Math 150 - Week-In-Review 1

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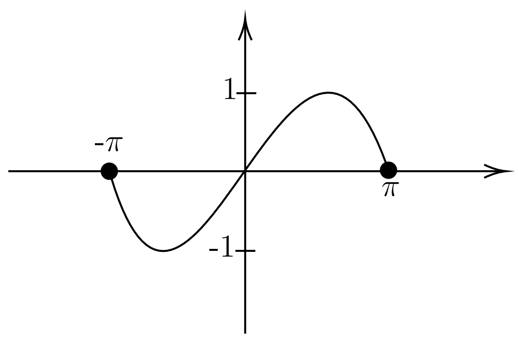
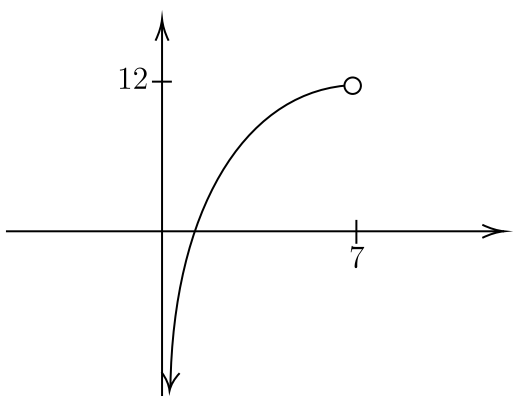
Problem Statements

1. Determine the domain and range of the following graphs.



Domain: $[-\pi, \pi]$

Range: $[-1, 1]$

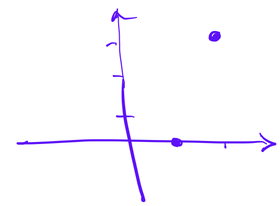


Domain: $(0, 7)$

Range: $(-\infty, 12)$

2. Identify and sketch the region given by $\{(t, t^2 - 1) |\ t = 1, t = 2\}$

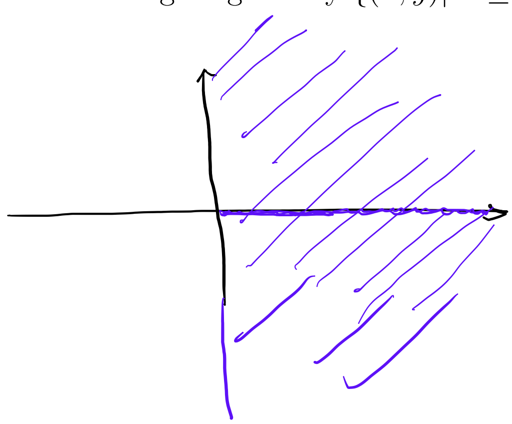
$t = 1 \longrightarrow (1, (1)^2 - 1) = (1, 0)$



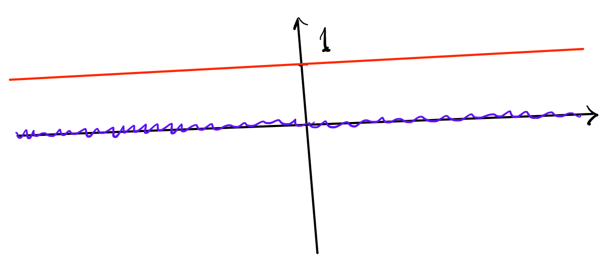
$t = 2 \longrightarrow (2, 4 - 1) = (2, 3)$

3. Identify and sketch the region given by $\{(x, y) |\ x \geq 0\} = \{(x, y) \mid x \geq 0, y \in \mathbb{R}\}$

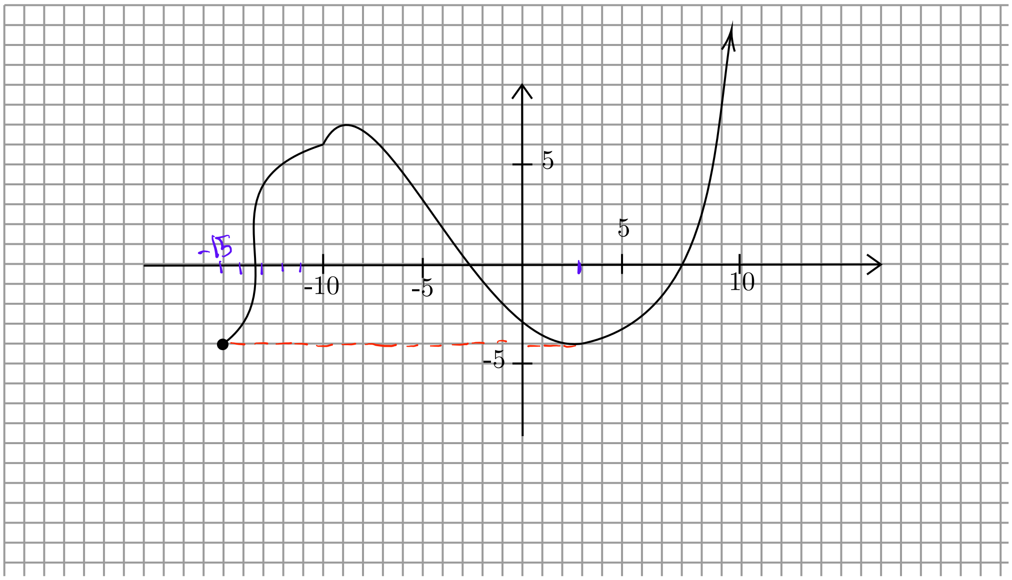
$x \geq 0$



4. Identify and sketch the region given by $\{(x, y) |\ y = 1\} = \{(x, 1) \mid x \in \mathbb{R}\}$



5. Find the absolute extreme points of the following functions if they exist. Also state the interval of increase and decrease.

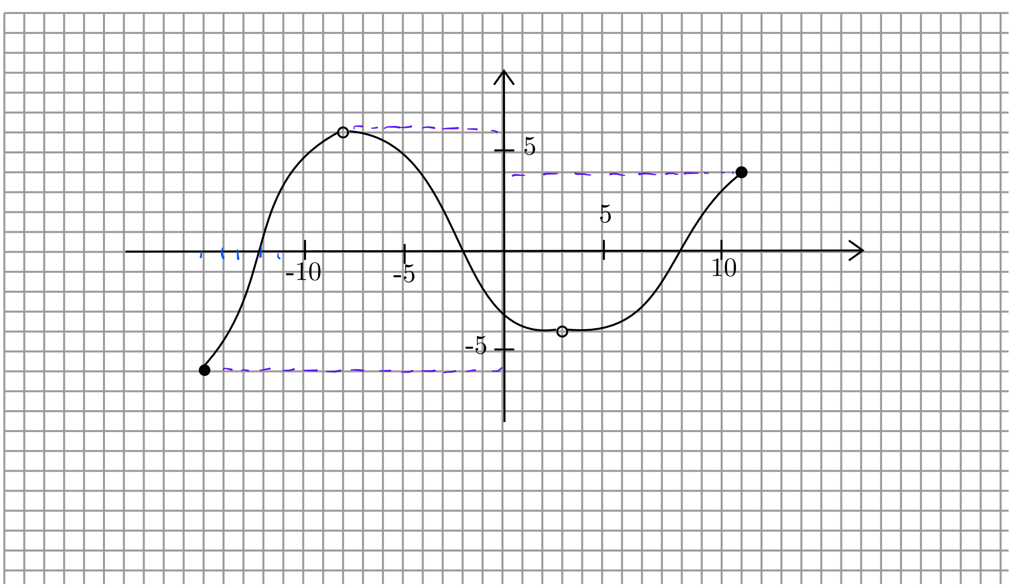


Abs. Max DNE

Abs Min −4

increase: (−15, −9), (3, +∞)

decrease: (−9, 3)



Abs Min −6

Abs Max DNE

increase: (−15, −8), (3, 11)

decrease: (−8, 3)

6. Which of the points $A(3,1)$, $B(-1,3)$ is closer to the point $C(-1,-1)$.

$$d_{AC} = \sqrt{(3-(-1))^2 + (1-(-1))^2} = \sqrt{(4)^2+(2)^2} = \sqrt{16+4} = \sqrt{20}$$

$$d_{BC} = \sqrt{(-1-(-1))^2 + (3-(-1))^2} = \sqrt{0+16} = 4$$

$$d_{BC} < d_{AC}$$

B is closer to C!

7. Test the following equation for symmetry. $y = x^3 - 9x$

Sym. about x-axis? keep x, change y with "$-y$" do we get the same equation?

$(-y) = x^3 - 9x \rightarrow -y = x^3 - 9x \longrightarrow y = -x^3 + 9x$ ✗ incorrect

Sym. about y-axis? keep y, change x with "$-x$" do we get the same equation?

$y = (-x)^3 - 9(-x) = -x^3 + 9x$ ✗ incorrect

Sym. about origin? change y with "$-y$" & change x with "$-x$" do we get the same equation?

$(-y) = (-x)^3 - 9(-x)$

$-y = -x^3 + 9x \longrightarrow y = x^3 - 9x$ ✓ So y is sym. about the origin

8. Determine whether the following functions are even, odd or neither.

(a) $g(x) = 1 - \sqrt[3]{x}$

$g(-x) = 1 - \sqrt[3]{-x} = 1 + \sqrt[3]{x} \neq g(x)$

$-g(x) = -(1 - \sqrt[3]{x}) = -1 + \sqrt[3]{x}$ so $g(-x) \neq -g(x)$

Neither!

(b) $g(x) = \sqrt[3]{x^2 - 1}$

$g(-x) = \sqrt[3]{(-x)^2 - 1} = \sqrt[3]{x^2 - 1} = g(x)$ ✓ even

(c) $h(x) = \dfrac{x^3}{x^4 + 2}$

$h(-x) = \dfrac{(-x)^3}{(-x)^4 + 2} = \dfrac{-x^3}{x^4 + 2} \neq h(x)$

$-h(x) = -\dfrac{x^3}{x^4 + 2} = \dfrac{-x^3}{x^4 + 2}$ So $h(-x) = -h(x)$ ✓ odd

9. Determine whether the following equations define y as a function of x.

(a) $\sqrt{y} - x = 5$

$\sqrt{y} = x + 5$

$y = (x+5)^2$ ✓ function

(b) $2x + |y| = 0$ $\rightarrow$ $|y| = -2x$

if $y \geq 0$: $y = -2x$

if $y < 0$: $-y = -2x \rightarrow y = 2x$

(in other words, $y = \pm 2x$)

Not a function!

10. Find an equation of the line through the points $(-1,-2)$ and perpendicular to the line $2x+5y+8=0$.

$$2x+5y+8=0 \longrightarrow 5y=-2x-8$$

$$\longrightarrow y=-\frac{2}{5}x-\frac{8}{5} \qquad \text{Slope: } -\frac{2}{5}$$

$$m=\frac{-1}{-2/5}=-1\times -5/2=5/2$$

$$y-(-2)=5/2(x-(-1))$$

$$y+2=5/2(x+1)$$

11. Find an equation of the line through the points $(10,-5)$ and $(6,-5)$.

$$m=\frac{-5+5}{10-6}=\frac{0}{4}=0 \longrightarrow \text{Constant line (Horizontal line)} \qquad y=-5$$

12. Find $\underbrace{\text{average rate of change}}_{\text{Slope}}$ of the equation $h(t) = \dfrac{4}{3+2t}$ on the interval $[-2, 3]$.

$h(3) = \dfrac{4}{3+6} = \dfrac{4}{9}$

$h(-2) = \dfrac{4}{3+2(-2)} = \dfrac{4}{3-4} = -4$

$$\text{Ave. R.O.C} = \frac{h(3) - h(-2)}{3-(-2)} = \frac{\frac{4}{9} - (-4)}{5} = \frac{\frac{4}{9}+4}{5} = \frac{\frac{4+36}{9}}{5}$$

$$= \frac{40}{9} \times \frac{1}{5} = \boxed{\frac{40}{45}}$$

13. If an object is dropped from a high cliff or a tall building, then the distance it has fallen after t second is given by the function $d(t) = 16t^2$. Find its average speed (average rate of change) over the interval $[a, a+h]$

$d(a) = 16a^2$

$d(a+h) = 16(a+h)^2 = 16(a^2+h^2+2ah)$

$$\text{Ave. R.O.C.} = \frac{d(a+h) - d(a)}{(a+h)-(a)} = \frac{\cancel{16a^2} + 16h^2 + 32ah - (\cancel{16a^2})}{h}$$

$$= \frac{16h^2 + 32ah}{h} = \frac{\cancel{h}(16h + 32a)}{\cancel{h}}$$

14. Solve the following.
    (a) $|x+3| = x^2 - 4x - 3$

$x+3 = \pm(x^2-4x-3)$

$x+3 = x^2-4x-3$

$0 = x^2-4x-3-x-3$

$x^2-5x-6=0$

$(x+1)(x-6)=0$

$x=6$

$x=-1$

$x+3 = -x^2+4x+3$

$-x^2+3x=0$

$x(-x+3)=0$

$x=0$

$x=3$

Check your answers:

$x=0$ $\quad |3| = -3$ ✗ extraneous

$x=3$ $\quad |3+3| = 9-12-3$

$6 = 9-15 = -6$ ✗

$\boxed{x=6}$ $\quad |6+3| = 36-4(6)-3$

$9 = 36-24-3$

$9=9$ ✓

$\boxed{x=-1}$

$|-1+3| = (-1)^2-4(-1)-3$

$|2| = 1+4-3$

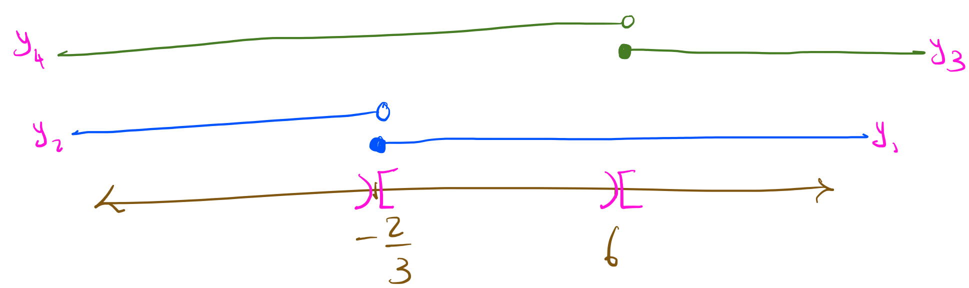
$|2| = 2$ ✓

(b) $|3x+2| \le |x-6| - 5 \iff |3x+2| - |x-6| + 5 \le 0$

First we solve for $|3x+2| - |x-6| + 5 = 0$

$$|3x+2| = \begin{cases} 3x+2 & \text{if } 3x+2 \ge 0 \\ -(3x+2) & \text{if } 3x+2 < 0 \end{cases} \longrightarrow \begin{cases} y_1 \quad 3x+2 & \text{if } x \ge -2/3 \\ y_2 \quad -3x-2 & \text{if } x < -2/3 \end{cases}$$

$$|x-6| = \begin{cases} x-6 & \text{if } x-6 \ge 0 \\ -(x-6) & \text{if } x-6 < 0 \end{cases} \longrightarrow \begin{cases} y_3 \quad x-6 & \text{if } x \ge 6 \\ y_4 \quad -x+6 & \text{if } x < 6 \end{cases}$$



we have three intervals $x < -2/3$, $-\frac{2}{3} \le x < 6$, $x \ge 6$

Case 1 if $x < -\frac{2}{3}$

$|3x+2| - |x-6| + 5 = y_2 - y_4 + 5 = -3x - 2 - (-x+6) + 5$

$= -3x - 2 + x - 6 + 5 = -2x - 8 + 5 = \boxed{-2x - 3}$

Case 2 if $-\frac{2}{3} \leq x < 6$

$$|3x+2| - |x-6| + 5 = y_1 - y_4 + 5 = 3x+2 - (-x+6) + 5$$

$$= 3x+2+x-6+5 = 4x-4+5 = \boxed{4x+1}$$

Case 3 if $x \geq 6$

$$|3x+2| - |x-6| + 5 = y_1 - y_3 + 5 = 3x+2 - (x-6) + 5$$

$$= 3x+2-x+6+5 = 2x+8+5 = \boxed{2x+13}$$

So to solve for $|3x+2| - |x-6| + 5 = 0$

if $x < -2/3$: $-2x-3=0 \rightarrow \boxed{x = -3/2}$

$\frac{-9}{6} = -\frac{3}{2} < -\frac{2}{3} = \frac{-4}{6}$ ✓

if $-\frac{2}{3} \leq x < 6$: $4x+1=0 \rightarrow \boxed{x = -\frac{1}{4}}$

$\frac{-3}{12} = -\frac{1}{4} > -\frac{2}{3} = \frac{-8}{12}$

$-\frac{2}{3} < -\frac{1}{4} < 6$ ✓ ✓

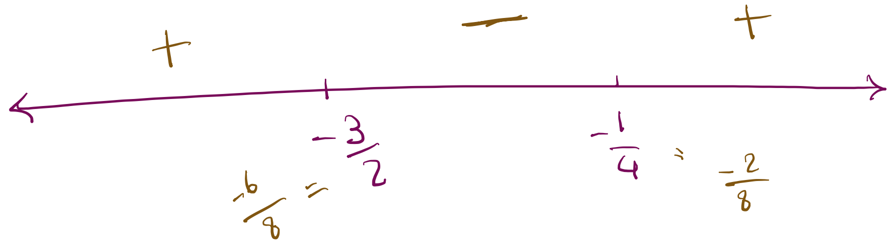
if $x \geq 6$: $2x+13=0 \rightarrow x = -\frac{13}{2}$

but $-\frac{13}{2} \not\geq 6$

doesn't satisfy the condition

So it's an extraneous solution

Finally to solve our inequality: $|3x+2|-|x-6|+5 \leq 0$



$$|-30+2| - |-16|+5 = |-28|-|16| > 0$$

$$\left|-\frac{3}{2}+2\right| - \left|-\frac{1}{2}-6\right| + 5 = \left|\frac{-3+4}{2}\right| - \left|\frac{-1-12}{2}\right| + 5 = \left|\frac{1}{2}\right| - \left|\frac{-13}{2}\right| + 5$$

$$= \frac{1}{2} - \frac{13}{2} + 5 = -\frac{12}{2} + \frac{10}{2} = -\frac{2}{2}$$

$$= -1$$

Solution $\left[-\frac{3}{2}, -\frac{1}{4}\right]$

15. Consider the function

$$h(x) = \begin{cases} -2x+5 & \text{, if } x < -1 \\ 2x^2 - 4 & \text{, if } x > -1. \end{cases}$$

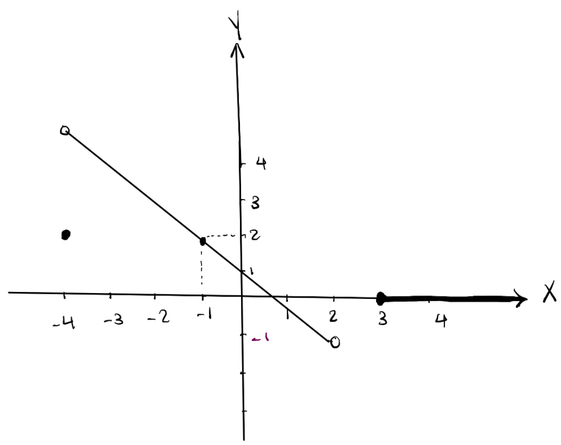
Find $h(-3)$, $h(-1)$, and $h(5)$.

$h(-3) = -2(-3)+5 = 0$

$h(-1)$ undefined

$h(5) = 2(5)^2 - 4 = 50 - 4 = 46$

16. Write a piecewise defined function for the graph below.



for $x \geq 3$ (Horiz.) Constant line $\boxed{y_1 = 0}$

for $x = -4 \to$ just a point $y_2: (-4, 2)$

for $-4 < x < 2$ linear equation two points $(2,-1)$ & $(-1,2)$ $m = \frac{2-(-1)}{-1-2} = \frac{3}{-3} = -1$

$y-(-1) = -1(x-2) \to y+1 = -x+2 \longrightarrow y_3 = -x+1$

$$f(x) = \begin{cases} 2 & x = -4 \\ -x+1 & -4 < x < 2 \\ 0 & x \geq 3 \end{cases}$$