Math 152 - Week-In-Review 4 (Exam 1 review)

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1. Evaluate the indefinite integral $\int x^3 \ln x \; dx$.

2. Evaluate the definite integral $\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right) \; dx$.

3. Evaluate the indefinite integral $\int x^2 e^x \, dx$

4. Evaluate the definite integral $\int_0^1 \frac{x}{\sqrt{1+x^2}} \, dx$

5. Evaluate the definite integral $\int_0^\pi e^{\cos t} \sin 2t \, dt$.

6. Evaluate the indefinite integral $\int e^{3x} \cos x \; dx$.

7. Evaluate the indefinite integral $\int x^5 \sqrt{x^3 + 1} \; dx$

8. Evaluate the indefinite integral $\int \sec^5 x \tan^3 x \; dx$.

9. Evaluate the indefinite integral $\int \frac{\sec\theta \tan\theta}{4 + \sec\theta} \; d\theta$

10. Evaluate $\int_0^{\pi/4} (\sec^2 x) e^{\tan x}\, dx$.

(a) $e^{\sqrt{2}/2} - 1$

(b) $e^{\sqrt{2}} - 1$

(c) $e^{1/2} - 1$

(d) $1 - e$

(e) $e - 1$

11. Compute $\int_0^{\pi/4} x \cos x\, dx$.

(a) $\frac{\sqrt{2}}{2}\left(\frac{\pi}{4} + 1\right)$

(b) $\frac{\sqrt{2}}{2}\left(\frac{\pi}{4} + 1\right) - 1$

(c) $\frac{\pi}{4} + \frac{\sqrt{2}}{2}$

(d) $\sqrt{2} - 1$

(e) $\frac{\pi\sqrt{2}}{8}$

(f) 0

12. Which of the following is the definite integral $\int_0^{\pi/2} \sin(2x)\cos(2x)\, dx$ equal to?

(a) 3/2

(b) 2/3

(c) 1/2

(d) 1

(e) 0

13. A conical tank is 3 feet tall, has a 2 foot radius at the top and is full of water with weight density ρg. The tank has an additional 1 foot spout at the top of the tank. Find the work required to pump all the water out of the spout.

14. A 6 meter long tank with a semi-circular cross section is full of water, with weight denisty $\rho g = 9800$ Newtons per cubic meter. The diameter of the semi-circle is 3 meters. There is a 0.5 meter nozzle at the top of the tank. Find the work required to pump all the water out of the tank through the nozzle.

15. A rope that is 20 feet long and weighs 2 pounds per foot supports a 160-lb weight while hanging over the side of a tall building. How much work, in ft-lb, would be required to pull the rope up 10 feet?

16. A spring has a natural length of 2 meters. If a force of 12 Newtons is required to hold the spring stretched to a length of 4 meters, find the work that would be required to stretch the spring from 3 to 7 meters.

17. Find the area between the curves $y = x^2 + 1$ and $y = x + 3$ from $x = 0$ to $x = 3$.

18. The region bounded by the curves $y = x - x^2$ anf the x-axis is rotated about the y-axis. Find the volume of the resultant solid.

19. Which of the following integrals gives the volume of the solid formed by rotating the region bounded by the $y = x^2$ and $y = \sqrt[3]{x}$ about the line $y = -1$?

(a) $2\pi \int_0^1 (y-1)(\sqrt{y} - y^3)\, dy$

(b) $\pi \int_0^1 (y^3 - \sqrt{y})^2\, dy$

(c) $2\pi \int_0^1 (y+1)(\sqrt{y} - y^3)\, dy$

(d) $\pi \int_0^1 \left((x^2-1)^2 - (\sqrt[3]{x} - 1)^2 \right) dx$

(e) $\pi \int_0^1 (x^2 - \sqrt[3]{x})(x+1)\, dx$

20. Which of the following integrals gives the volume of the solid formed by rotating the region bounded by the $y = x$ and $y = x^2$ about the line $x = 2$?

(a) $2\pi \int_0^1 (2-x)(x - x^2)\, dx$

(b) $2\pi \int_0^1 (2-y)(y - \sqrt{y})\, dy$

(c) $2\pi \int_0^1 (x-2)(x - x^2)\, dx$

(d) $\pi \int_0^1 (y - \sqrt{y})^2\, dy$

(e) $\pi \int_0^1 \left((2-x)^2 - (2-x^2)^2 \right) dx$