



Math 151
Week-In-Review 3
2.2 and 2.5
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Problem Statements

Section 2.3

1. Evaluate the following limits.

(a) $\lim_{x \rightarrow 8} (2x^2 - 10x + 5)$

(b) $\lim_{x \rightarrow 5^+} \frac{1}{x - 5}$

(c) $\lim_{x \rightarrow 5^+} \frac{x^2 - 25}{(x - 5)^2}$



(d) $\lim_{h \rightarrow 0} \frac{(5+h)^2 - 5^2}{h}$

(e) $\lim_{x \rightarrow 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$



(f) $\lim_{x \rightarrow 3} \left(\frac{3}{x+3} + \frac{\sqrt{7-x}-2}{\sqrt{4-x}-1} \right)$

2. Find $\lim_{x \rightarrow -4} f(x)$ and $\lim_{x \rightarrow 0} f(x)$ for the function below.

$$f(x) = \begin{cases} 5 - x & \text{if } x < -4 \\ x + 13 & \text{if } -4 \leq x \leq 0 \\ 12 & \text{if } x > 0 \end{cases}$$



3. Find $\lim_{x \rightarrow 2} g(x)$ for the function below.

$$g(x) = \begin{cases} \frac{8}{x} - 4 & \text{if } x < 2 \\ x + 3 & \text{if } x = 2 \\ \frac{x^2 + 2x - 8}{x^2 - 7x + 10} & \text{if } x > 2 \end{cases}$$



4. Evaluate the following limits.

(a) $\lim_{x \rightarrow 7^-} \frac{14 - 2x}{|x - 7|}$

(b) $\lim_{x \rightarrow 7^+} \frac{14 - 2x}{|x - 7|}$

(c) $\lim_{x \rightarrow 7} \frac{14 - 2x}{|x - 7|}$

(d) $\lim_{x \rightarrow 0^-} \frac{1}{x} - \frac{1}{|x|}$

(e) $\lim_{x \rightarrow 0^+} \frac{1}{x} - \frac{1}{|x|}$

(f) $\lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{|x|}$



5. Prove $\lim_{x \rightarrow 0^+} \sqrt{x} e^{\sin(\pi/x)} = 0$.



Section 2.5

6. Sketch the graph of a function that is continuous except for at the stated discontinuities.
- (a) Discontinuous, but continuous from the right, at 2

 - (b) Discontinuities at -1 and 4 , but continuous at the left at -1 and from the right at 4

 - (c) Removable discontinuity at 3 , jump discontinuity at 5

 - (d) Neither left nor right continuous at -2 , continuous only from the left at 2

 - (e) Infinite discontinuity at -3 , infinite discontinuity at 3 , but continuous from the right at 3



7. Prove $f(x)$ is continuous at 5 for the following function.

$$f(x) = \begin{cases} \sqrt{14-x} & \text{if } x < 5 \\ 8-x & \text{if } x = 5 \\ 23-4x & \text{if } x > 5 \end{cases}$$



8. Determine the values of x for which the following function is discontinuous.

$$f(x) = \begin{cases} -12 - x & \text{if } x \leq -4 \\ \frac{x^2 - 16}{x^2 + 9x + 20} & \text{if } -4 < x \leq 4 \\ \frac{x + 5}{x - 5} & \text{if } x > 4 \end{cases}$$



9. Determine the values of a and b that make $f(x)$ continuous everywhere.

$$f(x) = \begin{cases} x + 3 & \text{if } x < a \\ x^2 - 8x - 7 & \text{if } a \leq x < 5 \\ -4x + b & \text{if } x \geq 5 \end{cases}$$



10. Prove there is a solution to the equation $e^x = 3 - 2x$ on the interval $(0, 1)$.

11. Suppose f is continuous on $[-1, 4]$ and the only solutions to $f(x) = 100$ are $x = 0$ and $x = 4$.
If $f(1) = 50$, explain why $f(3) < 100$.