Fubini's Theorem If f is continuous on the rectangle

$$R = \{(x, y) \mid a \le x \le b, c \le y \le d\},$$

then

$$\iint\limits_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

Example 1 (15.1)**.** *Calculate the double integrals.*

(a) $\displaystyle\iint\limits_R \frac{y\,\cos x}{y^2+1}\,dA$, *where* $R = \{(x,y) | -\pi/2 \le x \le \pi/2,\, 0 \le y \le 2\}$

(b) $\int_0^{\pi/2} \int_{-1}^{1} (x + x^2 \sin y)\, dx\, dy$

(c) $\iint\limits_R xe^{-xy}\, dA$, *where* $R = [0, 1] \times [0, 2]$.

*If f is continuous on a **type I region** D described by*

$$D = \{(x, y) \mid a \le x \le b, g_1(x) \le y \le g_2(x)\}$$

then

$$\iint\limits_D f(x, y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx$$

*If f is continuous on a **type II region** D described by*

$$D = \{(x, y) \mid c \le y \le d, h_1(y) \le x \le h_2(y)\}$$

then

$$\iint\limits_D f(x, y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\, dx\, dy$$

Example 2 (15.2)**.** *Compute the double integral* $\iint\limits_D (2x + y^2)\, dA$*, where D is the triangular region with vertices $(-2, 0)$, $(0, 2)$ and $(2, 0)$.*

Example 3 (15.2)**.** *Find the volume of the tetrahedron bounded by the planes* $z = 0$, $x = 0$, $x = y$ *and* $x + y + z = 4$.

Example 4 (15.2)**.** *Find the volume of the solid bounded by the paraboloid* $z = 1 + 3x^2 + 3y^2$ *and the planes* $x = 0$, $y = 0$, $z = 0$ *and* $x + y = 1$.

Example 5 (15.2)**.** *Sketch the region and change the order of integration.*

(a) $\displaystyle\int_0^{16}\int_{\sqrt{x}}^{4} f(x,y)\,dy\,dx$

(b) $\displaystyle\int_{-3}^{3}\int_0^{\sqrt{9-y^2}} f(x,y)\,dx\,dy$

Example 6 (15.2)**.** *Evaluate the integral by reversing the order of integration.*

$$\int_0^1 \int_{\sqrt{x}}^1 x \, \sin(y^5 + 1) \, dy \, dx$$

If f is continuous on a polar rectangle R given by $0 \leq a \leq r \leq b$, $\alpha \leq \theta \leq \beta$, where $0 \leq \beta - \alpha \leq 2\pi$, then

$$\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

If f is continuous on a polar region of the form

$$D = \{(r,\theta) \mid \alpha \leq \theta \leq \beta, h_1(\theta) \leq r \leq h_2(\theta)\},$$

then

$$\iint_D f(x,y)\,dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta) r\, dr\, d\theta$$

Example 7 (15.3). *Evaluate the integral* $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{1+x^2+y^2}\, dy\, dx$

Example 8 (15.3)**.** *Evaluate the integral* $\displaystyle\iint_R \frac{x^2}{x^2+y^2}\,dA$*, where* R *is the region between the circles* $x^2+y^2=1$ *and* $x^2+y^2=4$*.*

Example 9 (15.3)**.** *Find the volume of the solid between the cone* $z = \sqrt{x^2 + y^2}$ *and the ellipsoid* $2x^2 + 2y^2 + z^2 = 12$.

Example 10 (15.3). *Evaluate the integral* $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$

