Math 152 - Week-In-Review 2

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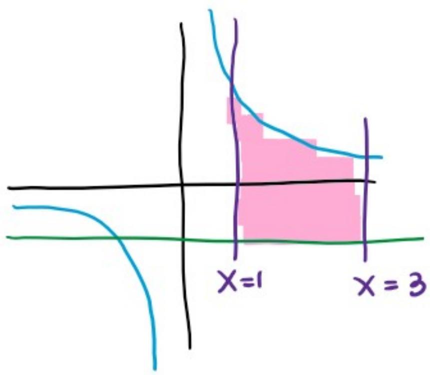
$\int dx \to T-B$

$\int dy \to R-L$

1. Find the area of the region bounded by the curves $y = 4/x$, $y = -1$, $\underbrace{x = 1 \text{ and } x = 3}_{\text{bounds}}$.



$$A = \int_1^3 \left(\frac{4}{x} - (-1)\right) dx$$

$$= \int_1^3 \left(\frac{4}{x} + 1\right) dx$$

$$= 4\ln|x| + x \Big|_{x=1}^{x=3}$$

$$= 4\ln(3) + 3 - [4\cancel{\ln(1)}^{0} + 1]$$

$$= 4\ln 3 + 2$$

2. Find the area bounded by the curves $y = x^2 + 2$, $\underbrace{y = 2x + 5}$, $x = 0$ and $x = 6$.

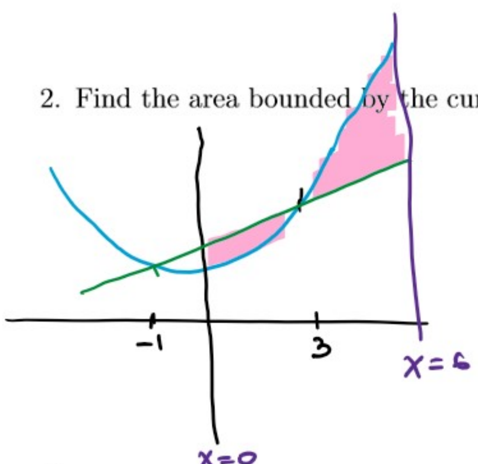
Intersection points

$$x^2 + 2 = 2x + 5$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0.$$

$$x = 3 \qquad x = -1$$

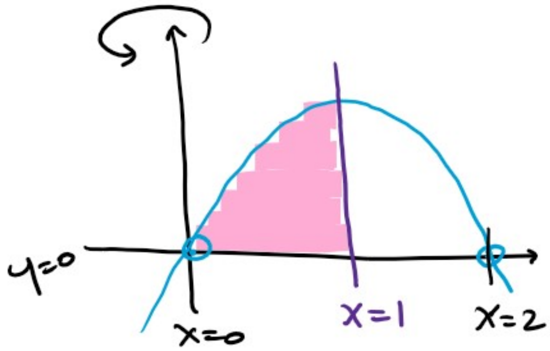


$$A = \int_0^3 (2x+5 - (x^2+2))\, dx + \int_3^6 (x^2+2 - (2x+5))\, dx$$

$$A = \int_0^3 (3 + 2x - x^2)\, dx + \int_3^6 (x^2 - 2x - 3)\, dx = 9 + 27 = 36$$

$y = 2x - x^2$ as a $f(y) \rightarrow \quad x = \sqrt{1-y} + 1$

3. Find the volume of the solid obtained by rotating the region bounded by the curves $y = 2x - x^2$, $y = 0$, $x = 0$ and $x = 1$, about the y-axis.



$2x - x^2 = 0$

$x(2-x) = 0$

$x = 0, \; x = 2$

Washers $\rightarrow \int dy \rightarrow$ R-L

Shells $\rightarrow \int dx \rightarrow$ T-B

$2\pi r h$

$r = x$

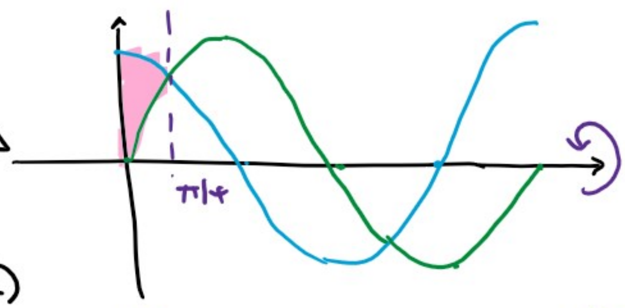
$h = 2x - x^2 - 0$

$$V = 2\pi \int_0^1 (x)(2x - x^2)\, dx$$

$$= 2\pi \int_0^1 (2x^2 - x^3)\, dx$$

$$= 2\pi \left[\frac{2x^3}{3} - \frac{x^4}{4} \Big|_0^1 \right] = 2\pi \left[\frac{2}{3} - \frac{1}{4} \right] = \frac{5\pi}{6}$$

4. Find the volume of the solid obtained by rotating the region bounded by the curves $y = \cos x$, $y = \sin x$, $x = 0$ and $x = \pi/4$, about the x-axis.



x=0

Trig Identity

$\cos^2 x - \sin^2 x = \cos(2x)$

Intersection

$\cos(x) = \sin(x)$

$1 = \tan(x)$

$\pi/4 = x$

Washers $\rightarrow \int dx \rightarrow$ T-B

$R = \cos(x) - 0 = \cos(x)$

$r = \sin(x) - 0 = \sin(x)$

$$V = \pi \int_0^{\pi/4} \left[(\cos x)^2 - (\sin x)^2 \right] dx = \pi \int_0^{\pi/4} \cos(2x)\, dx$$

$$V = \frac{\pi}{2}$$

sketch ① Shape of base (area between curves)
② Shape of cross section (ex: squares, Δ, semi circles)
③ Orientation of x-sec

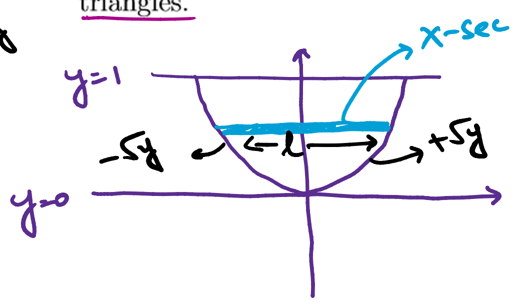
Texas A&M University College of Arts & Sciences

Math 152 - Fall 2024
WIR-2: 6.1, 6.2, 6.3

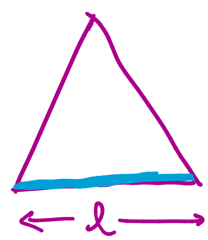
Volumes by slices   a) ⊥ x axis → $\int dx$ b) ⊥ y axis → $\int dy$

5. Find the volume of a solid whose base is the region bounded by the parabola $y = x^2$ and the line $y = 1$ and where the cross sections perpendicular to the y-axis are equilateral triangles.

$y = x^2$
$x = \pm\sqrt{y}$



→ $\int dy$ → (R−L)



$A = \frac{\sqrt{3}}{4}\ell^2$
$= \frac{\sqrt{3}}{4}(2\sqrt{y})^2$

$\ell = R - L$
$\ell = +\sqrt{y} - (-\sqrt{y})$
$= 2\sqrt{y}$

$A = \frac{\sqrt{3}}{\cancel{4}} \cdot \cancel{4} y = \sqrt{3}\, y$

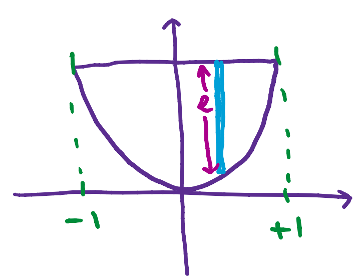
$$V = \int_{y=0}^{y=1} A\, dy$$

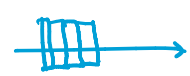
$$= \int_0^1 \sqrt{3}\, y\, dy = \sqrt{3} \cdot \frac{y^2}{2}\Big|_0^1$$

$$V = \sqrt{3}/2$$

6. Find the volume of a solid whose base is the region bounded by the parabola $y = x^2$ and the line $y = 1$ and where the cross sections perpendicular to the x-axis are semi circles.

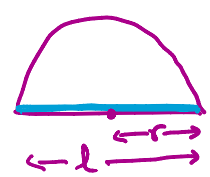
intersection points
$x^2 = 1$
$x = \pm 1$





$\int dx$ → T−B

$A = \frac{\pi r^2}{2}$



$r = \frac{\ell}{2}$

$\ell = 1 - x^2$
$r = \left(\frac{1-x^2}{2}\right)$
$A = \frac{\pi}{2}\left(\frac{1-x^2}{2}\right)^2$
$A = \frac{\pi(1-2x^2+x^4)}{8}$

$$V = \int_{-1}^{1} A\, dx$$

$$V = \frac{\pi}{8}\int_{-1}^{1}(1-2x+x^4)\, dx$$

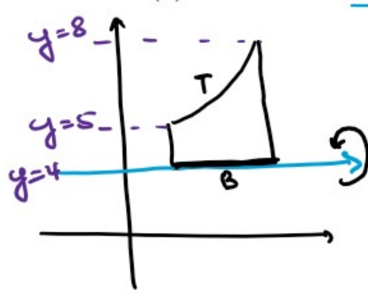
$$= 0.133\pi$$

$$= 0.133\pi$$

Disks $\rightarrow \pi r^2$
washers $\rightarrow \pi(R^2 - r^2)$ } $\int dx$; $\int dy$
Shells $\rightarrow 2\pi r h$ } $\int dy$; $\int dx$

7. Set up the integral(s) to find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2 + 4$, $y = 4$, $x = 1$, and $x = 2$

(a) about the line $y = 4$ using the method of disks.

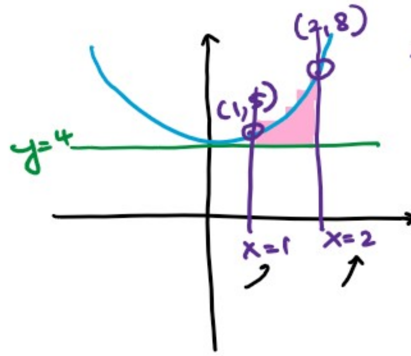


$\int dx$ πr^2

$r = T - B$

$r = x^2 + 4 - 4$

$r = x^2$



x^2+4 @ $x=1$
$y=5$
x^2+4 @ $x=2$
$y=8$

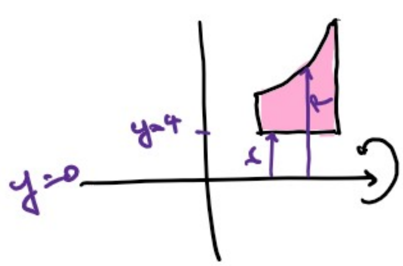
$$V = \pi \int_1^2 (x^2)^2\,dx$$

$$= \pi \int_1^2 x^4\,dx$$

$\int dx \rightarrow T - B$

(b) about the x-axis using the method of washers.

$1 \le x \le 2$



$\pi(R^2 - r^2)$

$R = (x^2+4) - (0) = x^2+4$

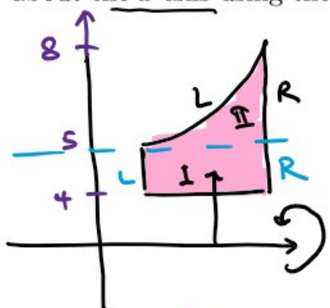
$r = 4 - 0 = 4$

$$V = \pi \int_1^2 \left[(x^2+4)^2 - (4)^2\right] dx$$

$\to \int dy \to R-L$ $\qquad$ $\to 2\pi r h$

(c) about the x-axis using the method of cylindrical shells.

$y=x^2+4$
$x^2=y-4$
$x=\sqrt{y-4}$



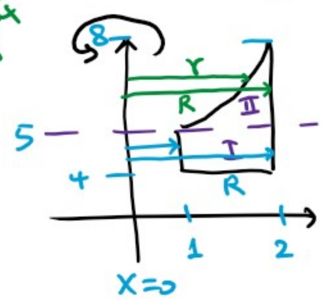
I $\quad h_1 = 2-1 = 1.$ $\quad r_1 = y$

π $\quad h_2 = 2-\sqrt{y-4}$ $\quad r_2 = y$

$$V = 2\pi\int_4^5 (y)(1)\,dy + 2\pi\int_5^8 (y)(2-\sqrt{y-4})\,dy$$

(d) about the y-axis using the method of washers. $\to \int dy$

$y=x^2+4$
$x=\sqrt{y-4}$



$\pi(R^2-r^2)$

I $(4 \le y \le 5)$
$R = 2-0=2$
$r = 1-0=1$

II $(5 \le y \le 8)$
$R = 2-0=2$
$r = \sqrt{y-4}-0$

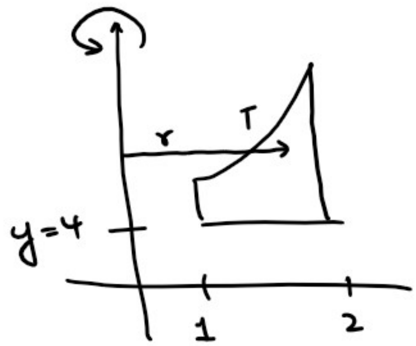
$4-(y-4) = -y$

$$V = \pi\int_4^5 (2^2 \cancel{-} 1^2)\,dy + \pi\int_5^8 (2^2 \cancel{-(\sqrt{y-4})^2})\,dy$$

(3)

(e) about the y-axis using the method of cylindrical shells.

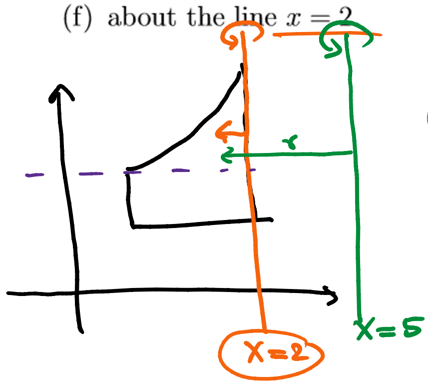
$\to \int dx \to T-B$
$2\pi r h$



$h = T-B = x^2+4-4 = x^2$
$r = x$

$$V = 2\pi\int_1^2 (x)(x^2)\,dx$$

(f) about the line $x = 2$.



Disks $\to \int dy \to R-L$

Shells $\to \int dx \to T-B$

$2\pi r h$

$h = x^2+4-4 = x^2$

$r = 2-x \quad (R-L)$

$$V = 2\pi \int_1^2 (2-x)(x^2)\, dx$$

(g) about the line $x = 5$.

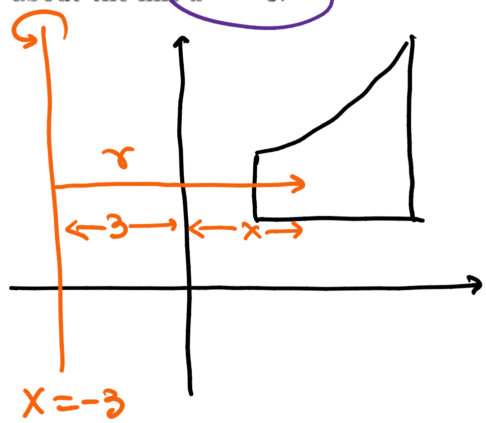
shells $\to \int dx$

$h = x^2+4-4 = x^2$

$r = 5-x$

$$V = 2\pi \int_1^2 (5-x)(x^2)\, dx$$

(h) about the line $x = -3$. $\longrightarrow$ Shells $\to \int dx$



$2\pi r h$

$h = T-B = x^2$

$r = R-L = x-(-3)$

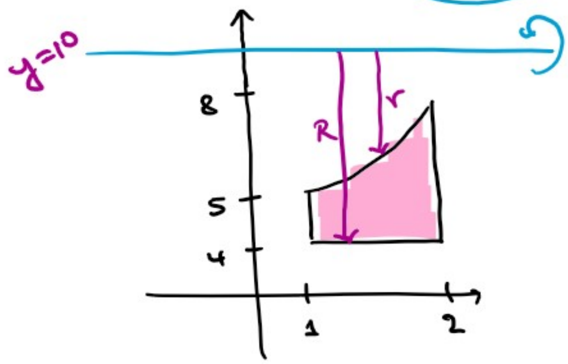
$r = x+3$

$$V = 2\pi \int_1^2 (x+3)(x^2)\, dx$$

(i) about the line $y = 10$.

Washers $\rightarrow \int dx \rightarrow \pi(R^2 - r^2)$

shells $\rightarrow \int dy$

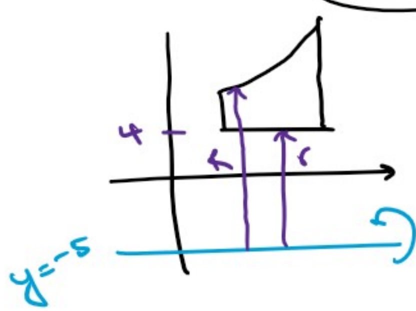


$R = 10 - 4 = 6$

$r = 10 - (x^2+4) = 6 - x^2$

$$V = \pi \int_1^2 \left[6^2 - (6 - x^2)^2\right] dx$$

(j) about the line $y = -5$. $\rightarrow$ Washers $\rightarrow \int dx \rightarrow T - B$

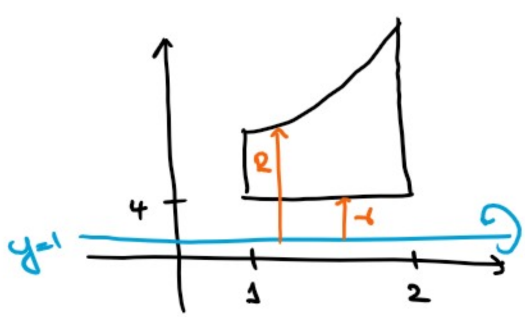


$R = (x^2+4) - (-5) = x^2 + 9$

$r = (4) - (-5) = 9$

$$V = \pi \int_1^2 \left[(x^2+9)^2 - 9^2\right] dx$$

(k) about the line $y = 1$. $\rightarrow$ Washers $\rightarrow \int dx \rightarrow T - B$



$R = (x^2+4) - 1 = x^2 + 3$

$r = (4) - (1) = 3$

$$V = \pi \int_1^2 \left[(x^2+3)^2 - 3^2\right] dx$$

8. Set up the integral(s) to find the volume of the solid obtained by rotating the region bounded by the curves $y = -x^2 + 7x - 10$ and $y = x - 2$

$y = -x^2 + 7x - 10 = 0$

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$x = 5$ $\quad$ $x = 2$

Intersection points

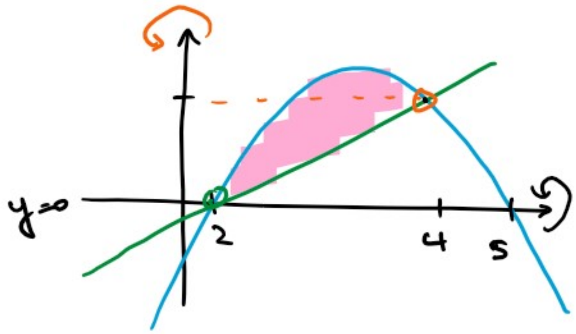
$-x^2 + 7x - 10 = x - 2$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$x = 4$ $\quad$ $x = 2$

(a) about the x-axis.



Washers $\rightarrow \int dx \rightarrow$ T−B

$R = -x^2 + 7x - 10$

$r = x - 2$

$$V = \pi \int_2^4 (-x^2+7x-10)^2 - (x-2)^2 \, dx$$

(b) about the y-axis.

Shells $\rightarrow \int dx \rightarrow$ T−B

$h = T - B = (-x^2 + 7x - 10) - (x - 2)$

$r = x$

$$V = 2\pi \int_2^4 (x)(-x^2 + 6x - 8) \, dx$$