3.2: SOLUTIONS OF LINEAR HOMOGENEOUS ODES

Review

- **Existence and uniqueness:** Consider the initial value problem

$$y'' + p(t)y' + q(t)y = g(t), \qquad y(t_0) = y_0, \qquad y'(t_0) = y_0'.$$

  If p, q, and g are ________________ on an open interval $I = (a, b)$ that contains the point t_0, then there is exactly one solution to the initial value problem and the solution exists throughout the entire interval I.

- **Principle of superposition:** If y_1 and y_2 are solutions to a homogeneous ODE, then

- A set of functions is called a **fundamental set of solutions** if adding them together with constants forms the general solution.

- The **Wronskian** of y_1 and y_2 is defined as

- Interpretation of the Wronskian:

- The Wronskian only needs to be checked at a single value of t in the interval where the solution exists.

Exercise 1

Is the following initial value problem guaranteed to have a unique solution? If so, on what interval is it guaranteed to exist?

$$y'' - \sec(t)y' + (t^2+1)y = \sqrt{3t-7}, \qquad y(3) = -3, \qquad y'(3) = 2.$$

Exercise 2

Is the following initial value problem guaranteed to have a unique solution? If so, on what interval is it guaranteed to exist?

$$tf'' + \sin(t)f' + \ln(t+2)f = 8, \qquad f(-1) = 7, \qquad f'(-1) = -4.$$

Exercise 3

Do $y_1(t) = e^t$ and $y_2(t) = e^{-3t}$ form a fundamental set of solutions for the following differential equation?

$$y'' - 3y' + 3y = 0.$$

Exercise 4

Do $y_1(t) = e^t$ and $y_2(t) = t+1$ form a fundamental set of solutions for the following differential equation?

$$ty'' - (t+1)y' + y = 0, \qquad t < 0.$$

Exercise 5

Do $y_1(t) = \cos(t)$ and $y_2(t) = \sin(t + \pi)$ form a fundamental set of solutions to the following differential equation?

$$y'' + y = 0.$$

Exercise 6

Show that $y(t) = c_1 t + c_2 t \ln(t)$ is the general solution to the differential equation,

$$t^2 y'' - ty' + y = 0, \qquad t > 0.$$

3.3 & 3.4: SECOND ORDER LINEAR ODES

Review

- A **second order linear ODE with constant coefficients** has the form

$$ay'' + by' + cy = g(t).$$

- It is **homogeneous** if $g(t) = 0$.
- Process for **solving** a second order homogeneous linear ODE:
  1. Look for solutions of the form $y(t) = e^{rt}$.
  2. Find the characteristic equation.
  3. Find the roots of the characteristic equation.
  4. The general solution is given by
     - Distinct real roots:
     - Complex roots:
     - Repeated real roots:
  5. If you have initial conditions, use them to solve for c_1 and c_2.

Exercise 7

Find the general solution to the differential equation

$$y'' + 10y' + 25y = 0.$$

Exercise 8

Find the general solution to the differential equation

$$y'' - 9y' + 20y = 0.$$

Exercise 9

Solve the initial value problem

$$f'' - 2f' + 8f = 0, \quad f(0) = 0, \quad f'(0) = 1.$$

Exercise 10

Find the general solution to the differential equation

$$4g'' + g = 0.$$

Exercise 11

Find the general solution to the differential equation

$$3y'' - 2y' - y = 0$$

Exercise 12

Solve the initial value problem

$$f'' - 4f' + 4f = 0, \qquad f(0) = 2, \qquad f'(0) = -1.$$