MATH 140: WEEK-IN-REVIEW 3
(2.3, 2.4 & REVIEW QUESTIONS OVER CHAPTERS 1 & 2)

Solve for (x,y)

1. Find the exact intersection point of the lines $\underline{7x - 11y = 25}$ and $\underline{-5x + 6y = -16}$

Addition Method

$7x - 11y = 25$ (times 6)

$-5x + 6y = -16$ (times 11)

$\Downarrow$

$42x - 66y = 150$

$-55x + 66y = -176$

$-13x = -26$

$x = 2 \Rightarrow -5(2) + 6y = -16$

$6y = -6 \Rightarrow \boxed{y=-1}$

Soln $\boxed{(x,y) = (2,-1)}$

Calculator

$$\left[\begin{array}{cc|c} x & y & \text{cons} \\ 7 & -11 & 25 \\ -5 & 6 & -16 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} x & y & \text{const} \\ 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right]$$

$x = 2$
$y = -1$

$\boxed{(x,y) = (2,-1)}$

2. Given the system

$$\left\{\begin{array}{l} -3x + 4y = 20 \Rightarrow y = \frac{3}{4}x + 5 \\ kx - 8y = -16 \Rightarrow y = \frac{k}{8}x + 2 \end{array}\right\} \text{slope-intercept form}$$

For what value(s) of k is/are there

(a) infinitely many solutions? $\boxed{m_1 = m_2 \text{ and } b_1 = b_2}$

$\frac{3}{4} = \frac{k}{8} \Rightarrow (3)(\overset{2}{\cancel{8}}) = (k)(\cancel{4})$

$\Rightarrow \boxed{k = 6}$

$b_1 = 5,\ b_2 = 2$

$b_1 \neq b_2$

* no value of k with infinitely many solutions *

(b) no solutions ? $\boxed{m_1 = m_2 \text{ and } b_1 \neq b_2}$

From part (a), $k = 6 \Rightarrow m_1 = \frac{3}{4}$ and $m_2 = \frac{6}{8} = \frac{3}{4}$ ✓

$b_1 = 5,\ b_2 = 2 \Rightarrow b_1 \neq b_2$ ✓

$\boxed{k = 6}$

(c) exactly one solution? $\boxed{m_1 \neq m_2}$

$\frac{3}{4} \neq \frac{k}{8} \Rightarrow (3)(8) \neq (4)(k)$

$\Rightarrow k \neq 6$

$\boxed{\text{* Exactly one solution if } k \neq 6 \text{, and } k \text{ is a real number *}}$

3. Solve the following system of equations using the substitution method.

$$\begin{cases} -2x+3y=-7 \\ 6y-4x=16 \end{cases}$$

* easiest to solve for x *

$$6y-4x=16 \Rightarrow \frac{-4x}{-4} = \frac{16}{-4} - \frac{6y}{-4}$$

$$\Rightarrow \quad x = -4+\frac{3}{2}y$$

$$-2\left(-4+\frac{3}{2}y\right)+3y=-7 \Rightarrow 8-\cancel{3y}+\cancel{3y}=-7$$

$$\boxed{8=-7}$$

* this is a contradiction! *

* system is inconsistent *

* no solutions *

$$\boxed{\text{* The system has no solutions *}}$$

4. Solve the following system of equations using the addition method.

$$\begin{cases} 3x-2y=7 \\ -6x+4y=-14 \end{cases}$$

(times 2)

$$\begin{array}{rcl} 6x-4y &=& 14 \\ -6x+4y &=& -14 \quad (\text{add}) \\ \hline 0 &=& 0 \end{array}$$

(identity)

* system is dependent *

* system has infinitely many solutions *

* solutions to be represented in parametric form *

$$3x-2y=7 \Rightarrow 3x=7+2y$$

$$x=\frac{7}{3}+\frac{2}{3}y$$

* Let $y=t$ (parametrize)

$$\boxed{\text{SOLN: } (x,y)=\left(\frac{7}{3}+\frac{2}{3}t,\ t\right),\ t \text{ is any real number}}$$

e.g. when $t=0$, $(x,y)=\left(\frac{7}{3},0\right)$ is one specific solution

5. A company has a profit function $P(x) = 12.5x - 14250$, where x represents the number of gadgets made and sold by the company, and the profit is given in dollars. Suppose that each item is sold for \$30.

$p = 30$

(a) Determine the company's cost and revenue functions.

Revenue price per item

$p = 30$, $R(x) = px$

$\boxed{R(x) = 30x}$

Cost production costs per unit

$C(x) = mx + F$ fixed costs

$= 17.5x + 14250$

$\boxed{C(x) = 17.5x + 14250}$

Profit profit per item fixed costs

$P(x) = 12.5x - 14250$

profit per item $= 12.5 =$ price per item $-$ production costs per item

$= 30 - m$

$m = 30 - 12.5$

$= 17.5$

(b) Determine the company's break-even point and explain the meaning of each coordinate of the break-even point in the context of the application.

* At the break-even point, $R(x) = C(x)$ *

OR $P(x) = 0$

$R(x) = C(x)$

$30x = 17.5x + 14250$

$12.5x = 14250$

$x = \frac{14250}{12.5} = 1140$ (BE quantity)

OR $P(x) = 0$

$12.5x - 14250 = 0$

$12.5x = 14250$

$x = \frac{14250}{12.5} = 1140$

$R(1140) = (30)(1140) = 34200$ (BE revenue)

$\boxed{(1140, 34200) \text{ BE point}}$

* It costs \$34,200 to make 1140 gadgets, and when 1140 gadgets are sold, \$34,200 is received in revenue, exactly covering the costs *

6. Determine the equilibrium point for a marketplace with demand and supply for x record players (in units of 100) given by $p(x) = -2.5x + 250$ and $p(x) = 3.5x + 40$, respectively, where $p(x)$ is in dollars. Then write a sentence explaining the meaning of the coordinates of the point found, in the context of the application.

DEMAND EQN : → negative slope

$p(x) = -2.5x + 250$

SUPPLY EQN: → positive slope

$p(x) = 3.5x + 40$

Demand = Supply (at market equilibrium)

$$-2.5x + 250 = 3.5x + 40$$

$$250 - 40 = 3.5x + 2.5x$$

* units of 100 !

$$\frac{210}{6} = \frac{6x}{6} \Rightarrow x = 35, \quad \boxed{x = 3500 \text{ record players}}$$

(equibrium quantity)

$$p(35) = -2.5(35) + 250$$

$$= 162.5 \quad \text{(equilibrium price)}$$

EQUILIBRIUM POINT

$$(\underbrace{35}_{\text{hundreds}}, \underbrace{162.5}_{\text{dollars}}) = (3500 \text{ record players}, \$162.5)$$

* At a price of \$162.5 (↳ per record player), suppliers will market 3500 record players, and at the same price of \$162.5 (↳ per record player), consumers will buy all 3500 record players. There is no shortage or surplus of record players in the market *

7. Given the system of equations below, write the corresponding augmented matrix

$$\begin{cases} 3x + 2y - 4z = 4 \\ 4y + z = 2 + 2x \\ 6z - 1 = 3x - 2y \end{cases}$$

* line up variables first *

$$\left.\begin{array}{rcl} 3x + 2y - 4z &=& 4 \\ -2x + 4y + z &=& 2 \\ -3x + 2y + 6z &=& 1 \end{array}\right\} \begin{array}{l} EQ\ 1 \to R_1 \\ EQ\ 2 \to R_2 \\ EQ\ 3 \to R_3 \end{array}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ C1 & C_2 & C_3 \end{array} \hookrightarrow$$

$$\begin{array}{ccc|cl} x & y & z & \text{const} & \\ 3 & 2 & -4 & 4 & R_1 \\ -2 & 4 & 1 & 2 & R_2 \\ -3 & 2 & 6 & 1 & R_3 \\ C1 & C2 & C_3 & & \end{array}$$

* augmented matrix *

8. What system of linear equations would result in the following augmented matrix? (Assuming variables x, y, and z)

$$\begin{array}{ccc|cl} x & y & z & \text{const.} & \\ 2 & 0 & -1 & 3 & R_1 \to EQ\ 1 \\ 1 & 4 & 9 & 0 & R_2 \to EQ\ 2 \\ -2 & 1 & 3 & -1 & R_3 \to EQ\ 3 \\ \downarrow & \downarrow & \downarrow & & \\ C1 & C2 & C_3 & & \end{array}$$

$$\begin{array}{lrcl} & \overset{C_1}{\downarrow}\quad \overset{C_2}{\downarrow}\quad \overset{C_3}{\downarrow} & & \\ EQ1: & 2x + 0y - z &=& 3 \\ EQ\ 2: & x + 4y + 9z &=& 0 \\ EQ\ 3: & -2x + y + 3z &=& -1 \end{array}\Bigg\}$$

* system of equations *

⇓ simplify

$$\boxed{\begin{array}{l} 2x - z = 3 \\ x + 4y + 9z = 0 \\ -2x + y + 3z = -1 \end{array}}$$

9. Perform the given row operations in the Gauss-Jordan Elimination Method, and show the resulting matrices.

$$\begin{array}{lrrrr} -2R_2: & 0 & -2 & 4 & -10 \\ +R_1: & 1 & 2 & -3 & 11 \end{array}$$

$$\boxed{R_1: \quad 1 \quad 0 \quad 1 \quad 1}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 11 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 1 & -1 \end{array}\right] \xrightarrow{-2R_2+R_1\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\begin{array}{lrrrr} -R_3: & 0 & 0 & -1 & 1 \\ +R_1: & 1 & 0 & 1 & 1 \end{array}$$

$$\boxed{R_1: \quad 1 \quad 0 \quad 0 \quad 2}$$

$$\xrightarrow{-R_3+R_1\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\begin{array}{lrrrr} 2R_3: & 0 & 0 & 2 & -2 \\ +R_2: & 0 & 1 & -2 & 5 \end{array}$$

$$\boxed{R_2: \quad 0 \quad 1 \quad 0 \quad 3}$$

$$\xrightarrow{2R_3+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

10. Determine whether the following matrices are in reduced row echelon form.

If YES, write the final simplified system and state the solution.

If NO, write the next best row operation you would use in the Gauss-Jordan Elimination Method.

RREF

1. zero rows, if any, at bottom
2. leading ones left-most entries in each row
3. leading ones in next rows to the right of leading rows above
4. leading ones are the only non-zero entries in their columns

1. X

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right]$

* not RREF.
* fails 1

* next step: interchange rows 3 and 4

$R_3 \Leftrightarrow R_4$

1. ✓
2. X

(b) $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -4 & 6 \end{array}\right]$

* not RREF
* fails 2

* next step: replace row 3 by $-\frac{1}{4}$ row 3

$-\frac{1}{4}R_3 \rightarrow R_3$

1. ✓
2. ✓
3. X

(c) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 1 & 0 & 7 \end{array}\right]$

* not RREF
* fails 3

* next step: interchange rows 2 and 3

$R_2 \Leftrightarrow R_3$

1. ✓
2. ✓
3. ✓
4. X

(d) $\left[\begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]$

* not RREF
* fails 4

* next step: replace 3 by 0 in row 1

$-3R_2 + R_1 \rightarrow R_1$

1. ✓
2. ✓
3. ✓
4. ✓

(e) $\begin{array}{c} \begin{array}{cccc} x & y & z & \text{const} \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & 5 & 20 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$

* RREF ✓
* passes all 4 conditions

$$\begin{cases} x + 3z = 10 \\ y + 5z = 20 \\ 0 = 0 \text{ (identity)} \end{cases} \Rightarrow \text{let } z = t \Rightarrow (x, y, z) = (10-3t, 20-5t, t)$$

dependent

SOLUTION

t is any real number

* using calculator *

11. Solve the following system of equations. If there are infinitely many solutions, find both the parametric solution and one specific solution. State whether the system is independent, inconsistent or dependent.

(a)

$$\boxed{\begin{aligned}3x+2y-4z&=-3\\-2x+4y+z&=5\\-x+10y-2z&=7\end{aligned}}$$

re-arranged

$$\begin{cases}3x+2y-4z=-3\\4y+z=5+2x\\10y-x=7+2z\end{cases}$$

* rearrange to match variables *

$$\begin{array}{c|ccc|c} & x & y & z & \text{const} \\ R1 & 3 & 2 & -4 & -3 \\ R_2 & -2 & 4 & 1 & 5 \\ R_3 & -1 & 10 & -2 & 7 \end{array} \xrightarrow{\text{RREF}} \begin{array}{ccc|c} x & y & z & \text{const.} \\ 1 & 0 & -9/8 & -11/8 \\ 0 & 1 & -5/16 & 9/16 \\ 0 & 0 & 0 & 0 \end{array}$$

$x - \frac{9}{8}z = -\frac{11}{8} \Rightarrow x = -\frac{11}{8} + \frac{9}{8}z$

$y - \frac{5}{16}z = \frac{9}{16} \Rightarrow y = \frac{9}{16} + \frac{5}{16}z$

$0 = 0$ (identity!)

* infinitely many solns *

* dependent system *

Let $z = t$ (parametrize)

$$\boxed{(x,y,z) = \left(-\tfrac{11}{8} + \tfrac{9}{8}t,\ \tfrac{9}{16} + \tfrac{5}{16}t,\ t\right) \text{ where } t \text{ is any real number}}$$

* one specific solution $(x,y,z) = \left(-\frac{11}{8}, \frac{9}{16}, 0\right)$ when $t = 0$ *

(b)

$$\boxed{\begin{aligned}3x+2y-4z&=-3\\-2x+4y+z&=5\\-3x+2y+6z&=15\end{aligned}} \Longleftrightarrow \begin{cases}3x+2y-4z=-3\\4y+z=5+2x\\6z-15=3x-2y\end{cases}$$

re-arranged

$$\begin{array}{c|ccc|c} & x & y & z & \text{const} \\ R_1 & 3 & 2 & -4 & -3 \\ R_2 & -2 & 4 & 1 & 5 \\ R_3 & -3 & 2 & 6 & 15 \end{array} \xrightarrow{\text{RREF}} \begin{array}{ccc|c} x & y & z & \text{const.} \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 3 \end{array} \Rightarrow \left.\begin{aligned}x&=2\\y&=1.5\\z&=3\end{aligned}\right\}$$

* exactly one solution

* independent systems

$$\boxed{\text{SOLN: } (x,y,z) = (2, 1.5, 3)}$$

(c)

$$\boxed{\begin{aligned}x-3y+z&=4\\-x+2y-5z&=3\\5x-13y+13z&=8\end{aligned}} \Longleftrightarrow \begin{cases}x+z=4+3y\\2y-5z=3+x\\5x+13z=8+13y\end{cases}$$

$$\begin{array}{c|ccc|c} & x & y & z & \text{const.} \\ R_1 & 1 & -3 & 1 & 4 \\ R_2 & -1 & 2 & -5 & 3 \\ R_3 & 5 & -13 & 13 & 8 \end{array} \xrightarrow{\text{RREF}} \begin{array}{ccc|c} x & y & z & \text{const.} \\ 1 & 0 & 13 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{array} \Rightarrow \begin{aligned}x+13z&=0\\y+4z&=0\\0&=1 \ \text{X}\end{aligned}$$

* contradiction

* inconsistent system

* no solutions

$$\boxed{\text{* The system has no solutions *}}$$

For the next two problems, set up a system of linear equations representing the given problem, then find a solution to the given problem.

12. A private school orders three types of plastic shapes for children: rectangles, squares, and triangles. Suppose the private school wants to have 210 squares, 194 rectangles, and 243 triangles for their math class, and they come in three different types of boxes: small, medium and large boxes. The small box contains 5 triangles, 2 rectangles and 4 squares. The medium box contains 12 rectangles, 14 triangles and 10 squares. The large box contains 20 squares, 18 rectangles and 19 triangles. Assuming that all the required shapes will be packaged without left overs, how many of each type of box will the private school order?

Variables : * read the question to decide which quantities to define as variables (the unknowns)

* how many of each type of box ⇒ define variables for box sizes

s = number of small boxes
m = number of medium boxes
l = number of large boxes

Equations : * look for constraints in the variables *

* constraints — total number of rectangles, squares, triangles

$$\overset{\text{small}}{2s} + \overset{\text{medium}}{12m} + \overset{\text{large}}{18l} = 194$$

(rectangles) : $2s + 12m + 18l = 194$ ← total rectangles

(triangles) : $5s + 14m + 19l = 243$ ← total number of triangles

(squares) : $4s + 10m + 20l = 210$ ← total number of squares

$$\left[\begin{array}{ccc|c} s & m & l & \text{const.} \\ 2 & 12 & 18 & 194 \\ 5 & 14 & 19 & 243 \\ 4 & 10 & 20 & 210 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 5 \end{array}\right] \Rightarrow \left.\begin{array}{l} s = 10 \\ m = 7 \\ l = 5 \end{array}\right\}$$

SOLN

$\boxed{\text{10 small boxes, 7 medium boxes, and 5 large boxes}}$

13. A company decides to spend \$5 million on radio, magazine and TV advertising. If the company spends as much money on TV advertising as on radio and magazine together, and the amount spent on magazines and TV combined equals three times that spent on radio, what is the amount to be spent on each type of advertising?

Variables * clue : amount spent on each type of advertising

r = amount spent on radio

m = amount spent on magazines

t = amount spent on TV

Equations

* total spending is \$ 5 million : $r+m+t=5$ (in millions)

* spends as much on TV as on radio + magazine together : $t = m+r$

* amount spend on magazines + TV combined equals 3 times spent on radio : $m+t = 3r$

$$\begin{array}{ccc|c} r & m & t & \text{Const} \\ 1 & 1 & 1 & 5 \\ -1 & -1 & 1 & 0 \\ -3 & 1 & 1 & 0 \end{array} \xrightarrow{\text{RREF}} \begin{array}{ccc|c} r & m & t & \\ 1 & 0 & 0 & 1.25 \\ 0 & 1 & 0 & 1.25 \\ 0 & 0 & 1 & 2.5 \end{array} \Rightarrow \left.\begin{array}{l} r = 1.25 \\ m = 1.25 \\ t = 2.5 \end{array}\right\}$$

SOLN: \$ 1.25 million is spent on radio advertising
\$ 1.25 million is spent on magazine advertising
\$ 2.50 million is spent on TV advertising

* check
$r+m+t = 5$ ✓
$t = m+r$ ✓
$m+t = 3r$ ✓

14. If A is a 3×4 matrix, B is a 3×4 matrix, and C is a 4×3 matrix, determine the size of $(3A+4B)^T-5C$, if possible.

$$\underbrace{(\underset{\substack{3\times 4\\ \checkmark}}{3A}+\underset{\substack{3\times 4\\ \checkmark}}{4B})^T}_{4\times 3} - \underbrace{5C}_{4\times 3} \quad \checkmark$$

$$\boxed{4\times 3}$$

15. Determine the values of w, x, and y given $\begin{bmatrix} 2 & w-1 \\ 2 & 4x \end{bmatrix} - \begin{bmatrix} y & -6 \\ -8 & 12 \end{bmatrix}^T = 2\begin{bmatrix} -1 & 9 \\ 4 & -4 \end{bmatrix}$

$\Downarrow$ simplify

$$\begin{bmatrix} 2 & w-1 \\ 2 & 4x \end{bmatrix} - \begin{bmatrix} y & -8 \\ -6 & 12 \end{bmatrix} = \begin{bmatrix} -2 & 18 \\ 8 & -8 \end{bmatrix}$$

$$\begin{bmatrix} 2-y & w-1-(-8) \\ 2-(-6) & 4x-12 \end{bmatrix} = \begin{bmatrix} -2 & 18 \\ 8 & -8 \end{bmatrix}$$

$$\begin{bmatrix} 2-y & w+7 \\ \underset{\checkmark}{8} & 4x-12 \end{bmatrix} = \begin{bmatrix} -2 & 18 \\ \underset{\checkmark}{8} & -8 \end{bmatrix} \Rightarrow \begin{aligned} 2-y &= -2 \Rightarrow y = 4 \\ w+7 &= 18 \Rightarrow w = 11 \\ 8 &= 8 \checkmark \\ 4x-12 &= -8 \Rightarrow x = 1 \end{aligned}$$

$$\boxed{\text{SOLN: } w = 11,\ x = 1,\ y = 4}$$

16. If A is a 2×4 matrix, B is a 2×4 matrix, and C is a 3×2 matrix, determine the size of CAB^T, if possible.

$$\underset{(3\times 2)}{C} \cdot \underset{(2\times 4)}{A} \cdot \underset{(4\times 2)}{B^T} \Rightarrow CAB^T \text{ is of size } \boxed{3\times 2}$$

$$(3\times \underbrace{2)\cdot(2}_{\checkmark}\times \underbrace{4)\cdot(4}_{\checkmark}\times 2)$$

possible

17. There are three food trucks in town which sell chicken. Last week, the east store sold 120 chicken fingers, 48 baskets of fries, 60 chicken sandwiches, and 60 cans of soda. The west store sold 105 chicken fingers, 72 baskets of fries, 21 chicken sandwiches, and 147 cans of soda. The north store sold 60 chicken fingers, 40 baskets of fries, 50 cans of soda, but no chicken sandwiches.

    (a) Write down a 4 × 3 matrix Q to express the sales information for these three food trucks last week.

$$Q = \begin{array}{c|ccc} & \text{East} & \text{West} & \text{North} \\ \hline \underline{\text{chicken fingers}} & 120 & 105 & 60 \\ \underline{\text{basket of fries}} & 48 & 72 & 40 \\ \underline{\text{chicken sandwiches}} & 60 & 21 & 0 \\ \underline{\text{cans of soda}} & 60 & 147 & 50 \end{array}$$

    (b) Suppose sales at the food trucks are expected to decrease by 18% next week, use a matrix to show the expected sales for next week.

$$\text{Expected sales matrix} = Q + 0.18Q = 1.18Q$$

expected sales next week

$$1.18Q = 1.18\begin{bmatrix} 120 & 105 & 60 \\ 48 & 72 & 40 \\ 60 & 21 & 0 \\ 60 & 147 & 50 \end{bmatrix} = \begin{bmatrix} 141.6 & 123.9 & 70.8 \\ 56.64 & 84.96 & 47.2 \\ 70.8 & 24.78 & 0 \\ 70.8 & 173.46 & 59 \end{bmatrix}$$

    (c) If each order of chicken fingers costs \$8.99, each basket of fries costs \$4.99, each chicken sandwich costs \$9.45, and a can of soda costs \$1.50, write down a pricing matrix P so that it can be multiplied by the matrix Q above to give last week's total revenue from each of the three stores.

* food items in $\underline{\text{rows}}$ ⇒ pricing in $\underline{\text{columns}}$

$$P = \begin{array}{cccc} [\,8.99 & 4.99 & 9.45 & 1.50\,] \\ \text{chicken fingers} & \text{basket of fries} & \text{chicken sandwich} & \text{can of soda} \end{array}$$

(1×4 matrix)

* Revenue matrix : PQ $(1\times 4)(4\times 3) \rightarrow (1\times 3)$

18. Compute $\begin{bmatrix} -2 & 3x & 3 \\ 6w & 0 & 2y \end{bmatrix} \begin{bmatrix} -6 & 3m \\ 3n & 4 \\ -p & 0 \end{bmatrix}$.

$$(2 \times \underbrace{3)\ (3}_{✓} \times 2) \rightarrow (2 \times 2)$$

$$= \begin{bmatrix} (-2)(-6) + (3x)(3n) + (3)(-p) & (-2)(3m) + (3x)(4) + (3)(0) \\ (6w)(-6) + (0)(3n) + (2y)(-p) & (6w)(3m) + (0)(4) + (2y)(0) \end{bmatrix}$$

$$= \begin{bmatrix} 12 + 9xn - 3p & -6m + 12x \\ -36w - 2yp & 18wm \end{bmatrix}$$

SOLUTION

19. Write the equation of the line that passes through the point $(-3, 7)$ and has a slope of $-\frac{2}{3}$.

* point - slope

$(x_1, y_1) = (-3, 7) \rightarrow$ point

$y - y_1 = m(x - x_1)$

$m = -2/3 \rightarrow$ slope

$$y - 7 = -\frac{2}{3}(x - (-3))$$

$$y - 7 = -\frac{2}{3}(x + 3)$$

$$y - 7 = -\frac{2}{3}x - 2$$

$$\boxed{y = -\frac{2}{3}x + 5}$$

20. You have a line which passes through the points $(3, -4)$ and $\left(\frac{1}{2}, \frac{2}{3}\right)$. If $\underbrace{x \text{ decreases by 6 units}}$, $\Delta x = -6$ what is the corresponding change in y? $\Delta y = ?$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2/3 - (-4)}{\frac{1}{2} - 3} = \frac{2/3 + 12/3}{\frac{1}{2} - \frac{6}{2}} = \frac{14}{3} \cdot \frac{-2}{5} = \boxed{\frac{-28}{15}}$$

* negative slope ⇒ if x decreases, then y will increase *
(or if x increases, then y will decrease)

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \Delta y = m \Delta x$$

$$= \left(-\frac{28}{\underset{5}{\cancel{15}}}\right)\left(-\overset{-2}{\cancel{6}}\right) = \frac{56}{5}$$

* If x decreases by 6 units, y will increase by $\frac{56}{5}$ units *

$V(0) = b = 23950$

21. An automobile purchased for use by the manager of a firm at a price of \$23,950 is to be depreciated using a linear model over ten years. What will the book value of the automobile be at the end of five years if the automobile has a scrap value of \$1,000 at the end of 10 years?

$V(5)$

$V(10) = 1000$

initial price

$$V(t) = mt + b = mt + 23950$$

rate of depreciation
(negative sign)

$$m = \frac{23950 - 1000}{0 - 10}$$

$$= -\frac{22{,}950}{10}$$

$$= -2{,}295$$

$$V(t) = -2{,}295t + 23950$$

$$V(5) = -2{,}295(5) + 23950 = 12\,475$$

* The automobile is worth $\boxed{\$12{,}475}$ after 5 years *

22. Tim sells lemonade at his lemonade stand. He makes the lemonade for $0.50 per cup. When he sells 20 cups in a day, then his profit is $15. When he sells 30 cups in a day, then his cost for that day is $40.

$P(20) = 15 \qquad C(30) = 40$

production cost per item

(a) Determine the linear cost function.

$$C(x) = \underset{\substack{\uparrow \\ \text{production} \\ \text{cost per} \\ \text{item}}}{m}x + \overset{\text{fixed costs}}{F} = 0.5x + F$$

$$C(30) = 0.5(30) + F = 40$$

$$15 + F = 40$$

$$F = 40 - 15 = 25$$

$\boxed{C(x) = 0.5x + 25}$ where x is the number of cups made and $C(x)$ is the cost in $

(b) Determine the linear revenue function.

$$R(x) = \overset{\text{price per cup}}{p}x$$

$$P(x) = (\overset{\text{price}}{p} - \overset{\text{production cost}}{m})x - \overset{\text{fixed costs}}{F}$$

$$= (p - 0.5)x - 25$$

$$P(20) = (p - 0.5)(20) - 25 = 15$$

$$20p = 15 + 25 + 10 = 50 \Rightarrow \overset{\text{\$2.50 per cup}}{p} = \frac{50}{20} = 2.5$$

$$\boxed{R(x) = 2.5x}$$

(c) Determine the linear profit function.

$$P(x) = (p - m)x - F$$

$$= (2.5 - 0.5)x - 25$$

$$= 2x - 25$$

$$\boxed{P(x) = 2x - 25}$$