TEXAS A&M UNIVERSITY
Math Learning Center

Math 251 - Fall 2024
"Hands On Grades Up"
Exam 1 Review
Thursday, Sept 5 and
Thursday, Sept 12,
6:30-8:30 PM
ZACH 340

Exam 1 Review: Covering sections 12.1-12.6, 13.1-13.4

PLEASE SCAN THE QR CODE BELOW

We will begin at 6:30 PM. A problem will be displayed on the wall monitors. Collaborate with your table on how to solve each problem. If you have a question, raise your hand. At the end of a predetermined number of minutes, the solutions will be displayed on the table monitors. Feel free to take a picture of the solution, as the solutions are not posted.

(1) Find the equation of the sphere given the points $P(2, 3, 4)$ and $Q(-4, 1, 2)$ are endpoints of a diameter of the sphere. In what way does this sphere intersect the yz-plane, or explain why it does not intersect the yz-plane.

(2) Consider the points $P = (-1, -2, 4)$, $Q = (3, -1, 1)$ and $R = (2, 0, 2)$.
   (a) Find the vector projection of **PQ** onto **PR**
   (b) Find the area of the triangle formed by the points P,Q, and R.

(3) Consider the plane P_1 given by the equation $x - y + 3z = 5$ and the plane P_2 given by the equation $x - y + 2z = 3$.
   (a) Find the angle between the planes P_1 and P_2.
   (b) Find any point on the line of intersection of the two planes.
   (c) Find a parametric equation for the line where the two planes intersect.

(4) Find the domain of $r(t) = \left\langle \ln(16 - t^2), \dfrac{t-1}{t^2-1}, \sqrt{t+3} \right\rangle$. Express your answer in interval notation.

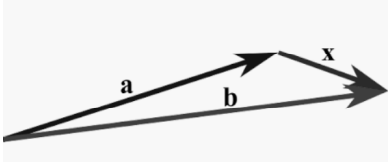
(5) Find $\lim_{t\to 0}\left\langle e^{-2t}, \frac{t^2}{\sin^2 t}, \cos(4t)\right\rangle$

(6) Let $\mathbf{r}(t) = \langle \cos(t^2), \sin(t^2), t^2 \rangle$.

(a) Find $\mathbf{T}(\sqrt{\pi})$, the unit tangent vector, at $t = \sqrt{\pi}$.

(b) Find the length of the curve from $(1, 0, 0)$ to $(1, 0, 2\pi)$.

(7) Use the figure below to answer the questions that follow.



(a) Write $\mathbf{x}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(b) If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $60°$, $|\mathbf{a}| = 7$, and $|\mathbf{b}| = 6$, find $\mathbf{a} \cdot \mathbf{b}$.

(c) If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $60°$, $|\mathbf{a}| = 7$, and $|\mathbf{b}| = 6$, find $|\mathbf{a} \times \mathbf{b}|$ and determine whether $\mathbf{a} \times \mathbf{b}$ is directed into or out of the page.

(8) Consider the lines $x = 1 + 2t$, $y = t$, $z = 4t + 1$ and $x = s$, $y = 2s - 2$, $z = 3s - 2$. Find the point of intersection of these two lines and find the equation of the plane that contains these two lines.

(9) Find parametric equations for the tangent line to the curve $x = 4t$, $y = t^2 - 1$, $z = \dfrac{4}{t}$ at $(8, 3, 2)$.

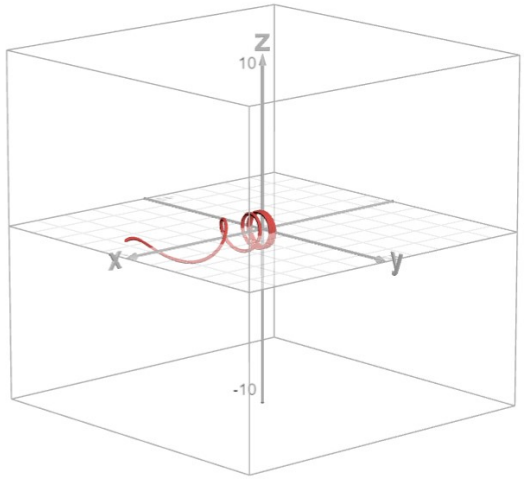
(10) If $\mathbf{r}'(t) = \left\langle \dfrac{4t}{t^2+1},\ \cos(2t),\ te^t \right\rangle$ and $\mathbf{r}(0) = \langle 1, 3, 2 \rangle$, find $\mathbf{r}(t)$.

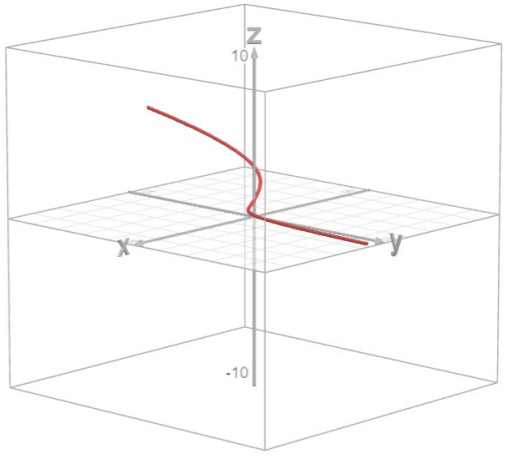
(11) Find parametric equations for the line that passes through $(2, -1, 5)$ and is
   (a) parallel to the line $x = 3t - 1$, $y = 4t + 6$, $z = t$.
   (b) perpendicular to the plane $8x - 11y = 2z + 6$.

(12) The curves $\mathbf{r_1}(t) = \langle 5t, t^2, t^3 \rangle$ and $\mathbf{r_2}(v) = \langle \sin v, \sin(2v), 3v \rangle$ intersect at the origin. Find the angle of intersection.

(13) Match the space curve with its graph

A. $x = t$, $y = t^5$, $z = e^{-t}$ B. $x = e^t$, $y = \sin(5t)$, $z = \cos(5t)$





(14) Find the curvature of $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ at $t = 1$

(15) Match the Quadric Surface with its graph

A. $y = x^2 + z^2$ B. $x^2 - y^2 - z^2 = 5$ C. $z^2 = y^2 + x^2$