Session 7: Review Exam #2

1. Find the derivatives of the following functions. You do not need to simplify your answers.

   (a) $f(x) = (2x + 1)\sqrt{x^2 + 1}$

   (b) $f(x) = \dfrac{1}{e^x + e^{-x}}$

   (c) $f(x) = e^{2x} \ln\left(2x^3 + x\right)$

(d) $f(x) = 4^{x^4+5}$

(e) $f(x) = \dfrac{\sqrt{x^2 + 4x - e}}{3^{2x^4+x}}$

(f) $f(x) = \left(x^2 + 6x + 1\right)^4 + e^{2\pi} - \log_4\left(2x^3 - \sqrt{x}\right)$

2. Sketch a graph of a function that satisfies the following conditions.

- Domian: $(-\infty, \infty)$
- Range: $[2, \infty)$
- Continuous on $(-\infty, \infty)$
- $f'(x) > 0$ on $(-3, 0) \cup (3, \infty)$
- $f'(x) < 0$ on $(-\infty, -3) \cup (0, 3)$
- $f'(0)$ is undefined
- $f''(x) > 0$ on $(-\infty, 0) \cup (0, 6)$
- $f''(x) < 0$ on $(6, \infty)$
- $\lim_{x \to \infty} f(x) = 4$
- $\lim_{x \to -\infty} f(x) \longrightarrow \infty$
- $f(-3) = 2,\ f(0) = 5$

3. Given a function $f(x)$ with domain $(-\infty, 2) \cup (2, \infty)$, $f'(x) = \dfrac{(x-7)(x+3)}{(x-2)^2}$, and $f''(x) = \dfrac{50}{(x-2)^3}$, find all intervals where $f(x)$ is

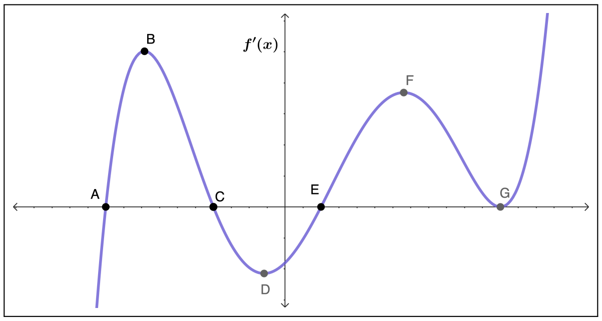
(a) decreasing and concave down.

(b) decreasing and concave up.

(c) increasing and concave down.

(d) increasing and concave up.

4. A graph of $f'(x)$ is given below for a function $f(x)$ whose domain is $(-\infty, \infty)$.



Use the graph to find:

(a) the x-values for all local extrema of $f(x)$.

(b) determine the x-values at which $f(x)$ has points of inflection.

(c) determine the intervals where $f(x)$ is concave down.

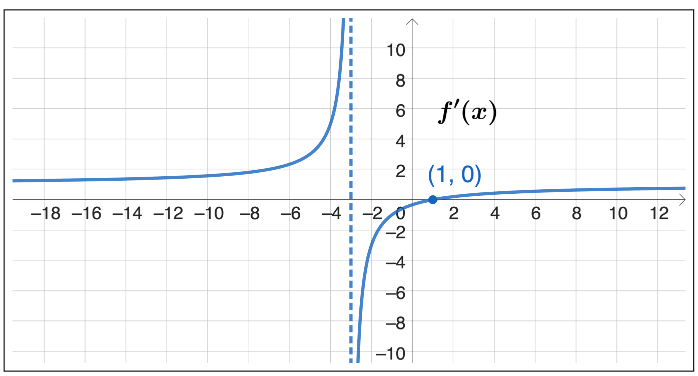
5. The length of a rectangle is increasing at a rate of 3 centimeters per second, and the width is increasing at a rate of 2 centimeters per second. How fast is the area of the rectangle increasing when the length is 10 centimeters and the width is 5 centimeters?

6. Two cars start from the same point. One car travels north at 60 kilometers per hour, and the other car travels east at 80 kilometers per hour. How fast is the distance between the two cars increasing 2 hours later?

7. A triangle has a height that is increasing at a rate of 2 cm/second, and its area is increasing at a rate of 4 cm^2/second. Find the rate at which the length of the base of the triangle is changing when the height of the triangle is 4 cm and its area is 20 cm^2.

8. Given $f(x) = x^{2/3} - x^2$, find all (a) partition numbers for $f'(x)$, (b) critical values, (c) the intervals where $f(x)$ is increasing, (d) the intervals where $f(x)$ is decreasing, and (e) all local extrema. Round any answers to two decimal places if necessary.

9. Given the graph of $f'(x)$ below and that the domain of $f(x)$ is $(-\infty, -3) \cup (-3, \infty)$, find all (a) partition numbers for $f'(x)$, (b) critical values, (c) the intervals where $f(x)$ is increasing, (d) the intervals where $f(x)$ is decreasing, and (e) all local extrema.



10. Given $f(x) = (x^2 + 3x)^{2/3}$, find the following. Round answers to two decimal places if necessary.

    (a) the equation of the tangent line to $f(x)$ at $x = -2$.

    (b) all critical values for $f(x)$.

    (c) all values of x where the line tangent to $f(x)$ is horizontal.

11. Find the approximate cost of producing the 50^{th} item if a company's cost function is $C(x) = \sqrt{x}(x - 10)$ where $C(x)$ is in dollars and x is the number of items produced.

12. Use implicit differentiation to find the slope of the line tangent to the graph of $19 - 3x^2 + 9^x = 4\sqrt{y}$ at $(0, 25)$.

13. Find $\dfrac{dy}{dx}$ for the curve $5e^{xy} - 6x^2 = 8y^3 + 7$.

14. Suppose that $x = x(t)$ and $y = y(t)$ are both functions of t. If $x^2 + y^2 = 40$, and $\frac{dx}{dt} = 3$ when $x = 2$ and $y = 6$, what is $\frac{dy}{dt}$?