Section 4.1

- The **General Antiderivative** of $f(x)$ on an interval is $F(x) + C$, where C is any real number constant, if $\dfrac{d}{dx}(F(x) + C) = f(x)$.

- The collection of all antiderivatives of a function, $f(x)$, is called the **indefinite integral**, and is denoted by $\displaystyle\int f(x)\,dx$ (the indefinite integral of $f(x)$ with respect to x). If we know one function $F(x)$ for which $F'(x) = f(x)$, then $\displaystyle\int f(x)\,dx = F(x) + C$.

- **Rules of Integration:**

  - $\displaystyle\int k\,dx = kx + C$, where k is any real number

  - $\displaystyle\int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + C$, where n is any real number with $n \neq -1$

  - $\displaystyle\int x^{-1}\,dx = \int \dfrac{1}{x}\,dx = \ln|x| + C$

  - $\displaystyle\int b^x\,dx = \dfrac{1}{\ln b}\cdot b^x + C$ where b is any positive real number

  - $\displaystyle\int e^x\,dx = e^x + C$

  - $\displaystyle\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

  - $\displaystyle\int k\cdot f(x)\,dx = k\int f(x)\,dx$

1. Evaluate the following:

   (a) $\displaystyle\int x^4\,dx$

   (b) $\displaystyle\int \left(\dfrac{1}{5}e^x + 6\sqrt{x^3}\right)\,dx$

(c) $\displaystyle\int \left(\frac{4}{x^7} - 5x^{-1} - \frac{1}{\sqrt[4]{x^3}} + 3x^{-3} + 7^x - 8 \right) \, dx$

(d) $\displaystyle\int \frac{4\sqrt{x} + 3x^7 - 4}{2\sqrt[5]{x^2}} \, dx$

(e) $\displaystyle\int (4x^2 + 7)(7x - 9x^4) \, dx$

2. Find $f(x)$ if $f'(x) = \dfrac{3e^{-2x} + 4e^{-x}}{2e^{-2x}}$ and $f(0) = 5$.

3. Find the cost of producing 10 items if the marginal cost, in dollars per item, is given by $f(x) = 150 - 0.01e^x$, where x is the number of items produced. Assume the fixed costs are \$100.

Section 4.2

- We use u-substitution when our integrand is the result (or nearly the result) of the Chain Rule. We follow the process outlined below:

    - Select u (look for function of x where you normally have just x)
    $$\int u^n \cdot (\text{other stuff})\ \ dx \text{ OR } \int b^u \cdot (\text{other stuff})\ \ dx \text{ OR } \int \frac{1}{u} \cdot (\text{other stuff})\ \ dx$$

    - Take the derivative of u using $\dfrac{du}{dx}$ notation.

    - Bring dx to the right hand side.

    - Bring any constant multiples to the left-hand side.

    - Substitute to replace all terms with x's.

    - Integrate with u's.

    - Return x's into the problem.

4. Evaluate the following integrals:

(a) $\displaystyle\int \frac{8x + 21x^2}{4x^2 + 7x^3}\, dx$

(b) $\displaystyle\int \frac{20x^7 - 15x}{(3x^8 - 9x^2)^{13}}\, dx$

(c) $\displaystyle\int \frac{64x^3 - 32x^7 + 2e^x}{\sqrt[7]{(8x^4 - 2x^8 + e^x)^5}}\, dx$

(d) $\displaystyle\int \frac{4 \cdot 2^{-6/x^2}}{x^3}\,dx$

(e) $\displaystyle\int \frac{5\sqrt{\ln x}}{3x}\,dx$

5. Find $f(x)$ if $f\,'(x) = (10x + 45)\sqrt[3]{x^2 + 9x + 27}$ and $f(0) = 310$.