



Math 151
Week-In-Review 14

5.4, 5.5
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Problem Statements

1. Find the general indefinite integral.

(a) $\int \left(3x^2 + 8x - 5 + \frac{2}{x^3} + \frac{3}{x} + \sqrt[5]{x^2} \right) dx$

(b) $\int (11e^x - 3^x + 2 \cdot 5^x + 2^{3x} - 7^{x+5}) dx$

(c) $\int (2 \sin x + 4 \cos x - 6 \sec^2 x + 8 \sec x \tan x) dx$

(d) $\int \left(3 \csc^2 x - 5 \csc x \cot x + \frac{7}{x^2 + 1} - \frac{9}{\sqrt{1 - x^2}} \right) dx$



2. A water balloon is being filled at a constant rate of $100 \text{ cm}^3/\text{s}$.
- (a) Find an equation representing the volume of water in the balloon after t seconds.

 - (b) How much water is added to the balloon in the interval from $t = 1$ to $t = 3$?

 - (c) Suppose instead that the balloon immediately springs a leak, and water begins to drain from the balloon at a rate of $0.5V$. (That is, more water in the balloon results in water leaking out of the balloon faster). Write a *differential equation* representing the total rate of change of the volume of water in the balloon.

 - (d) Verify $V = 200 - 200e^{-0.5t}$ is a solution to the differential equation.

 - (e) Find $V'(t)$.

 - (f) What is the net change in the amount of water in the balloon in the interval from $t = 1$ to $t = 3$?

 - (g) What is the net change in the amount of water in the balloon in the interval from $t = 101$ to $t = 103$?



3. The velocity function for a particle moving along a straight line is given by $v(t) = 3t^2 - 3t - 6$. On the interval $1 \leq t \leq 3$, determine

(a) The displacement of the particle.

(b) The total distance traveled by the particle.



4. If $h(t)$ represents a person's heart rate in beats per minute t minutes into a workout, what does $\int_0^{30} h(t) dt$ represent?

5. If $s(m)$ represents the slope of a trail at a distance of m miles from the trailhead, what does $\int_2^4 s(m) dm$ represent?



6. Evaluate

(a) $\frac{d}{dx} [(x^2 + 1)^{100}]$

(b) $\int 100(x^2 + 1)^{99} \cdot 2x \, dx$

(c) $\int x^3(x^4 + 1)^{99} \, dx$

(d) $\int x^4 \cdot x^3 \, dx$



7. Evaluate

(a) $\int \sqrt{2x+1} \, dx$

(b) $\int \sqrt{2x+1} \, dx$

(c) $\int 4 \cos(x) \sqrt{\sin(x)+1} \, dx$

(d) $\int x^3 \sqrt{x^2+1} \, dx$



8. Evaluate

(a) $\int \tan x \cdot \sec^2(x) dx$

(b) $\int \tan x \cdot \sec^2(x) dx$

9. Evaluate

(a) $\int e^{8x} dx$

(b) $\int_{1/8}^2 e^{8x} dx$

(c) $\int_{1/8}^1 e^{8x} dx$



10. Evaluate

(a) $\int_0^1 t^2(2t^3 + 3)^{20} dt$

(b) $\int_1^2 x(2x + 5)^{10} dx$



(c) $\int_1^4 \frac{1}{(x+1)\sqrt{x}} dx$

(d) $\int_0^1 \frac{1}{(1+\sqrt{x})^4} dx$