MATH 140: WEEK-IN-REVIEW 1 (1.1 & 1.2)

1.

$$A = \begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 1 \\ 0 & -1 \\ 2 & 4 \end{bmatrix} \quad D = \begin{bmatrix} 4 & 3 & 0 \\ 2 & -1 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 2 & -2 & 4 \\ 4 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

(2x2) (2x3) (3x2) (2x3) (3x3)

(a) *Using the matrices above, determine whether or not the following operations are possible. If the operation is possible, give the size (dimensions) of the resulting matrix and then perform the operation. If the operation is not possible, explain why not.*

(i) $5A$ * scalar multiplication * dimension is unchanged

(2x2)

$$5A = 5\begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ -10 & 5 \end{bmatrix}$$

(ii) $C + D$ C is 3x2 & D is 2x3 have different sizes

(3x2) + (2x3)

so C + D is not possible

(iii) D^T D is 2x3 ⇒ D^T = 3x2

(3x2)

$$D^T = \begin{bmatrix} 4 & 2 \\ 3 & -1 \\ 0 & 1 \end{bmatrix}$$

(iv) $3B - C^T$ * B is 2x3 ⇒ 3B is also 2x3

(2x3) (2x3)

* C is 3x2 ⇒ C^T is 2x3

$$\underbrace{3B - C^T}_{\text{is possible}} = \overbrace{\begin{bmatrix} 0 & 3 & 0 \\ -3 & -6 & 3 \end{bmatrix}}^{3B} - \overbrace{\begin{bmatrix} 3 & 0 & 2 \\ 1 & -1 & 4 \end{bmatrix}}^{C^T} = \begin{bmatrix} -3 & 3 & -2 \\ -4 & -5 & -1 \end{bmatrix}$$

$$\boxed{\begin{aligned} 3B &= 3\begin{bmatrix} 0 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 0 \\ -3 & -6 & 3 \end{bmatrix} \\ C^T &= \begin{bmatrix} 3 & 0 & 2 \\ 1 & -1 & 4 \end{bmatrix} \end{aligned}}$$

$$A = \begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 1 \\ 0 & -1 \\ 2 & 4 \end{bmatrix} \quad D = \begin{bmatrix} 4 & 3 & 0 \\ 2 & -1 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 2 & -2 & 4 \\ 4 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

(2x2) (2x3) (3x2) (2x3) (3x3)

(v) AB

$(2\times 2)\cdot(2\times 3)$ ✓

the interior dimensions match, so multiplication is possible & AB has size 2x3

$$\begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} (2)(0)+(3)(-1) & (2)(1)+(3)(-2) & (2)(0)+(3)(1) \\ (-2)(0)+(1)(-1) & (-2)(1)+(1)(-2) & (-2)(0)+(1)(1) \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -4 & 3 \\ -1 & -4 & 1 \end{bmatrix}$$

(vi) BA

$(2\times 3)\cdot(2\times 2)$ X

the interior dimensions don't match so BA is not possible

$$A = \begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 1 \\ 0 & -1 \\ 2 & 4 \end{bmatrix} \quad D = \begin{bmatrix} 4 & 3 & 0 \\ 2 & -1 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 2 & -2 & 4 \\ 4 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

(2x2) (2 x 3) (3x2) (2x3) (3x3)

(vii) $3BCA$

* multiplication by the number 3 does not change dimension *

$(2 \times 3) \cdot (3 \times 2) \cdot (2 \times 2)$ ✓ ✓

* interior dimensions match, so 3BCA is possible, and has size 2x2

2x3

$$3B = 3\begin{bmatrix} 0 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 0 \\ -3 & -6 & 3 \end{bmatrix}$$

3B

$$3BC = \underbrace{\begin{bmatrix} 0 & 3 & 0 \\ -3 & -6 & 3 \end{bmatrix}}_{3B\ (2\times 3)} \underbrace{\begin{bmatrix} 3 & 1 \\ 0 & -1 \\ 2 & 4 \end{bmatrix}}_{C\ (3\times 2)} = \overbrace{\begin{bmatrix} 0 & -3 \\ -3 & 15 \end{bmatrix}}^{3BC\ (2\times 2)}$$

$$\begin{bmatrix} (0)(3)+(3)(0)+(0)(2) & (0)(1)+(3)(-1)+(0)(4) \\ (-3)(3)+(-6)(0)+(3)(2) & (-3)(1)+(-6)(-1)+(3)(4) \end{bmatrix}$$

solution

$$3BCA = \underbrace{\begin{bmatrix} 0 & -3 \\ -3 & 15 \end{bmatrix}}_{\substack{3BC \\ (2\times 2)}} \underbrace{\begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix}}_{\substack{A \\ (2\times 2)}} = \underbrace{\begin{bmatrix} 6 & -3 \\ -36 & 6 \end{bmatrix}}_{\substack{3BCA \\ (2\times 2)}}$$

(viii) Determine if $L = (EC)^T$ is possible. If possible, state the size of L and then find the entry ℓ_{23}. ⇝ 2 is the row number, 3 is the column number

$L = (EC)^T$ * EC is possible and has dimensions 3x2 *

$(3\times 3)(3\times 2)$ ✓

2x3

* Hence $L = (EC)^T$ is also possible *

transpose swaps indices

$$\ell_{23} = (EC)_{32} = (0)(1) + (2)(-1) + (-1)(4) = -2 - 4 = -6$$

* obtained from 3rd row of E & 2nd column of C *

* no need to compute the entire matrix $L = (EC)^T$! *

(viii) Using the given matrices, determine the value of $2c_{32} - a_{21} + e_{13}$

3rd row, 2nd column etc

* $c_{32} = 4$
* $a_{21} = -2$
* $e_{13} = 4$

$$2c_{32} - a_{21} + e_{13} = (2)(4) - (-2) + 4$$
$$= 8 + 2 + 4$$
$$= 14$$

2. Solve the following matrix equation for X. What is the size of X?

$$7X + \begin{bmatrix} 3 & 1 \\ 0 & -1 \\ 2 & 4 \end{bmatrix} = 2X - \begin{bmatrix} -3 & -4 \\ 2 & 0 \\ 5 & 1 \end{bmatrix}$$

$$7X - 2X = -\begin{bmatrix} 3 & 1 \\ 0 & -1 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -3 & -4 \\ 2 & 0 \\ 5 & 1 \end{bmatrix}$$

* bring X to one side *
and simplify

$$\Rightarrow \quad 5X = \begin{bmatrix} -3 & -1 \\ 0 & 1 \\ -2 & -4 \end{bmatrix} - \begin{bmatrix} -3 & -4 \\ 2 & 0 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} -3-(-3) & -1-(-4) \\ 0-2 & 1-0 \\ -2-5 & -4-1 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -2 & 1 \\ -7 & -5 \end{bmatrix}$$

$$\Rightarrow \quad X = \frac{1}{5}\begin{bmatrix} 0 & 3 \\ -2 & 1 \\ -7 & -5 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 3/5 \\ -2/5 & 1/5 \\ -7/5 & -1 \end{bmatrix}}_{X\ (3\times 2)}$$

3. Solve the following matrix equations for w, x, y and z. If this is not possible, then explain why not.

(a)

$$\underbrace{\begin{bmatrix} (w+3) & 4 \\ -7 & z \end{bmatrix}}_{(2\times 2)} + \underbrace{\begin{bmatrix} 2x & y \\ -5 & 1 \end{bmatrix}^T}_{(2\times 2)} = \underbrace{\frac{1}{3}\begin{bmatrix} 12 & (x-6) \\ -18 & (4z-1) \end{bmatrix}}_{(2\times 2)}$$

* simplify *

$$\begin{bmatrix} w+3 & 4 \\ -7 & z \end{bmatrix} + \begin{bmatrix} 2x & -5 \\ y & 1 \end{bmatrix} = \begin{bmatrix} 4 & (x-6)/3 \\ -6 & (4z-1)/3 \end{bmatrix}$$

$$\begin{bmatrix} w+3+2x & -1 \\ -7+y & z+1 \end{bmatrix} = \begin{bmatrix} 4 & (x-6)/3 \\ -6 & (4z-1)/3 \end{bmatrix}$$

* matrix equality *

1 $w+3+2x = 4$

2 $-1 = (x-6)/3$

3 $-7+y = -6$

4 $z+1 = (4z-1)/3$

$\Downarrow$

2 $x-6 = -3 \Rightarrow x = 3$

1 $w+3+2(3) = w+9 = 4 \Rightarrow w = -5$

3 $-7+y = -6 \Rightarrow y = 1$

4 $z+1 = (4z-1)/3 \Rightarrow 3(z+1) = 4z-1$

$\Rightarrow 3z+3 = 4z-1 \Rightarrow z = 4$

$$\boxed{w = -5,\ x = 3,\ y = 1,\ z = 4}$$

solution

(b)

$$\underset{(2\times2)}{\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}}\underset{(2\times2)}{\begin{bmatrix}-3x & 0\\ 2 & 6\end{bmatrix}} - 2\underset{(2\times2)}{\begin{bmatrix}(y+2) & x\\ 5 & (3z+5)\end{bmatrix}} = \underset{(2\times2)}{\begin{bmatrix}-5 & -3w\\ 3x-1 & 14\end{bmatrix}^T}$$

$$\boxed{\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}\begin{bmatrix}-3x & 0\\ 2 & 6\end{bmatrix} = \begin{bmatrix}-3x-2 & -6\\ 4 & 12\end{bmatrix}}$$

$$\Downarrow$$

$$\begin{bmatrix}-3x-2 & -6\\ 4 & 12\end{bmatrix} - \begin{bmatrix}2(y+2) & 2x\\ 10 & 2(3z+5)\end{bmatrix} = \begin{bmatrix}-5 & 3x-1\\ -3w & 14\end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix}-3x-2-2(y+2) & -6-2x\\ -6 & 12-2(3z+5)\end{bmatrix} = \begin{bmatrix}-5 & 3x-1\\ -3w & 14\end{bmatrix}$$

* matrix equality *

① $-3x-2-2(y+2) = -5$

② $-6-2x = 3x-1$

③ $-6 = -3w$

④ $12-2(3z+5) = 14$

$\Rightarrow$

② $5x = -5 \Rightarrow x = -1$

① $(-3)(-1)-2-2(y+2) = -5 \Rightarrow -3-2y = -5$

$\Rightarrow 2y = 2 \Rightarrow y = 1$

③ $w = 2$

④ $12-2(3z+5) = 12-6z-10 = 2-6z = 14$

$-6z = 12$

$z = -2$

$$\boxed{\text{Solution: } w = 2,\ x = -1,\ y = 1,\ z = -2}$$

4. In August before school starts, a bookstore sells 200 boxes of blue pens, 175 pencil sets, and 100 notebooks. In September, the bookstore sells 80 notebooks, 100 pencil sets, and 125 boxes of blue pens.

3 rows , 2 columns

(a) Organize this information into a 3×2 matrix Q. Label all rows and columns.

$$Q_{(3\times 2)} = \begin{array}{c|cc} & \text{August} & \text{September} \\ \hline \text{blue pens} & 200 & 125 \\ \text{pencil sets} & 175 & 100 \\ \text{notebooks} & 100 & 80 \end{array}$$

(b) If the bookstore sells boxes of blue pens for \$5.00, notebooks for \$ 3.50, and pencil sets for \$ 7.00, write a matrix P that could be used to multiply matrix Q by, in order to have the August and September revenue from the three products. Again, label all rows and columns.

* require P to multiply $Q \Rightarrow P$ is 1×3

$$P_{(1\times 3)} = \text{price} \begin{bmatrix} 5 & 7 & 3.5 \end{bmatrix}$$
$$\begin{array}{ccc} \text{blue pens} & \text{pencil sets} & \text{notebooks} \end{array}$$

(c) How much revenue does the bookstore bring in, in August? In September?

$$\underset{\substack{\text{(revenue)} \\ 1\times 2}}{R} = \underset{(1\times 3)\cdot(3\times 2)}{P \cdot Q} = \begin{bmatrix} 5 & 7 & 3.5 \end{bmatrix} \begin{bmatrix} 200 & 125 \\ 175 & 100 \\ 100 & 80 \end{bmatrix}$$

revenue for August; revenue for September

$$= \begin{bmatrix} (5)(200) + (7)(175) + (3.5)(100) & (5)(125) + (7)(100) + (3.5)(80) \end{bmatrix}$$

$$= \begin{bmatrix} 1675 & 1353 \end{bmatrix}$$

* August revenue = \$1,675
* Sept revenue = \$1,353