MATH 150 - WEEK-IN-REVIEW 6

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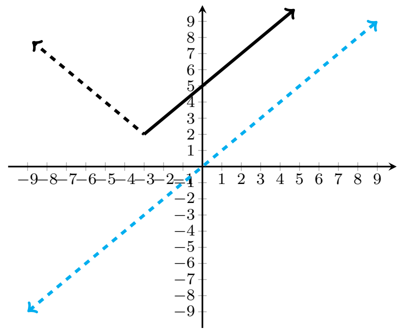
SECTIONS 5.1, 5.2 AND 5.3

1. Determine whether the function $h(x) = |x + 3| + 2$ where $x \geq -3$ has an inverse.

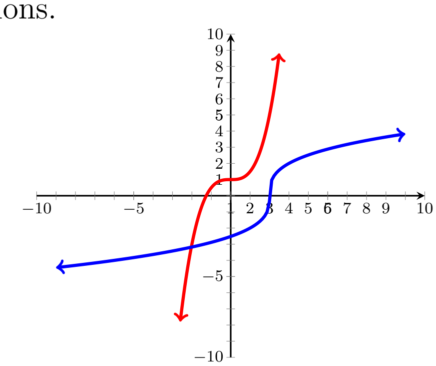


one - to - one ✓

invertible

★★ Note: if $f(x)$ & $g(x)$ are inverse $f(g(x)) = x$ for $x \in \text{dom}(g(x))$
$g(f(x)) = x$ for $x \in \text{dom}(f(x))$

2. Graphically and algebraically verify whether $f(x) = 2\sqrt[3]{x-2}$ and $g(x) = \dfrac{x^3+2}{2}$ are inverse functions.



$f(g(x)) = 2\sqrt[3]{g(x) - 2} = 2\sqrt[3]{\frac{x^3+2}{2} - 2}$

$= 2\sqrt[3]{x^3 - 2} \neq x$

3. Verify whether $f(x) = \dfrac{-3x+4}{x-2}$ and $g(x) = \dfrac{2x+4}{x+3}$ are inverse of eachother.

$$f(g(x)) = \frac{-3(g(x))+4}{g(x)-2} = \frac{-3\left(\frac{2x+4}{x+3}\right)+4}{\frac{2x+4}{x+3}-2} = \frac{\frac{-6x-12}{x+3} + 4\times\frac{x+3}{x+3}}{\frac{2x+4}{x+3} - 2\times\frac{x+3}{x+3}}$$

$$= \frac{\frac{-6x-12+4(x+3)}{x+3}}{\frac{2x+4-2(x+3)}{x+3}} = \frac{\frac{-6x-12+4x+12}{x+3}}{\frac{2x+4-2x-6}{x+3}} = \frac{-2x}{-2} = \boxed{x} \checkmark$$

$$g(f(x)) = \frac{2(f(x))+4}{f(x)+3} = \frac{2\left(\frac{-3x+4}{x-2}\right)+4}{\frac{-3x+4}{x-2}+3} = \frac{\frac{-6x+8}{x-2} + 4\times\frac{x-2}{x-2}}{\frac{-3x+4}{x-2} + 3\times\frac{x-2}{x-2}} = \frac{\frac{-6x+8+4(x-2)}{x-2}}{\frac{-3x+4+3(x-2)}{x-2}}$$

$$= \frac{-2x}{2} = \boxed{x} \checkmark$$

4. If $f(x)$ is a one-to-one function with domain $(-\infty, 2) \cup (2, \infty)$, range $(-\infty, \infty)$, $f(1) = 5$ and $f(-2) = 8$. Assume $g(x)$ is inverse of this function. Evaluate its domain, range and $g(5) - g(8)$.

Domain of $g(x)$: $(-\infty, \infty)$

Range of $g(x)$: $(-\infty, 2) \cup (2, \infty)$

$g(5) - g(8) = 1 - (-2) = 3$

5. Simplify each of the following:

(a) $\log_8(0.25)$ $= \log_8\left(\frac{25}{100}\right) = \log_8\left(\frac{1}{4}\right) = \log_8 4^{-1} = \log_{2^3} 2^{-2} = \log_{2^3}(2^3)^{-\frac{2}{3}} = \frac{-2}{3}$

or using $\log_{b^m} a^n = \frac{n}{m}\log_b a$

(b) $11^{\log_{11}(5)} + 2$ $= 5 + 2 = 7$

(c) $\log(10^{-4})$ $= -4$

(d) $e^{\ln(\frac{1}{\pi})}$ $= \frac{1}{\pi}$

(e) $\log_2(8) + \log_9(3)$ $= 3 + \log_9(9)^{\frac{1}{2}} = 3 + \frac{1}{2} = \frac{7}{2}$

(f) $\ln(\frac{1}{e})$ $= \ln(1) - \ln e = 0 - 1 = \boxed{-1}$

(g) $10^{\log(13)}$ $= 13$

6. Express the following equations in exponential form.

(a) $\log_6(z) = 1$ $\iff$ $z = 6$

(b) $\log_7(3y) = 2$ $\iff$ $3y = 7^2$

(c) $\log(3) = 2t$ $\iff$ $3 = 10^{2t}$

(d) $\ln(x-1) = -1$ $\iff$ $x - 1 = e^{-1}$

7. Express the following equations in logarithmic form.

(a) $7^3 = 343$ $\Rightarrow$ $\log_7 7^3 = \log_7(343)$ $\Rightarrow$ $3 = \log_7 343$

(b) $e^{0.5x} = t$ $\iff$ $0.5x = \ln t$

(c) $10^{-4x} = 0.1$ $\iff$ $-4x = \log(0.1)$

8. Find domain of the following.

(a) $\log_5(8-2x)$

$8-2x>0 \longrightarrow 8>2x \longrightarrow \frac{8}{2}>x \longrightarrow x<4$

$x \in (-\infty, 4)$

(b) $\ln(\underbrace{x-x^2})$

$x(1-x) > 0$

Signchart: $-$ | 0 | $+$ | 1 | $-$

$x=0$
$x=1$

$x \in (0,1)$

** Note: domain of exponential function: $a^{g(x)}$ is the same as domain of $g(x)$

(c) $e^{\sqrt{8x^2-2x-3}}$

Square root: $\underbrace{8x^2-2x-3} \geq 0$

$8x^2-6x+4x-3 = 2x(4x-3)+4x-3 = (2x+1)(4x-3)$

$x=-\frac{1}{2}$
$x=\frac{3}{4}$

Signchart: $+$ | $-\frac{1}{2}$ | $-$ | $3/4$ | $+$

$x \in (-\infty, -\frac{1}{2}] \cup [\frac{3}{4}, \infty)$

(d) $e^{\frac{2x-7}{x+1}}$

denom.: $x+1 \neq 0 \longrightarrow x \neq -1$

$x \in (-\infty, -1) \cup (-1, \infty)$

9. Determine the properties of the function $g(x) = -2e^{x-5} + 3$ and use the properties to graph the function.

Domain: $(-\infty, +\infty)$

x-intercept(s): $(5+\ln\frac{3}{2}, 0)$ $\quad -2e^{x-5}+3=0 \rightarrow e^{x-5} = \frac{3}{2} \rightarrow x-5 = \ln\frac{3}{2} \quad x = 5+\ln\frac{3}{2}$

y-intercept(s): $(0, -2e^{-5}+3) = (0, \frac{-2}{e^5}+3)$

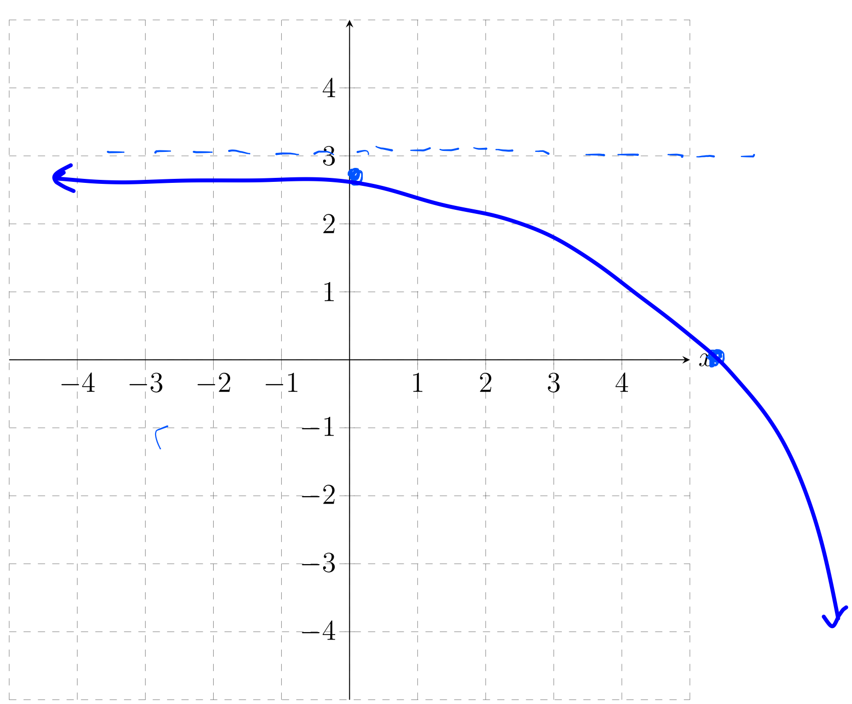
Horizontal Asymptote(s): As $x \to -\infty$ $\quad -2\underbrace{e^{\overbrace{x-5}^{\to -\infty}}}_{\to 0} + 3 \to 3$ $\quad \boxed{y=3}$

As $x \to +\infty$ $\quad -2e^{x-5}+3 \to -\infty$

Vertical Asymptote(s): None

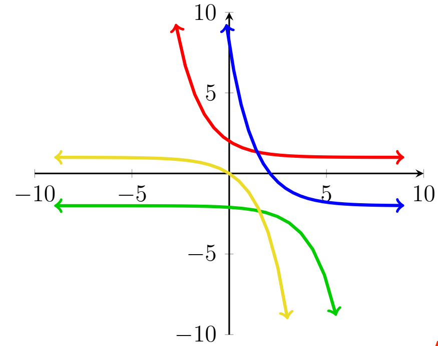
$e>1$

transformation: 1) right 5
2) vertic. stretch
3) reflect about x-axis
4) up 3



10. Determine the following for the function $g(x) = 3^{-0.7x} + 1$, then choose the graph that matches the function.

Domain: $(-\infty, \infty)$



Intercept(s): y-intercept $(0, 2)$

reasoning below

x-intercept: None

Red Graph ✓

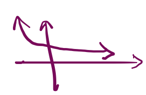
Asymptote(s):

No vertical asy.

Horizontal asy. $y = 1$ ← any vertical shift

Note: $g(x) = \left((3)^{-1}\right)^{0.7x} + 1 = \left(\frac{1}{3}\right)^{0.7x} + 1$

Since $0 < \frac{1}{3} < 1$ → graph:



work for intercepts:

y-intercept: $(0, 2)$

$g(0) = 3^{-0.7(0)} + 1 = 3^{0} + 1 = 2$

x-intercept: $3^{-0.7x} + 1 = 0 \rightarrow 3^{-0.7x} = -1$ (No solution)

$\ln(3)^{-0.7x} = \ln(-1)$

not in domain

11. Determine the properties of the function $f(x) = -\log_2(2x-5)+3$ and use the properties to graph the function.

Domain: $\left(\frac{5}{2}, +\infty\right)$

$2x-5>0$
$x>\frac{5}{2}$

x-intercept(s): $-\log_2(2x-5)+3=0 \longrightarrow \log_2(2x-5)=3 \quad 2x-5=8 \quad x=\frac{13}{2}$

$\left(\frac{13}{2}, 0\right)$

y-intercept(s): None

Horizontal Asymptote(s): None

Vertical Asymptote(s): $x=\frac{5}{2}$

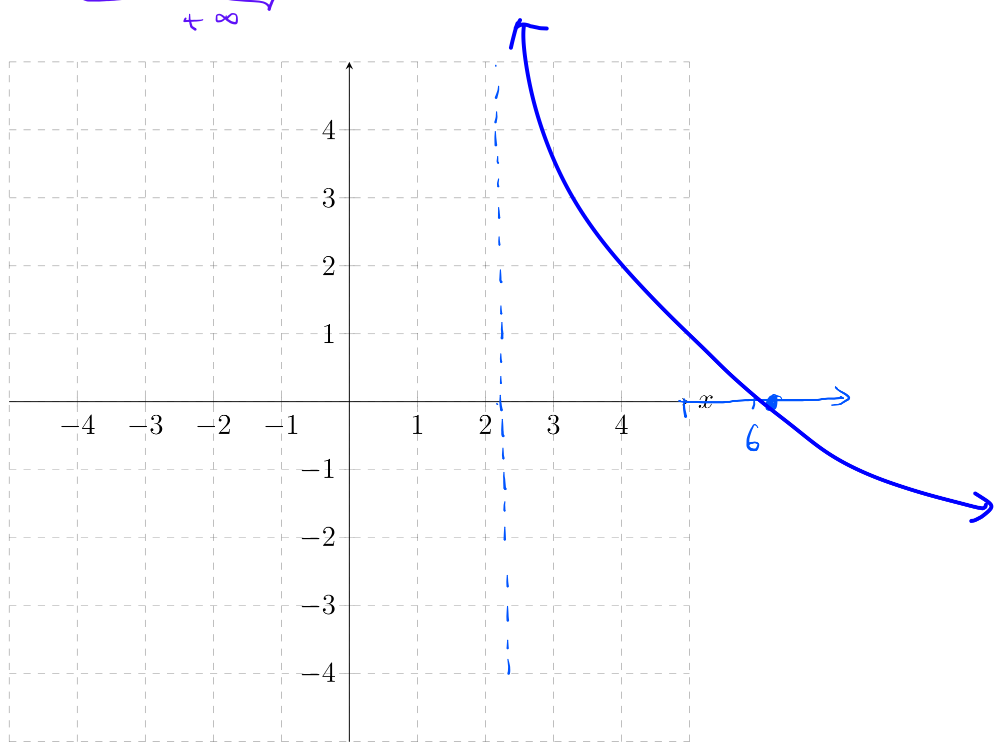
$2x-5 \to 0^+ \quad 2x \to 5+0^+ = 5^+ \quad x \to \frac{5}{2}^+$

As $x \to \infty$ $\quad -\log_2(\overbrace{2x-5}^{\to\infty})+3 \longrightarrow -\infty$

As $x \to \frac{5}{2}^+$ $\quad \underbrace{-\underbrace{\log_2(\underbrace{2x-5}_{\to 0^+})}_{-\infty}}_{+\infty}+3 \longrightarrow \infty$

transformation $\log_2$ $b>1$

1. 5 units right
2. Horiz. Shrink
3. reflect about x-axis
4. 3 units up



12. Determine the properties of the function $g(x) = 2\ln(-x+3)$ and use the properties to graph the function.

$-x+3>0$

$3>x$

Domain: $(-\infty, 3)$

x-intercept(s): $2\ln(-x+3)=0 \rightarrow -x+3=1 \quad -x=-2 \quad x=2$

$(2;0)$

y-intercept(s): $(0, 2\ln 3)$

Horizontal Asymptote(s): None

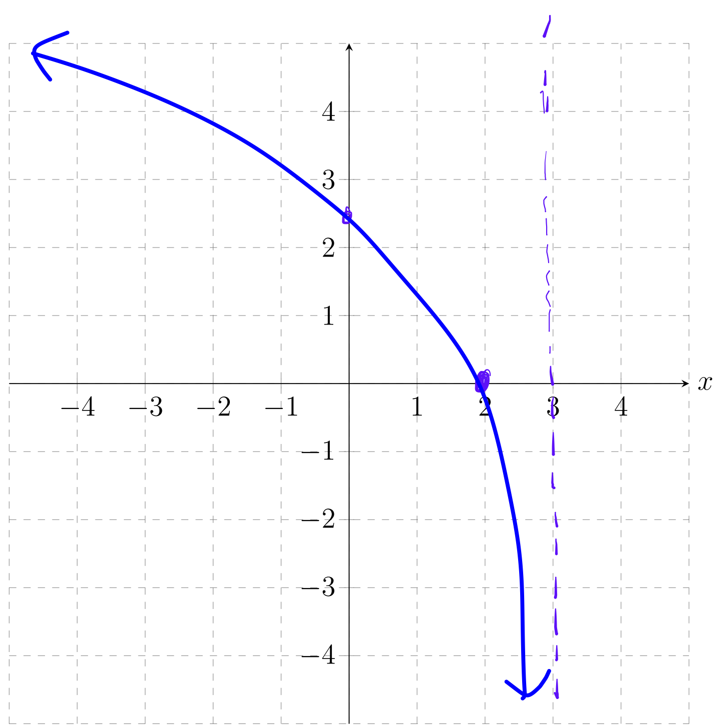
Vertical Asymptote(s): $x=3$ (argument $=0$)

as $x \to 3^-$ $\quad 2\ln(\overbrace{-x+3}^{\to 0}) \longrightarrow -\infty$

as $x \longrightarrow -\infty$ $\quad 2\ln(\underbrace{-x+3}_{\to\infty}) \longrightarrow +\infty$

Transformation from ln

1. 3 units left
2. reflect y-axis
3. Vertical stretch



13. Use the laws of logarithms to expand each expression.

(a) $\ln\left(\dfrac{ab}{c\sqrt[3]{d}}\right)$ $= \ln(ab) - \ln(c\sqrt[3]{d})$

$= \ln a + \ln b - \left[\ln c + \ln(d)^{\frac{1}{3}}\right]$

$= \ln a + \ln b - \ln c - \frac{1}{3}\ln(d)$

(b) $\log_5\left(\dfrac{s^3\sqrt{t}}{(t^2+1)^4}\right)$ $= 3\log_5 s + \frac{1}{2}\log_5 t - 4\log_5(t^2+1)$

(c) $\log\left(\sqrt{x\sqrt{y\sqrt{z}}}\right)$ $= \frac{1}{2}\log(x\sqrt{y\sqrt{z}}) = \frac{1}{2}\left(\log x + \log\sqrt{y\sqrt{z}}\right)$

$= \frac{1}{2}\left(\log x + \frac{1}{2}\log(y\sqrt{z})\right) = \frac{1}{2}\left(\log x + \frac{1}{2}(\log y + \log\sqrt{z})\right)$

$= \frac{1}{2}\left(\log x + \frac{1}{2}(\log y + \frac{1}{2}\log z)\right)$

$= \frac{1}{2}\left(\log x + \frac{1}{2}\log y + \frac{1}{4}\log z\right) = \frac{1}{2}\log x + \frac{1}{4}\log y + \frac{1}{8}\log z$