Note: This review is based only on sections 14.7 & 14.8 and 16.7 - 16.9. Students are encouraged to review the previous week's WIR sessions that cover the final exam topics and all other resources provided by their professor.

Example 1 (14.7)**.** *Find the local maximum and minimum values and saddle points of the function*

$$f(x, y) = x^3 + y^3 - 3x^2 - 3y^2 - 9y + 2.$$

Example 2 (14.7)**.** *Find three positive numbers whose sum is 50 and the sum of whose squares is the minimum.*

Example 3 (14.7)**.** *Find the absolute maximum and minimum values of the function*

$$f(x, y) = x + y - xy$$

on D, *where* D *is a triangular region with vertices* $(-3, 0), (3, 0),$ *and* $(0, 3)$.

Example 4 (14.8)**.** *Use Lagrange multipliers' method to find the extreme values of the function*

$$f(x, y) = x^2 y$$

subject to the constraint $x^2 + y^2 = 9$.

Suppose f is a continuous function defined on a surface S that is given by a vector valued function

$$\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$$

defined over a region D in the uv-plane. Then the surface integral of f over S is

$$\iint_S f(x,y,z)\,dS = \iint_D f(\mathbf{r}(u,v))|\mathbf{r}_u \times \mathbf{r}_v|\,dA$$

In particular, if the surface S is given by a function $z = g(x,y)$ defined over a region D in the xy-plane, then

$$\iint_S f(x,y,z)\,dS = \iint_D f(x,y,g(x,y))|\mathbf{r}_x \times \mathbf{r}_y|\,dA,$$

where $\mathbf{r}(x,y) = \langle x,\, y,\, g(x,y)\rangle$ is a parametric representation of S defined over D.

Example 5 (16.7)**.** *Evaluate $\displaystyle\iint_S xz\,dS$, where S is the part of the plane $x + 2y + z = 6$ that lies in the first octant.*

Example 6 (16.7)**.** *Evaluate* $\iint\limits_S 2z^2\,dS$, *where* S *is the part of the surface* $y = x^2 + z^2$ *between the planes* $y = 1$ *and* $y = 4$.

Surface integral of a vector field on parametric surface: *If a surface S is given by the vector function $\mathbf{r}(u,v)$ defined on the parameter domain D, then the surface integral of a continuous vector field $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$ is*

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v)\, dA \qquad (= \textit{Flux})$$

Here the orientation of S is induced by $\mathbf{r}_u \times \mathbf{r}_v$.

Surface integral of a vector field on a surface that is a graph of a function:. *If a surface S is given by a graph of a function $z = g(x,y)$, then a parametrization of the surface is $\mathbf{r}(x,y) = x\,\mathbf{i} + y\,\mathbf{j} + g(x,y)$ then*

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_D \mathbf{F} \cdot (\mathbf{r}_x \times \mathbf{r}_y)\, dA \qquad (= \textit{Flux})$$

where D is the projection of the surface S onto the xy-plane.

Example 7 (16.7)**.** *Compute* $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S}$, *where* $\mathbf{F}(x,y,z) = y\,\mathbf{i} - x\,\mathbf{j} + 2z\,\mathbf{k}$ *and* S *is the part of the sphere* $x^2+y^2+z^2=1$, $z \geq 0$, *oriented downward.*

Example 8 (16.7)**.** *Compute* $\iint\limits_S \mathbf{F} \cdot d\mathbf{S}$, *where* $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + 5\,\mathbf{k}$ *and* S *is the boundary of the cylinder* $x^2+z^2 = 1$ *and planes* $y = 0$ *and* $x+y = 2$ *with outward orientation.*

Example 9 (16.8)**.** *Use the Stokes' Theorem to evaluate* $\iint_S \text{curl}\mathbf{F} \cdot d\mathbf{S}$*, where*

$$\mathbf{F}(x, y, z) = \left\langle z\, e^{xy},\, -x^2\, \cos(yz),\, xz \sin^2 y \right\rangle$$

and S is the part of the sphere $x^2 + y^2 + z^2 = 9$, $y \geq 0$, oriented in the direction of positive y-axis.

Example 10 (16.8). *Use the Stokes' Theorem to evaluate* $\displaystyle\oint_C \mathbf{F}\cdot d\mathbf{r}$, *where*

$$\mathbf{F}(x,y,z) = \langle 2y + e^x\,, xy,\, 2+2z\rangle$$

and C *is the triangle with vertices* $(1,0,0)$, $(0,1,0)$, $(0,0,2)$ *with positive orientation.*

Example 11 (16.8)**.** *Use the Stokes' Theorem to evaluate* $\displaystyle\oint_C \mathbf{F} \cdot d\mathbf{r}$, *where*

$$\mathbf{F}(x, y, z) = \langle 2y\,, x,\, xz\rangle$$

and C *is the boundary of the paraboloid* $z = 1 - x^2 - y^2$ *in the first octant with positive orientation from eagle's view.*

Example 12 (16.9)**.** *(Example 8 above) Use the Divergence Theorem to compute* $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S}$, *where* $\mathbf{F}(x,y,z) = x\,\mathbf{i} + y\,\mathbf{j} + 5\,\mathbf{k}$ *and* S *is the boundary of the cylinder* $x^2+z^2=1$ *and planes* $y=0$ *and* $x+y=2$ *with outward orientation.*

Example 13 (16.9). *Use the Divergence Theorem to compute* $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$, *where*

$$\mathbf{F}(x, y, z) = \left\langle x^2 + yz,\, \sin x - 2yz,\ z^2 + 3 \right\rangle$$

and S *is the boundary of the tetrahedron with vertices* $(0, 0, 0)$, $(2, 0, 0)$, $(0, 2, 0)$, $(0, 0, 2)$, *with outward orientation.*