

$u = 3+5x^4$

$du = 20x^3\,dx$ need $15x^3\,dx$

so mult by $\frac{15}{20}$

$\frac{15}{20}du = \frac{15}{\cancel{20}}(\cancel{20}x^3)dx$

$\frac{3}{4}du = 15x^3\,dx$

$= \frac{3}{4}\ln|u| + c$

$\boxed{= \frac{3}{4}\ln|3+5x^4| + c}$

(f) $\int \frac{e^x + 5e^{-5x}}{\underbrace{(e^x - e^{-5x})}_{u}{}^4}\,dx$ du $= \int \frac{du}{u^4} = \int u^{-4}\,du = -\frac{1}{3}u^{-3} + c$

$\boxed{= -\frac{1}{3}(e^x - 5e^{-5x})^{-3} + c}$

$u = e^x - e^{-5x}$

$du = e^x - e^{-5x}(-5)\,dx$

$du = e^x + 5e^{-5x}\,dx$

(g) $\int \frac{x}{\sqrt{x-2}}\,dx$ **(challenging)

$= \int \frac{x}{\underbrace{(x-2)}_{u}{}^{1/2}}\,dx = \int \frac{u+2}{u^{1/2}}\,du = \int \frac{u}{u^{1/2}} + \frac{2}{u^{1/2}}\,du$

$u = x-2 \longrightarrow u+2 = x$

$du = dx$

$= \int u^{1/2} + 2u^{-1/2}\,du$

$= \frac{2}{3}u^{3/2} + 4u^{1/2} + c$

$\boxed{= \frac{2}{3}(x-2)^{3/2} + 4(x-2)^{1/2} + c}$

6. A contaminated lake is treated with a bactericide. The rate of increase in harmful bacteria t days after the treatment is given by the function

$$\frac{dN}{dt} = \frac{-3000t}{(1+t^2)}$$

for $0 \leq t \leq 8$. $N(t)$ is the number of bacteria per milliliter of water.

(a) Find the absolute minimum value of $\frac{dN}{dt}$.

(b) If the initial count was 8000 bacteria per milliliter, find $N(t)$ and then find the bacteria count after 8 days.

(c) When is the bacteria count 3821 bacteria per milliliter? (Round answers to two decimal places.)

a) $\frac{d}{dt}\left(\frac{dN}{dt}\right) = \frac{d}{dt}\left(\frac{-3000t}{(1+t^2)}\right) = \frac{(1-t^2)(-3000)-(-3000t)(2t)}{(1+t^2)^2}$

$= \frac{-3000+3000t^2+6000t}{(1+t^2)^2}$

$0 = \frac{3000(t^2-2t-1)}{(1+t^2)^2}$

$0 = 3000(t-1)^2$

$t = 1$

t	$\frac{dN}{dt}$
0	0
1	-1500 abs. min.
8	-369.23

b) $\int \frac{dN}{dt} = \int -3000t\underbrace{(1+t^2)}_{u}^{-1}\,dt = \int -1500\,u^{-1}\,du = -1500 \ln|u| + c$

$u = 1+t^2$

$du = 2t\,dt$

$-1500\,du = -3000t\,dt$

$N(t) = -1500\ln|1+t^2| + c$

$N(0) = 8000$

$8000 = -1500\ln|1+0^2| + c$

$8000 = -1500\underbrace{\ln|1|}_{0} + c$

$8000 = c$

$\boxed{N(t) = -1500\ln|1+t^2| + 8000}$

$\boxed{N(8) \approx 1738.42 \text{ bacteria / milliliter}}$

c) $3821 = -1500\ln|1+t^2| + 8000$

$-4179 = -1500\ln|1+t^2|$

$2.786 = \ln|1+t^2|$

$e^{2.786} = 1+t^2$

$e^{2.786} - 1 = t^2$

$t \approx 3.9$

$\boxed{\text{3.9 days after treatment}}$

7. The weekly marginal profit function for Shoe Fly, a company that makes insect themed footwear, is given by $P'(x) = 30 + 50xe^{-0.01x^2}$ dollars per pair of shoes when x pairs of shoes are sold each week. Find Shoe Fly's weekly profit function when x pairs of shoes are sold each week if it is known they have a $\underbrace{\text{profit loss of \$2300}}_{P(0) = -2300}$ when no shoes are sold.

$$\int P'(x) = \int 30 + 50xe^{-.01x^2}\,dx$$

$$= \int 30\,dx + \underbrace{\int 50xe^{-.01x^2}\,dx}$$

$$u = -.01x^2$$

$$du = -.02x\,dx$$

$$\frac{50}{-.02}\,du = \frac{50}{\cancel{-.02}}\left(\cancel{-.02}\right)x\,dx$$

$$-2500\,du = 50x\,dx$$

$$\int -2500e^u\,du = 2500e^u + C$$

$$= -2500e^{-.01x^2} + C$$

$$P(x) = 30x - 2500e^{-.01x^2} + C$$

$$-2300 = 30(0) - 2500\underbrace{e^{-.01(0)}}_{e^0 = 1} + C$$

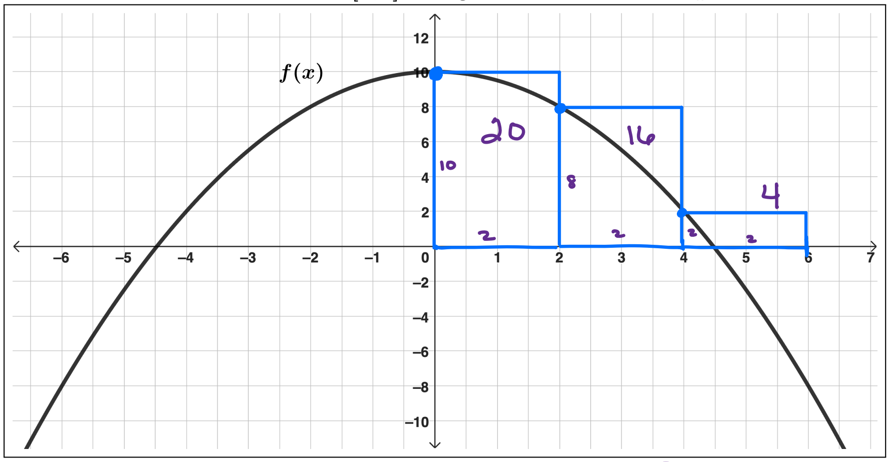
$$-2300 = -2500 + C$$

$$200 = C$$

$$\boxed{P(x) = 30x - 2500e^{-.01x^2} + 200}$$

8. Use the graph of $f(x)$ below to calculate the following:

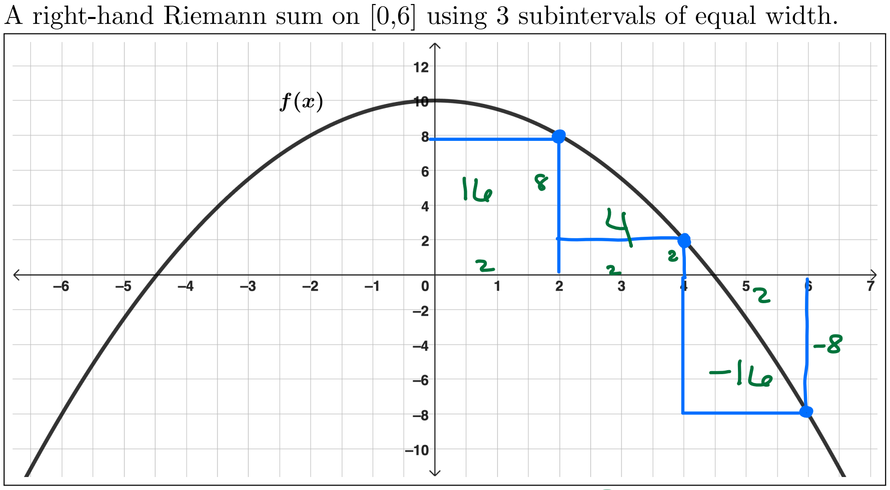
(a) A left-hand Riemann sum on [0,6] using 3 subintervals of equal width.



$\frac{\Delta x}{3} = \frac{6}{3} = 2$

$20 + 16 + 4 = 40$

(b) A right-hand Riemann sum on [0,6] using 3 subintervals of equal width.



$16 + 4 - 16 = 4$

9. The table below gives the velocity of a runner (in feet per second) for the first 6 seconds of her race. Use the table, estimate the distance traveled by the runner from $t = 0$ to $t = 6$ using a right-hand Riemann sum with 4 equal subintervals. (Rounded answers to one decimal.)

t (sec)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
$s(t)$ (ft/sec)	0	3.2	5.6	6.7	8.8	9.2	11.1	12.0	16.3	20.5	21.2	24.7	22.9

$$\frac{\Delta t}{4} = \frac{6}{4} = \frac{3}{2} = 1.5$$

$$RS = 1.5(6.7 + 11.1 + 20.5 + 22.9)$$
$$= 1.5(61.2)$$
$$= 91.8 \text{ feet}$$

10. Estimate $\int_2^5 (9 - x^2)\, dx$ using left-side Riemann sums and 6 sub-intervals of equal width.

$$\Delta x \ \frac{5-2}{6} = \frac{3}{6} = \frac{1}{2}$$

2   2.5   3   3.5   4   4.5   5

$9-2^2$ $9-2.5^2$ $9-3^2$ $9-3.5^2$ $9-4^2$ $9-4.5^2$

5   2.75   0   −3.25   −7   −11.25

$$LS = \frac{1}{2}(5 + 2.75 + 0 - 3.25 - 7 - 11.25)$$
$$= \frac{1}{2}(-13.75)$$
$$= -6.875$$