# Math 152 - Exam 2 Review

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$n$ = integer

1. Given the sequence $\{a_n\} = \left\{2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \cdots\right\}$,
$= \left\{ \frac{2}{1}, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \cdots \right\}$
$= \left\{ \frac{2}{1^2}, \frac{3}{2^2}, \frac{4}{3^2}, \frac{5}{4^2}, \cdots \right\}$

   (a) find the $n_{th}$ term.

   $a_n = \frac{(n+1)}{n^2}$

   n starts at 1.

   (b) find the limit of the sequence.

   $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{n+1}{n^2}\right) \sim \frac{n}{n^2} = \frac{1}{n} = 0$

   (c) Is this a monotonic sequence?

   $2 > \frac{3}{4} > \frac{4}{9} > \frac{5}{16} \cdots$ ie $a_n > a_{n+1}$

   Yes, sequence is decreasing & hence monotonic

   (d) Is this a bounded sequence?

   $a_1 = 2$, $\lim_{n\to\infty} a_n = 0$, sequence is decreasing

   $0 \leq a_n \leq 2$ → Hence sequence is bounded.

2. Is the sequence given by $a_n = \frac{\ln n}{n}$ increasing or decreasing? Find the limit of the sequence.

   $a_1 = \frac{\ln(1)}{1} = 0$ ; $a_2 = \frac{\ln 2}{2}$ ; $a_3 = \frac{\ln 3}{3}$, ....

   $a_n = \frac{\ln(n)}{n} \Rightarrow f(x) = \frac{\ln(x)}{x}$ then $f'(x) = \frac{x \cdot \left(\frac{1}{x}\right) - \ln(x) \cdot 1}{x^2} = \frac{1 - \ln(x)}{x^2}$

   $\ln(1) = 0$, $\ln(e) = 1$, $\ln(x > e) > 1$ ; $e \sim 2.7$

   for $a_n$, $\frac{1 - \ln(n)}{n^2} < 0$ for $n \geq 3$, $a_n$ is decreasing
   (−)ve Slope

   $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\ln(n)}{n} \sim \frac{\infty}{\infty} \xrightarrow{L'H} \lim_{n\to\infty} \frac{\left(\frac{1}{n}\right)}{1} = \frac{1}{n} = 0$

Find the limit of the sequence.

3. $a_n = \dfrac{n^2 + 2n - 5}{2n^2 + 1}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2+2n-5}{2n^2+1} \sim \frac{\infty}{\infty}$$

$$\xrightarrow{L'H} \frac{2n+2}{4n} \sim \frac{\infty}{\infty}$$

$$\xrightarrow{L'H} \frac{2}{4} = \frac{1}{2}$$

$$\lim_{n\to\infty} a_n \approx \frac{n^2}{2n^2} = \frac{1}{2}$$

4. $\left(1 + \dfrac{3}{n}\right)^{2n}$ $= a_n = y$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(1+\frac{3}{n}\right)^{2n} \sim \left(1+\frac{3}{\infty}\right)^{2(\infty)} \sim 1^\infty$$

Indeterminate power.

$$\lim_{n\to\infty} \ln y = \ln\left(1+\frac{3}{n}\right)^{2n} = \underbrace{(2n)}\underbrace{\ln\left(1+\frac{3}{n}\right)} \sim$$

$(\infty)(0) \to$ indeterminate product

$$= \frac{\ln\left(1+\frac{3}{n}\right)}{\left(\frac{1}{2n}\right)} \sim \frac{0}{0}$$

$$\xrightarrow{L'H} \frac{\left(\frac{1}{1+\frac{3}{n}}\right)\left(0-\frac{3}{n^2}\right)}{\left(-\frac{1}{2n^2}\right)} = \lim_{n\to\infty} \left(-\frac{3}{\cancel{n^2}}\right)\left(-2\cancel{n^2}\right)\left(\frac{1}{1+\cancel{\frac{3}{n}}^{\,0}}\right)$$

$$= (6)\cdot\left(\frac{1}{1+0}\right) = 6 = \ln y.$$

$y = e^6$ Ans.

5. $a_n = \dfrac{4n + (-1)^n}{n}$

$$a_n = \frac{4\cancel{n}}{\cancel{n}} + \frac{(-1)^n}{n} = 4 + \frac{(-1)^n}{n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}(4) + \lim_{n\to\infty} \frac{(-1)^n}{n}$$

$$= 4 + \cancel{\frac{(-1)^n}{\infty}}^{\,0}$$

$$= 4$$

$|4n + (-1)^n|$ is not one term

$\{a_n\} = \{2, \frac{1}{2}, -1, 2, \frac{1}{2}, -1, \ldots\}$

6. Given the recursive sequence $\{a_n\}$ where $a_1 = 2$ and $a_{n+1} = 1 - \dfrac{1}{a_n}$, find the limit of the sequence if it converges.

$a_1 = 2$

$a_2 = 1 - \frac{1}{2} = \frac{1}{2}$

$a_3 = 1 - \frac{1}{1/2} = 1 - 2 = -1$

$a_4 = 1 - \frac{1}{(-1)} = 1 + 1 = 2$

If the sequence converges

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1} = L$$

Solve for L

$$\lim_{n\to\infty}\left(a_{n+1} = 1 - \frac{1}{a_n}\right)$$

$$L = 1 - \frac{1}{L} \quad \text{or} \quad L = \frac{L-1}{L} \quad \text{or} \quad L^2 = L - 1$$

$$\text{or} \quad L^2 - L + 1 = 0$$

$$L = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{-3}}{2}$$

not real

Hence sequence diverges

7. Find the limit of the sequence $a_n = \dfrac{(-1)^n(2n^2+2)}{3n^2+1}$ or show that it diverges.

This is an alternating sequence.

$$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty}\left(\frac{2n^2+2}{3n^2+1}\right) \sim \frac{2\cancel{n^2}}{3\cancel{n^2}} = \frac{2}{3} \neq 0$$

limit of this sequence is $(-1)^n \cdot \frac{2}{3}$ $\to +\frac{2}{3}$, $\to -\frac{2}{3}$

$\therefore$ this alternating sequence diverges pay attention to the starting value of n for a series



8. Find the $3^{rd}$ partial sum $S_3$ of the series $\sum a_n = \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$.

$$S_3 = a_1 + a_2 + a_3$$

$$= \frac{\cos(\pi)}{1} + \frac{\cos(2\pi)}{2} + \frac{\cos(3\pi)}{3}$$

$$= (-1) + \left(\frac{1}{2}\right) + \left(-\frac{1}{3}\right)$$

$$= \frac{-5}{6}$$

9. If the $\boxed{\text{nth partial sum}}$ of a series is given by $\boxed{\underset{\text{sum}}{S_n} = 3 - n2^{-n}}$,

$$S_n = 3 - \frac{n}{2^n}$$

(a) find $a_n$

term

$$a_n = S_n - S_{n-1}$$

$$= \left(3 - \frac{n}{2^n}\right) - \left(3 - \frac{n-1}{2^{n-1}}\right)$$

$$= -\frac{n}{2^n} + \frac{(n-1)}{2^{n-1}}$$

$$a_n = \frac{n-2}{2^n}$$

(b) find $\sum_{n=1}^{\infty} a_n$ $= S = \lim_{n\to\infty} S_n$

$$= \lim_{n\to\infty}\left(3 - \frac{n}{2^n}\right)$$

$$= 3 - \lim_{n\to\infty}\left(\frac{n}{2^n}\right)$$

$$\to \frac{\infty}{\infty} \xrightarrow{L'H} \frac{1}{2^n.(\ln 2)} = 0$$

$$\therefore S = 3 - 0$$

$$= 3$$

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots = s = \lim_{n\to\infty} S_n$$



10. Find the nth partial sum $S_n$ of the series $\sum_{n=1}^{\infty} \frac{3}{n^2+n}$ and then determine if the series converges or diverges.

PFD

$$\sum_{n=1}^{\infty} \frac{3}{n^2+n} = \sum_{n=1}^{\infty} \frac{3}{n(n+1)} = \sum_{n=1}^{\infty} \left[ \frac{A}{n} + \frac{B}{n+1} \right]$$

$$\frac{3}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} \quad \text{or} \quad 3 = A(n+1) + B(n)$$

$$A = 3 \qquad B = -3$$

$$\sum_{n=1}^{\infty} \frac{3}{n^2+n} = \sum_{n=1}^{\infty} \left[ \frac{3}{n} - \frac{3}{n+1} \right] \longrightarrow \text{Telescoping series.}$$

$$a_1 = \frac{3}{1} - \cancel{\frac{3}{2}}$$

$$a_2 = \cancel{\frac{3}{2}} - \cancel{\frac{3}{3}}$$

$$a_3 = \cancel{\frac{3}{3}} - \cancel{\frac{3}{4}}$$

$$a_4 = \cancel{\frac{3}{4}} - \cancel{\frac{3}{5}}$$

$$\vdots$$

$$a_{n-2} = \cancel{\frac{3}{n-2}} - \cancel{\frac{3}{n-1}}$$

$$a_{n-1} = \cancel{\frac{3}{n-1}} - \cancel{\frac{3}{n}}$$

$$a_n = \cancel{\frac{3}{n}} - \frac{3}{n+1}$$

$$S_n = \sum_{n=1}^{n} a_n = 3 - \frac{3}{n+1}$$

$$S_n = 3 - \frac{3}{n+1}$$

$$s = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left[ 3 - \frac{3}{n+1} \right]$$

$$\boxed{s = 3}$$

$\therefore$ Series converges since the sum is finite

$$\left(\frac{2}{5}\right)^{n-1} = \left(\frac{2}{5}\right)^{n} \cdot \left(\frac{2}{5}\right)^{-1} = \left(\frac{2}{5}\right)^{n} \cdot \frac{5}{2}$$



11. Find the sum $s$ of the series $\displaystyle\sum_{n=3}^{\infty} 10\left(\frac{2}{5}\right)^{n-1}$. → geometric series

converges if $|r|<1$

form $\sum ar^n = \sum a(\,)^n$ ⇑ $r$

$r = \frac{2}{5}$

$|r|<1$ → series converges

$a_1 = 10\left(\frac{2}{5}\right)^{3-1} = 10\left(\frac{2}{5}\right)^2$

$= \frac{8}{5}$

OR, $a_1 \quad a_2 \quad a_3$

$\frac{a_2}{a_1} = \frac{a_3}{a_2} = r$

$$S = \frac{a_1}{1-r} = \frac{8/5}{1-2/5} = \frac{8/5}{3/5}$$

$$S = \frac{8}{3}$$ Ans.

12. Compute the sum of the series $\displaystyle\sum_{n=0}^{\infty}\left\{\left(\frac{1}{2}\right)^n + \left(\frac{2}{3}\right)^n\right\}$ → 2 geometric series

$$= \underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n}_{\left(r=\frac{1}{2}\right)<1} + \underbrace{\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n}_{\left(r=\frac{2}{3}\right)<1}$$ → both converge

$a_1 = \left(\frac{1}{2}\right)^0 = 1 \qquad a_1 = \left(\frac{2}{3}\right)^0 = 1$

$$= \left(\frac{1}{1-\frac{1}{2}}\right) + \left(\frac{1}{1-\frac{2}{3}}\right)$$

$$= \frac{1}{1/2} + \frac{1}{1/3}$$

$$= 2 + 3$$

$$s = 5$$

Do the following series converge or diverge? If they converge, find the sum of the series.

Options
1. $S_n$
2. Telescoping series
3. Geometric Series
4. Test for Divergence
5. Integral test

13. $\sum_{n=1}^{\infty} \frac{1}{4+e^{-n}}$ $= \sum_{n=1}^{\infty} a_n$

TOD:

$$\lim_{n\to\infty} \left( a_n = \frac{1}{4+e^{-n}} \right) = \frac{1}{4+\cancel{e^{-\infty}}^{0}} = \frac{1}{4} \neq 0$$

Series divergences by TOD.

14. $\sum_{n=1}^{\infty} \frac{2^n+1}{e^n}$ $= \sum_{n=1}^{\infty} \frac{2^n}{e^n} + \sum_{n=1}^{\infty} \frac{1}{e^n}$

$$= \underbrace{\sum_{n=1}^{\infty} \left(\frac{2}{e}\right)^n}_{\left(r=\frac{2}{e}\right)<1} + \underbrace{\sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n}_{\left(r=\frac{1}{e}\right)<1}$$

→ both converge

$a_1 = \frac{2}{e}$ $\qquad$ $a_1 = \frac{1}{e}$

$$= \left(\frac{2/e}{1-2/e}\right) + \left(\frac{1/e}{1-1/e}\right) = \frac{(2/e)}{\left(\frac{e-2}{e}\right)} + \frac{(1/e)}{\left(\frac{e-1}{e}\right)}$$

$$\boxed{S = \frac{2}{e-2} + \frac{1}{e-1}}$$

15. $\sum_{n=1}^{\infty} \frac{e^n}{n^3}$ $= \sum a_n$

TOD: $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{e^n}{n^3}\right)$

$$\xrightarrow{L'H} \frac{e^n}{3n^2} \xrightarrow{L'H} \frac{e^n}{6n} \xrightarrow{L'H} \frac{e^n}{6} \to \infty$$

Series diverges by TOD.

$a_n \longrightarrow f(x)$

state $f(x)$ is positive, continuous, decreasing on $[n, \infty)$

evaluate $\int_n^\infty f(x)\,dx$

16. Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ converges or diverges.

$a_n = n^2 e^{-n^3} \longrightarrow f(x) = x^2 e^{-x^3}$ is (+)ve, cont, decreasing on $[1, \infty)$

Int. test : $\int_1^\infty f(x)dx = \int_1^\infty x^2 e^{-x^3}\,dx$

$u = -x^3$

$du = -3x^2 dx$

$-\frac{du}{3} = x^2 dx$

$= \int e^u \cdot \frac{du}{-3} = -\frac{1}{3} e^u$

$= -\frac{1}{3} e^{-x^3} \Big|_1^\infty$

$= -\frac{1}{3}\left[\cancel{\frac{1}{e^\infty}}^{\,0} - \frac{1}{e^1}\right] = +\frac{1}{3e} \rightarrow$ finite

Since $\int_1^\infty x^2 e^{-x^3} dx$ converges, $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ also converges by the Integral Test.

17. Explain why the Integral Test can NOT be used to determine whether the series $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{1+n^2}$ is convergent.

$a_n = \frac{\cos(n\pi)}{1+n^2} \longrightarrow f(x) = \frac{\cos(\pi x)}{1+x^2}$

$-1 \le \cos(\pi x) \le 1$

$f(x)$ is not positive or decreasing on $[1, \infty)$ hence we cannot use the Integral Test here.

$$S_n + \int_{n+1}^{\infty} f(x)dx \le S \le \underbrace{S_n + \int_{n}^{\infty} f(x)dx}_{\text{max error}}$$



$S_{10} = a_1 + a_2 + a_3 + \cdots + a_{10}$

18. Given that the $\underbrace{\text{10th partial sum}}$ for the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is $\underbrace{s_{10} = 1.64522}$,

(a) Find the error when using the 10th partial sum to approximate the sum of the series.

$a_n = \frac{1}{n^2} \to f(x) = \frac{1}{x^2}$

$S = S_{10} + R_{10}$

$R_{10} = \int_{10}^{\infty} \frac{1}{x^2} dx$

$= -\frac{1}{x} \Big|_{10}^{\infty}$

$= -\left[\frac{1}{\infty} - \frac{1}{10}\right]$

$= +\frac{1}{10} = 0.1$

$S = S_{10} + R_{10}$

$= 1.64522 + 0.1$

$\therefore$ Error $= 0.1$

(b) How many terms $n$ would be required so that the error $\underbrace{s \approx s_n \text{ is less than } 0.001}_{\text{solve for } n}$?

$R_n = \int_{n}^{\infty} f(x)dx$

$\int_{n}^{\infty} \frac{1}{x^2} dx < 0.001$

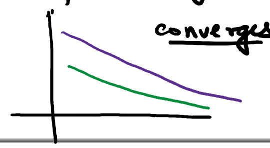
$= -\frac{1}{x} \Big|_{n}^{\infty} < 0.001$

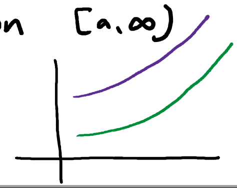
$= -\left(\cancel{\frac{1}{\infty}}^{0} - \frac{1}{n}\right) < 0.001$

$= \frac{1}{n} < 0.001$

$= n > \frac{1}{0.001}$ or $n > 1000$ Ans.

$f(x) \geq g(x)$ on $[a, \infty)$




Use the Comparison test to determine whether the following integrals converge or diverge.

19. $\displaystyle\int_1^{\infty} \frac{dx}{\sqrt{x^3+1}}$

$$\frac{1}{\sqrt{x^3+1}} \longrightarrow \frac{1}{\sqrt{x^3}} = \frac{1}{x^{3/2}}$$

$$\int_1^{\infty} \frac{1}{x^{3/2}}\,dx$$

converges by p series
$p > 1$

$5 > 2$
$\frac{1}{5} < \frac{1}{2}$

$$\left\{ \begin{array}{l} \sqrt{x^3+1} > \sqrt{x^3} \\ \dfrac{1}{\sqrt{x^3+1}} < \dfrac{1}{\sqrt{x^3}} \end{array} \right.$$

$\to$ larger fn

By comparison test, since larger fn converges, the smaller fn will also converge.

20. $\displaystyle\int_1^{\infty} \frac{\cos^2 x}{x^2}\,dx$

$-1 \leq \sin(x) \leq 1$ $\qquad$ $-1 \leq \cos(x) \leq 1$

$0 \leq \sin^2(x) \leq 1$ $\qquad$ $0 \leq \cos^2(x) \leq 1$

$$\frac{\cos^2(x)}{x^2} = \frac{[0,1]}{x^2} \quad \text{ie} \quad \int_1^{\infty} \frac{1}{x^2}\,dx \text{ converges by p-series}$$

$$\frac{0}{x^2} \leq \frac{\cos^2(x)}{x^2} \leq \frac{1}{x^2}$$

Since $\displaystyle\int_1^{\infty} \frac{1}{x^2}\,dx$ converges and is larger

then $\displaystyle\int_1^{\infty} \frac{\cos^2(x)}{x^2}\,dx$ will also converge.

21. $\displaystyle\int_1^\infty \frac{2+\cos x}{\sqrt{x^4+x^2}}\,dx$

[-1,1]

$$\frac{2+\cos(x)}{\sqrt{x^4+x^2}} \sim \frac{2+\cos(x)}{\sqrt{x^4}} = \frac{2+\cos(x)}{x^2}$$

[-1,1]

$$\frac{1}{x^2} \le \frac{2+\cos(x)}{x^2} \le \frac{3}{x^2}$$

Since $\displaystyle\int_1^\infty \frac{3}{x^2}\,dx$ converges and is larger,

$\displaystyle\int_1^\infty \frac{2+\cos(x)}{\sqrt{x^4+x^2}}\,dx$ will also converge.

22. $\displaystyle\int_1^\infty \frac{2+e^{-x}}{x}\,dx$

$$= \underbrace{\int_1^\infty \frac{2}{x}\,dx}_{\text{diverges}} + \int_1^\infty \frac{e^{-x}}{x}\,dx$$

$$e^{-x} = \frac{1}{e^x} > 0$$

$$\frac{2+e^{-x}}{x} \ge \frac{2}{x}$$

Since $\displaystyle\int_1^\infty \frac{2}{x}\,dx$ diverges and is smaller,

$\displaystyle\int_1^\infty \frac{2+e^{-x}}{x}\,dx$ also diverges.

23. Is the integral $\int_{-1}^{2} \frac{x}{(x+1)^2}\,dx$ convergent or divergent?

$f(x) = \frac{x}{(x+1)^2}$

D: $x \neq -1$ $\downarrow$ lower bound.

$\lim_{x \to (-1)^+}$

$$\int_{(-1)^+}^{2} \frac{x}{(x+1)^2}\,dx \qquad u = x+1,\quad du = dx,\quad x = u-1$$

$$= \int \frac{u-1}{u^2}\,du = \int \frac{u}{u^2}\,du - \int \frac{1}{u^2}\,du$$

$$= \ln|u| - \left(-\frac{1}{u}\right)$$

$$= \ln|x+1| + \frac{1}{(x+1)}\Bigg|_{(-1)^+}^{2}$$

$$= \left[\ln(3) + \frac{1}{3}\right] - \left[\ln(0^+) + \frac{1}{0^+}\right]$$

$(-\infty + \infty)$ Indeterminate difference $\to$ L'H

$$\lim_{x\to(-1)^+}\left[\ln|x+1| + \frac{1}{(x+1)}\right]$$

$$\lim_{x\to(-1)^+}\left[\frac{(x+1)\ln|x+1| + 1}{(x+1)}\right]$$

But, $\lim_{x\to(-1)^+} \underbrace{(x+1)\ln(x+1)}_{(0)(-\infty)} \to 0$

rewrite as, $\frac{\ln(x+1)}{\frac{1}{(x+1)}} \sim \frac{-\infty}{\infty}$

$$\xrightarrow{L'H} \frac{\left(\frac{1}{(x+1)}\right)}{-\left(\frac{1}{(x+1)^2}\right)} = -(x+1) = 0$$

Then, $\frac{0+1}{0^+} \to \infty$.

Hence, $\int_{-1}^{2} \frac{x}{(x+1)^2}\,dx$ diverges

24. Is the integral $\int_{-\infty}^{0} \frac{1}{3-4x}\,dx$ convergent or divergent?

$$\int_{-\infty}^{0} \frac{1}{3-4x}\,dx = \frac{\ln|3-4x|}{-4}\Bigg|_{-\infty}^{0}$$

$$= -\frac{1}{4}\left[\ln 3 - \underbrace{\ln(3+\infty)}_{\infty}\right]$$

$$\to \infty$$

$\therefore$ Integral diverges.

25. Use partial fractions to evaluate the integral $\int \frac{4x}{x^3+x^2+x+1}\,dx$

$$\int \frac{4x}{x^3+x^2+x+1}\,dx = \int \frac{4x}{x^2(x+1)+(x+1)}\,dx$$

$$= \int \frac{4x}{(x^2+1)(x+1)}\,dx \longrightarrow \text{PFD}$$

$$\frac{4x}{(x^2+1)(x+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+1)}$$

$$4x = A(x^2+1) + (Bx+C)(x+1)$$

$$= Ax^2 + A + Bx^2 + Bx + Cx + C$$

$$\therefore x^2(0) + x(4) + 0 = x^2(A+B) + x(B+C) + (A+C)$$

Then $A+B=0$, $A=-B$ | $B+C=4$, Since $B=C$, $B=C=2$, $A=-2$ | $A+C=0$, $A=-C$, But $A=-B$ $\therefore B=C$

$$= \int \frac{-2}{x+1}\,dx + \int \frac{2x+2}{(x^2+1)}\,dx$$

usub: $u=x^2+1$, $du=2x\,dx$, $2\int \cdot \frac{1}{2}\frac{du}{u} = \ln|u|$

$$= -2\int \frac{dx}{x+1} + \underbrace{2\int \frac{x}{x^2+1}\,dx} + 2\int \frac{1}{x^2+1}\,dx$$

$$= -2\ln|x+1| + \ln(x^2+1) + 2\arctan(x) + C$$

no absolute value needed for sum of squares which is always positive.

26. Evaluate the integral $\int_0^3 \frac{x}{\sqrt{36-x^2}}\,dx$ $\longrightarrow$ radical $+x^2$
trigonometric substitution

$\sqrt{36-x^2} \sim \sqrt{a^2-x^2}$

$x = a\sin\theta$, $a = \sqrt{36} = 6$.

Sub: $x = 6\sin\theta$

$dx = 6\cos\theta\, d\theta$

$\sqrt{36-x^2} = 6\cos\theta$

If we change bounds

$\boxed{x = 6\sin\theta}$

upper bound $x = 3$

$\sin\theta = \frac{3}{6} = \frac{1}{2}$

$\theta = \pi/6$.

Lower bound.

$x = 0$

$\sin\theta = \frac{0}{6} = 0$

$\theta = 0$.

$$= \int_{\theta=0}^{\theta=\pi/6} \frac{(6\sin\theta)(6\cos\theta\, d\theta)}{(6\cos\theta)}$$

$$= 6\int_0^{\pi/6} \sin\theta\, d\theta$$

$$= -6\cos\theta \Big|_0^{\pi/6}$$

$$= -6\left[\cos\left(\frac{\pi}{6}\right) - \cos(0)\right]$$

$$= -6\left[\frac{\sqrt{3}}{2} - 1\right]$$

$$= -3\sqrt{3} + 6$$ Ans

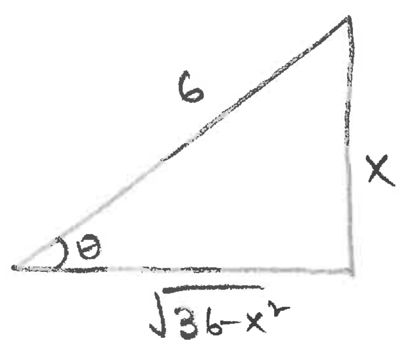
without changing bounds.

$$6\int \sin\theta\, d\theta = -6\cos\theta$$

$$= -6\,\frac{\sqrt{36-x^2}}{6}$$

$$= -\sqrt{36-x^2}\Big|_0^3$$

$$= -\left[\sqrt{36-9} - \sqrt{36}\right] = -\left[\sqrt{27} - \sqrt{36}\right] = -3\sqrt{3} + 6$$




$x = 6\sin\theta$

$\frac{x}{6} = \sin\theta$

$\therefore \cos\theta = \frac{\sqrt{36-x^2}}{6}$.

Same Ans.