Math 150 - Week-In-Review 4 Solutions

Saud Hussein

Section 3.2 – Properties of Rational Functions

1. **Describe the end behavior of the graph of each rational function and the behavior near any vertical asymptotes using approaching notation. Find the domain and the location of any holes for each rational function.**

   (a) $f(x) = \dfrac{x-2}{x^2-4}$

   Solution:

   If $x \neq 2$, then

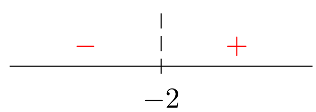
$$f(x) = \frac{x-2}{x^2-4} = \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \frac{1}{x+2}.$$

   Therefore, there is a vertical asymptote at $x = -2$ and a hole at $\left(2, \dfrac{1}{4}\right)$. This also means the domain of f is

$$(-\infty, -2) \cup (-2, 2) \cup (2, \infty).$$

   The degree of the polynomial in the denominator is greater than the degree of the polynomial in the numerator, so there is a horizontal asymptote at $y = 0$.

   Drawing a sign diagram of f around $x = -2$,



   Using the sign diagram, for the horizontal asymptote $y = 0$, as

$$x \to -\infty, \quad f(x) \to 0^-,$$

$$x \to \infty, \quad f(x) \to 0^+,$$

   and for the vertical asymptote $x = -2$, as

$$x \to -2^-, \quad f(x) \to -\infty,$$

$$x \to -2^+, \quad f(x) \to \infty.$$

   (b) $f(x) = \dfrac{2x^2+5}{-x^2-x+2}$

   Solution:

   Since

$$f(x) = \frac{2x^2+5}{-x^2-x+2} = \frac{2x^2+5}{-(x-1)(x+2)},$$

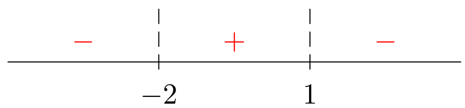
Copyright © 2024 Math Learning Center

there are vertical asymptotes at $x = -2$ and $x = 1$. This also means the domain of f is

$$(-\infty, -2) \cup (-2, 1) \cup (1, \infty).$$

The degrees of the polynomials in the denominator and numerator are the same, so there is a horizontal asymptote at the ratio of the leading coefficients, $y = -2$.

Drawing a sign diagram of f around $x = -2$ and $x = 1$,



Using the sign diagram, for the vertical asymptote $x = -2$, as

$$x \to -2^-, \quad f(x) \to -\infty,$$

$$x \to -2^+, \quad f(x) \to \infty,$$

and for the vertical asymptote $x = 1$, as

$$x \to 1^-, \quad f(x) \to \infty,$$

$$x \to 1^+, \quad f(x) \to -\infty.$$

For the horizontal asymptote $y = -2$, there is no easy way to determine the approaching end behavior when the horizontal asymptote $y \neq 0$. Evaluating f at large positive and negative x values, we see that as

$$x \to -\infty, \quad f(x) \to -2^-,$$

$$x \to \infty, \quad f(x) \to -2^+.$$

(c) $f(x) = \dfrac{2(x+1)^2(x-3)}{(x+3)^2(x-2)}$

Solution:

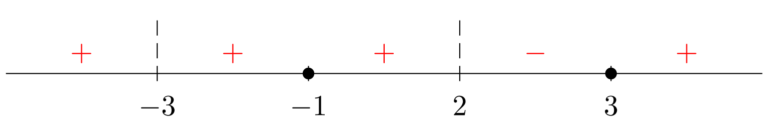
Since the denominator is factored, there are vertical asymptotes at $x = -3$ and $x = 2$.

Therefore, the domain of f is

$$(-\infty, -3) \cup (-3, 2) \cup (2, \infty).$$

The degrees of the polynomials in the denominator and numerator are the same, so there is a horizontal asymptote at the ratio of the leading coefficients, $y = 2$.

Drawing a sign diagram of f around the horizontal intercepts at $x = -1$ and $x = 3$, and around the vertical asymptotes $x = -3$ and $x = 2$,



Using the sign diagram, for the vertical asymptote $x = -3$, as

$$x \to -3^-, \quad f(x) \to \infty,$$
$$x \to -3^+, \quad f(x) \to \infty,$$

and for the vertical asymptote $x = 2$, as

$$x \to 2^-, \quad f(x) \to \infty,$$
$$x \to 2^+, \quad f(x) \to -\infty.$$

For the horizontal asymptote $y = 2$, again, there is no easy way to determine the approaching end behavior when the horizontal asymptote $y \neq 0$. Evaluating f at large positive and negative x values, we see that as

$$x \to -\infty, \quad f(x) \to 2^+,$$
$$x \to \infty, \quad f(x) \to 2^-.$$

The approaching behavior can sometimes be determined by graphing f, as we do for this function in Section 3.3 problem 1, part d.

2. **Determine the equation of any slant asymptotes for each rational function.**

(a) $f(x) = \dfrac{3x^2 + 2}{x - 5}$

Solution:
Since the degree of the polynomial in the numerator is greater than the degree of the polynomial in the denominator, there is no horizontal asymptote.

The degree is one larger, so there does exist a slant asymptote instead.

Using synthetic division,

$$\begin{array}{r|rrr} 5 & 3 & 0 & 2 \\ & \downarrow & 15 & 75 \\ \hline & 3 & 15 & \boxed{77} \end{array}$$

Therefore, as $x \to \pm\infty$,

$$f(x) = \frac{3x^2 + 2}{x - 5} = 3x + 15 + \frac{77}{x - 5} \to y = 3x + 15.$$

(b) $f(x) = \dfrac{x^2 - x - 6}{x - 1}$

Solution:
Since the degree of the polynomial in the numerator is greater than the degree of the polynomial in the denominator, there is no horizontal asymptote.

The degree is one larger, so there does exist a slant asymptote instead.

Using synthetic division,

1	1	−1	−6
	↓	1	0
	1	0	−6

Therefore, as $x \to \pm\infty$,

$$f(x) = \frac{x^2 - x - 6}{x - 1} = x - \frac{6}{x-1} \to y = x.$$

3. **A large mixing tank contains 100 gallons of water, into which 5 pounds of sugar has been mixed. A tap opens pouring 10 gallons a minute of water into the tank at the same time sugar is poured into the tank at a rate of one pound a minute. Find the concentration $C(t)$ (lb. per gallon) of sugar in the tank after t minutes and explain the physical meaning of the horizontal and vertical asymptotes of the graph of $C(t)$.**

Solution:

The water and sugar in the tank changes according to the linear functions

$$w(t) = 10t + 100, \qquad s(t) = t + 5,$$

so,

$$C(t) = \frac{s(t)}{w(t)} = \frac{t+5}{10t+100}.$$

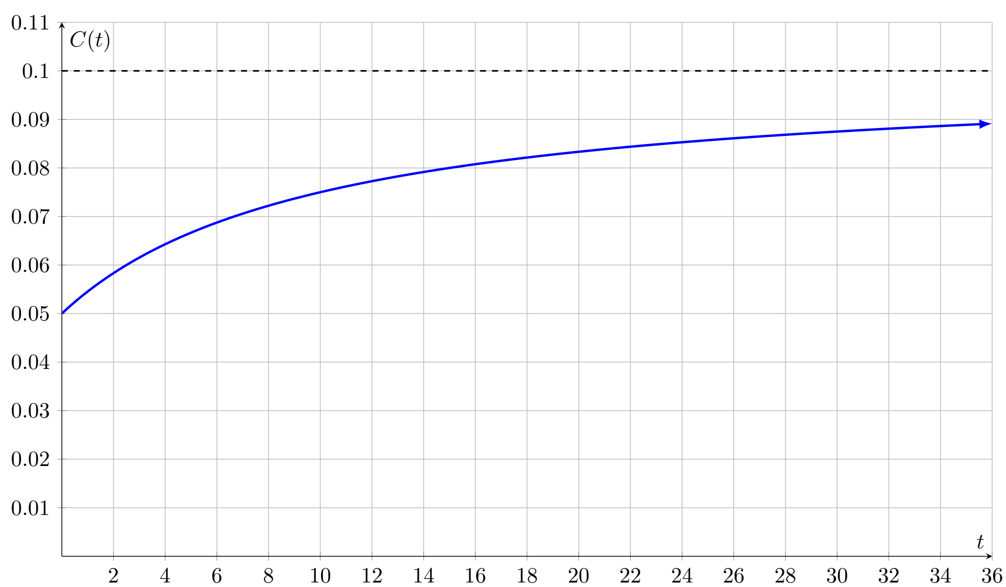
Since

$$10t + 100 = 0 \implies t = -10,$$

the vertical asymptote $t = -10$ has no physical meaning. Also, as $t \to \infty$,

$$C(t) = \frac{t+5}{10t+100} \approx \frac{t}{10t} = \frac{1}{10}.$$

Therefore, there is a horizontal asymptote at $C = 1/10$, meaning that the concentration of sugar in the tank will eventually be 1/10 of a pound of sugar for every gallon of water. The graph of the function confirms this behavior.



Section 3.3 – Graphs of Rational Functions

1. **Sketch the graph of each rational function.**

   (a) $f(x) = \dfrac{x^2 + 2x - 3}{x^2 - 1}$

   Solution:
   Since
   $$f(0) = \frac{-3}{-1} = 3,$$
   the vertical intercept is $(0, 3)$.

   Factoring the numerator, if $x \neq 1$, then
   $$f(x) = \frac{x^2 + 2x - 3}{x^2 - 1} = \frac{\cancel{(x-1)}(x+3)}{\cancel{(x-1)}(x+1)}.$$

   Therefore, there is a vertical asymptote at $x = -1$ and a hole at
   $$\left(1, \frac{4}{2}\right) = (1, 2).$$

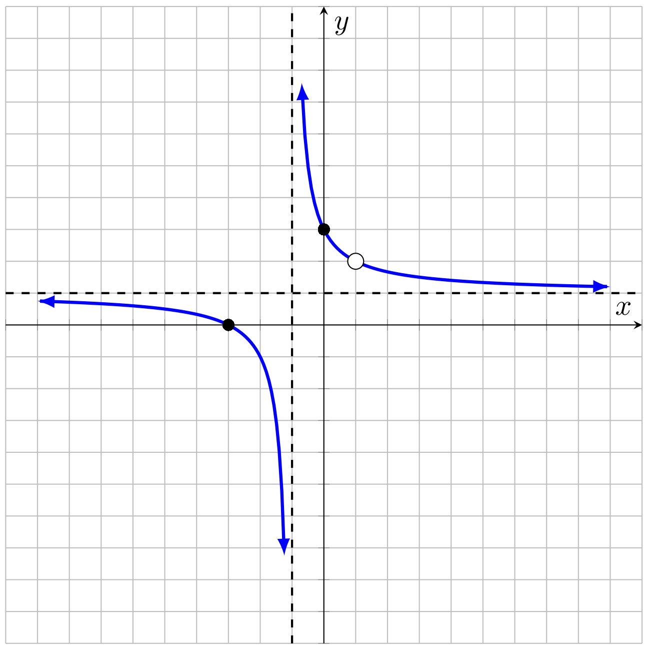
   The degrees of the polynomials in the denominator and numerator are the same, so there is a horizontal asymptote at the ratio of the leading coefficients, $y = 1$.

   Also, if $x \neq 1$,
   $$\begin{aligned} 0 = f(x) &= \frac{x+3}{x+1} \\ 0 &= x + 3. \end{aligned}$$

   Therefore, there is a horizontal intercept at $x = -3$, with the graph behaving like a line near $x = -3$, since the factor $(x + 3)$ is linear.

   Since $(x+1)$ in the denominator of f is linear, the graph will have the opposite behavior on either side of the vertical asymptote $x = -1$.

   We can also draw a sign diagram of f around $x = -3$ and -1 to help sketch the graph.



(b) $f(x) = \dfrac{27(x-2)}{(x+3)(x-3)^2}$

Solution:
Since

$$f(0) = \frac{27(-2)}{(3)(-3)^2} = -\frac{54}{27} = -2,$$

the vertical intercept is $(0, -2)$.

Since the denominator is factored, there are vertical asymptotes at $x = -3$ and $x = 3$.

The degree of the polynomial in the denominator is greater than the degree of the polynomial in the numerator, so there is a horizontal asymptote at $y = 0$.

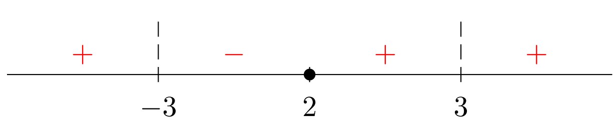
Also, if $f(x) = 0$, then

$$0 = f(x) = 27(x-2).$$

Therefore, there is a horizontal intercept at $x = 2$, with the graph behaving like a line near $x = 2$, since the factor $(x-2)$ is linear.

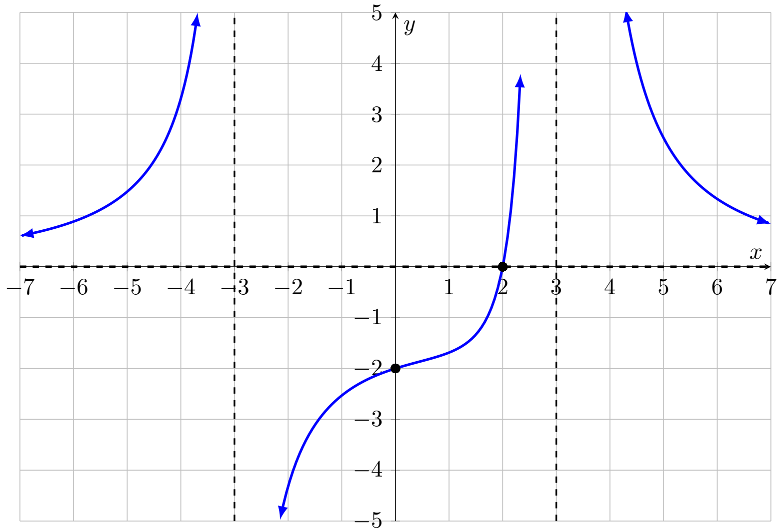
Since $(x-3)^2$ in the denominator is quadratic, the graph will have the same behavior on both sides of the vertical asymptote $x = 3$. Also, $(x+3)$ is linear, so the graph will have the opposite behavior on either side of the vertical asymptote $x = -3$.

We can also draw a sign diagram of f around $x = -3$, 2, and 3 to help sketch the graph.



Copyright © 2024 Math Learning Center

By first drawing the branch of the graph between the two vertical asymptotes, we can now sketch the complete graph.



(c) $f(x) = \dfrac{3(x^2 + 2x - 8)}{(x+1)(x^2 - 7x + 10)}$

Solution:
Since

$$f(0) = \frac{3(-8)}{(1)(10)} = -\frac{24}{10} - 2.4,$$

the vertical intercept is $(0, -2.4)$.

Factoring the numerator and denominator, if $x \neq 2$, then

$$f(x) = \frac{3(x^2 + 2x - 8)}{(x+1)(x^2 - 7x + 10)} = \frac{3\cancel{(x-2)}(x+4)}{(x+1)\cancel{(x-2)}(x-5)} = \frac{3(x+4)}{(x+1)(x-5)}.$$

Therefore, there are vertical asymptote at $x = -1$ and $x = 5$, and there is a hole at

$$\left(2, \frac{3(6)}{(3)(-3)}\right) = \left(2, -\frac{18}{9}\right) = (2, -2)\,.$$

The degree of the polynomial in the denominator is greater than the degree of the polynomial in the numerator, so there is a horizontal asymptote at $y = 0$.

Also, if $x \neq 2$,

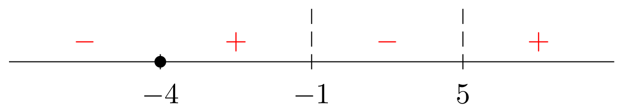
$$0 = f(x) = \frac{3(x+4)}{(x+1)(x-5)}$$
$$0 = 3(x+4).$$

Therefore, there is a horizontal intercept at $x = -4$, with the graph behaving like a line near $x = -4$, since the factor $(x + 4)$ is linear.

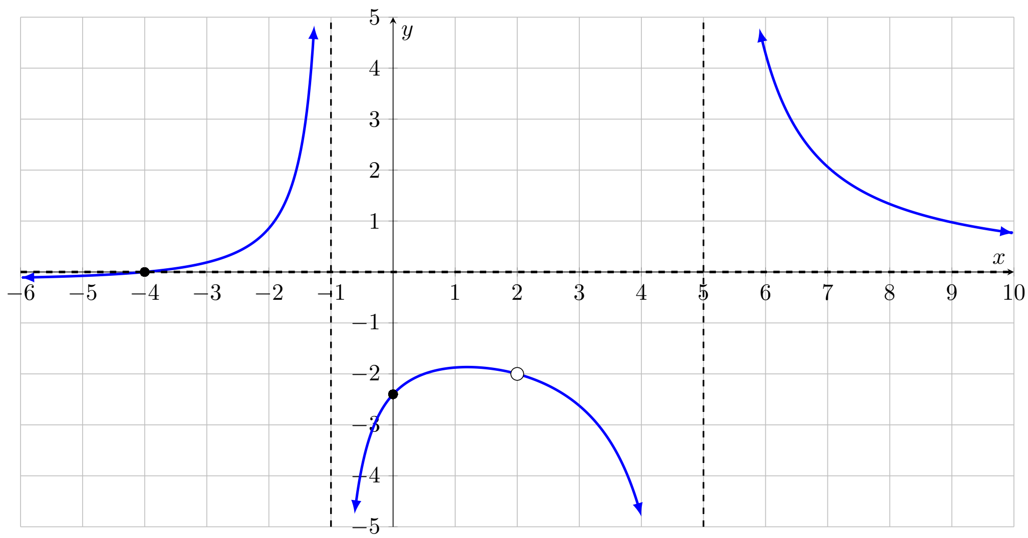
Copyright © 2024 Math Learning Center

Since $(x+1)$ and $(x-5)$ in the denominator of f are both linear, the graph will have the opposite behavior on either side of the vertical asymptotes $x=-1$ and $x=5$.

We can also draw a sign diagram around $x=-4$, -1, and 5 to help sketch the graph.



By first drawing the branch of the graph between the two vertical asymptotes, we can now sketch the complete graph.



(d) $f(x) = \dfrac{2(x+1)^2(x-3)}{(x+3)^2(x-2)}$

Solution:
Since

$$f(0) = \frac{2(1)^2(-3)}{(3)^2(-2)} = \frac{6}{18} = \frac{1}{3},$$

the vertical intercept is $(0, 1/3)$.

Also, if $f(x)=0$, then

$$0 = f(x) = 2(x+1)^2(x-3).$$

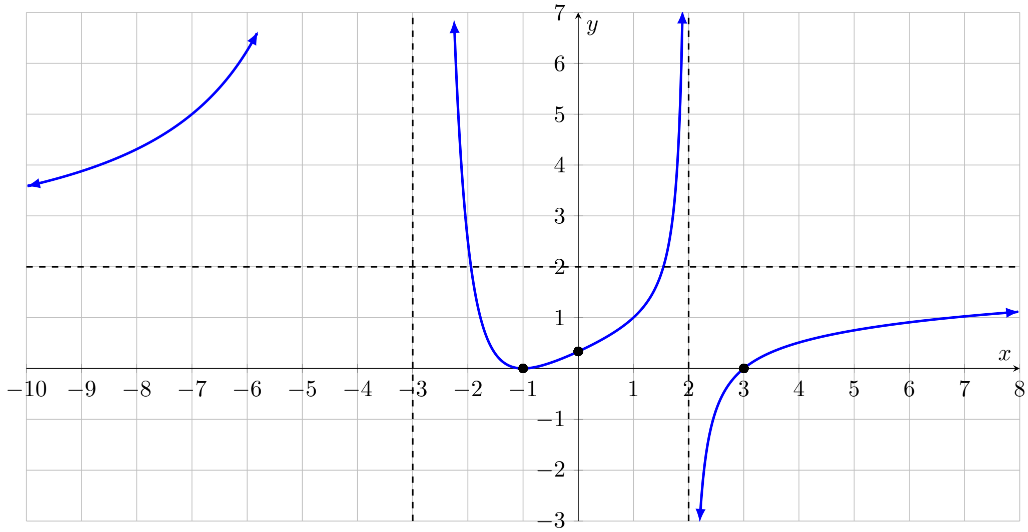
Therefore, there are horizontal intercepts at $x=-1$ and $x=3$, with the graph behaving like a line near $x=3$, since the factor $(x-3)$ is linear, and like a parabola near $x=-1$, since the factor $(x+1)^2$ is quadratic.

Since $(x+3)^2$ in the denominator is quadratic, the graph will have the same behavior on both sides of the vertical asymptote $x=-3$. Also, $(x-2)$ is linear, so the graph will have the opposite behavior on either side of the vertical asymptote $x=2$.

Copyright © 2024 Math Learning Center

In problem 1, part c, in Section 3.2, we determined the end behavior and used a sign diagram of f to confirm the observations described here.

By first drawing the branch of the graph between the two vertical asymptotes, we can now sketch the complete graph.



(e) $f(x) = \dfrac{-2x(x-3)}{(x-4)(x+3)}$

Solution:
Since

$$f(0) = \frac{(0)(-3)}{(-4)(3)} = 0,$$

the vertical intercept is $(0, 0)$.

Since the denominator is factored, there are vertical asymptotes at $x = -3$ and $x = 4$.

The degrees of the polynomials in the denominator and numerator are the same, so there is a horizontal asymptote at the ratio of the leading coefficients, $y = -2$.

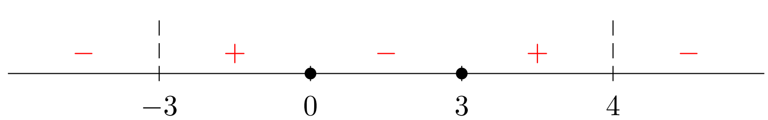
Also, if $f(x) = 0$, then

$$0 = f(x) = -2x(x-3).$$

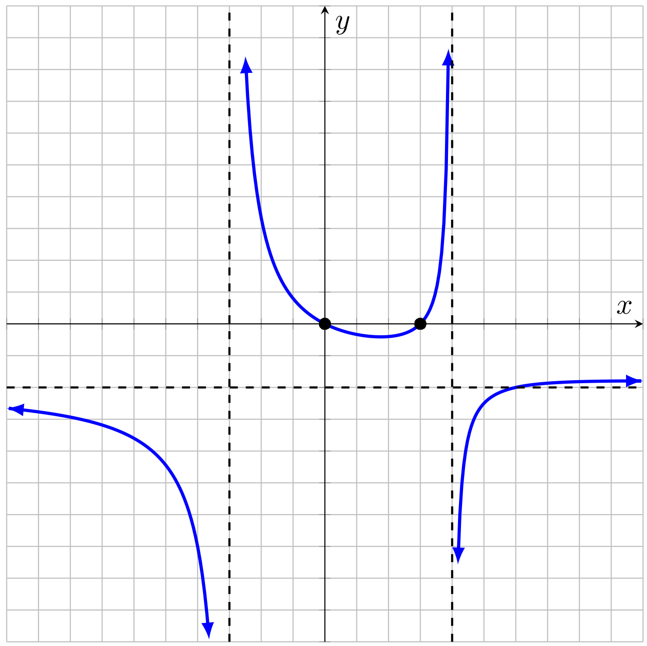
Therefore, there are horizontal intercepts at $x = 0$ and $x = 3$, with the graph behaving like a line near $x = 0$ and $x = 3$, since both factors x and $(x-3)$ are linear.

Since the factors $(x-4)$ and $(x+3)$ in the denominator are both linear, the graph will have the opposite behavior on either side of the vertical asymptotes at $x = -3$ and $x = 4$.

We can also draw a sign diagram around $x = -3$, 0, 3, and 4 to help sketch the graph.



By first drawing the branch of the graph between the two vertical asymptotes, we can now sketch the complete graph.



(f) $f(x) = \dfrac{(x+2)^2(x-5)}{(x-3)(x+1)(x+4)}$

Solution:
Since

$$f(0) = \frac{(2)^2(-5)}{(-3)(1)(4)} = \frac{20}{12} = \frac{5}{3},$$

the vertical intercept is $(0, 5/3)$.

Since the denominator is factored, there are vertical asymptotes at $x = -4$, -1, and 3.

The degrees of the polynomials in the denominator and numerator are the same, so there is a horizontal asymptote at the ratio of the leading coefficients, $y = 1$.

Also, if $f(x) = 0$, then

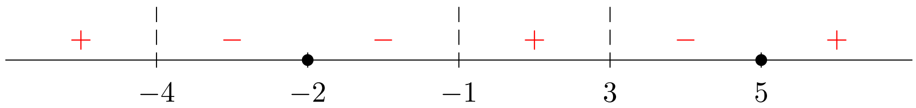
$$0 = f(x) = (x+2)^2(x-5).$$

Therefore, there are horizontal intercepts at $x = -2$ and $x = 5$, with the graph behaving like a line near $x = 5$, since the factor $(x-5)$ is linear, and like a parabola near $x = -2$, since the factor $(x+2)^2$ is quadratic.

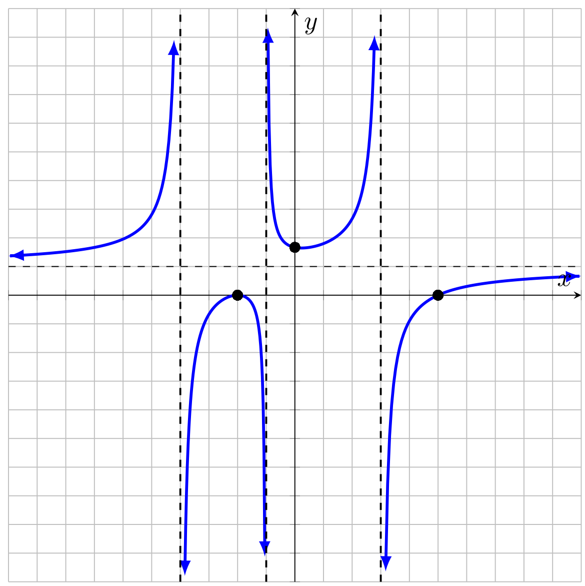
Since the factors $(x-3)$, $(x+1)$, and $(x+4)$ in the denominator are linear, the graph will have the opposite behavior on either side of the vertical asymptotes at $x = -4$, -1, and 3.

We can also draw a sign diagram around $x = -4$, -2, -1, 3, and 5 to help sketch f.

Copyright © 2024 Math Learning Center



By first drawing the branch of the graph between the two vertical asymptotes that include the vertical intercept, we can now sketch the complete graph.



2. **An airplane flies 200 miles into a 30 mph headwind and then flies another 200 miles with a 20 mph tailwind.**

   (a) **Find an equation for the time t of the trip as a function of the speed s of the plane.**

   Solution: Let s be the average speed of the plane, in mph, throughout the trip. Since distance traveled is the product of speed and time, time is distance over speed, so

$$t(s) = \frac{200}{s-30} + \frac{200}{s+20} = \frac{400(s-5)}{(s-30)(s+20)}.$$

   (b) **Graph the function found in part (a) and describe what the horizontal and vertical asymptotes of the function physically mean.**

   Solution:

   Since

$$t(0) = \frac{400(-5)}{(-30)(20)} = \frac{2000}{600} = \frac{10}{3},$$

   the vertical intercept is $(0, 10/3)$.

   Since the denominator is factored, there are vertical asymptotes at $s = -20$ and $s = 30$.

   The degree of the polynomial in the denominator is greater than the degree of the polynomial in the numerator, so there is a horizontal asymptote at $t = 0$.

Copyright © 2024 Math Learning Center

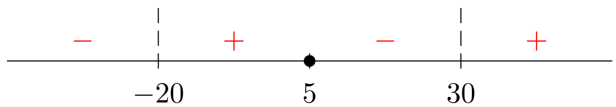
Also, if $t(s) = 0$, then

$$0 = t(s) = 400(s - 5).$$

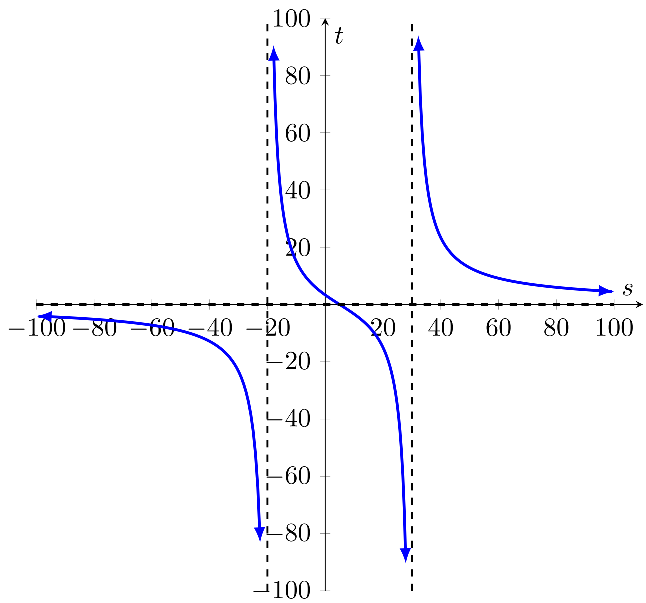
Therefore, there is a horizontal intercept at $s = 5$, with the graph behaving like a line near $s = 5$, since the factor $(s - 5)$ is linear.

Since the factors $(s - 30)$ and $(s + 20)$ in the denominator are linear, the graph will have the opposite behavior on either side of the vertical asymptotes at $s = -20$ and $s = 30$.

We can also draw a sign diagram around $s = -20$, 5, and 30 to help sketch the graph.



By first drawing the branch of the graph between the two vertical asymptotes, we can now sketch the complete graph.



Since negative speed and time does not make sense, only the portion of the graph in the first quadrant has meaning.

Also, the only branch that makes sense here is the curve to the right of the vertical asymptote $s = 30$, since flying with an average speed less than 30 mph would not be fast enough to overcome the initial 30 mph headwind.

The vertical asymptote $s = 30$ means if the plane flies with an average speed slighly larger than 30 mph, the first part of the trip with a 30 mph headwind will take a long time.

The horizontal asymptote at $t = 0$ means as the speed of the plane increases, the time of the trip decreases, approaching zero hours, in theory.

Copyright © 2024 Math Learning Center

Section 3.4 – Solving Rational Equations

1. **Solve the following rational equations.**

(a) $\dfrac{1}{x-2} = \dfrac{3}{x+2} - \dfrac{6x}{x^2-4}$

Solution: Since $x^2 - 4 = (x-2)(x+2)$, the excluded values are $x = -2$ and 2.

To clear the denominators,

$$\begin{aligned}(x-2)(x+2)\cdot\frac{1}{x-2} &= \left(\frac{3}{x+2} - \frac{6x}{(x-2)(x+2)}\right)(x-2)(x+2)\\ x+2 &= 3(x-2) - 6x\\ x+2 &= -3x-6\\ 4x &= -8\\ x &= -2.\end{aligned}$$

Since $x = -2$ is an excluded value, there are no solutions to this rational equation.

(b) $\dfrac{3}{x-1} - \dfrac{2}{x-8} = \dfrac{1}{x^2-9x+8}$

Solution: Since $x^2 - 9x + 8 = (x-1)(x-8)$, the excluded values are $x = 1$ and 8.

To clear the denominators,

$$\begin{aligned}(x-1)(x-8)\left(\frac{3}{x-1} - \frac{2}{x-8}\right) &= \frac{1}{(x-1)(x-8)}\cdot(x-1)(x-8)\\ 3(x-8) - 2(x-1) &= 1\\ 3x - 24 - 2x + 2 &= 1\\ x - 22 &= 1\\ x &= 23.\end{aligned}$$

Therefore, $x = 23$ is the solution to the rational equation.

(c) $\dfrac{x}{x+4} = \dfrac{32}{x^2-16} + 5$

Solution: Since $x^2 - 16 = (x-4)(x+4)$, the excluded values are $x = -4$ and 4.

To clear the denominators,

$$\begin{aligned}
(x-4)(x+4)\cdot\frac{x}{x+4} &= \left(\frac{32}{(x-4)(x+4)}+5\right)(x-4)(x+4)\\
x(x-4) &= 32+5(x-4)(x+4)\\
x^2-4x &= 32+5(x^2-16)\\
x^2-4x &= 32+5x^2-80\\
0 &= 4x^2+4x-48\\
0 &= x^2+x-12\\
0 &= (x-3)(x+4).
\end{aligned}$$

Since $x=-4$ is an excluded value, the only solution to this rational equation is $x=3$.

(d) $\dfrac{x}{x+3}=\dfrac{18}{x^2-9}+4$

Solution: Since $x^2-9=(x-3)(x+3)$, the excluded values are $x=-3$ and 3.

To clear the denominators,

$$\begin{aligned}
(x-3)(x+3)\cdot\frac{x}{x+3} &= \left(\frac{18}{(x-3)(x+3)}+4\right)(x-3)(x+3)\\
x(x-3) &= 18+4(x-3)(x+3)\\
x^2-3x &= 18+4(x^2-9)\\
x^2-3x &= 18+4x^2-36\\
0 &= 3x^2+3x-18\\
0 &= x^2+x-6\\
0 &= (x-2)(x+3).
\end{aligned}$$

Since $x=-3$ is an excluded value, the only solution to this rational equation is $x=2$.

(e) $\dfrac{x+3}{3x}+\dfrac{x}{24}=\dfrac{1}{x}$

Solution: First, take note of the excluded value $x=0$.

To clear the denominators,

$$\begin{aligned}
24x\cdot\left(\frac{x+3}{3x}+\frac{x}{24}\right) &= \frac{1}{x}\cdot 24x\\
8(x+3)+x^2 &= 24\\
8x+24+x^2 &= 24\\
x^2+8x &= 0\\
x(x+8) &= 0.
\end{aligned}$$

Since $x=0$ is an excluded value, the only solution to this rational equation is $x=-8$.

Copyright © 2024 Math Learning Center

(f) $\left|\dfrac{4x}{x^2-x-12}\right| = \left|\dfrac{2}{x-4}\right|$

Solution: Since $x^2 - x - 12 = (x-4)(x+3)$, the excluded values are $x = 4$ and -3.

The absolute value equation leads to two rational equations, and to clear the denominators in the first equation,

$$\begin{aligned}(x-4)(x+3)\cdot\frac{4x}{(x-4)(x+3)} &= \frac{2}{x-4}\cdot(x-4)(x+3)\\ 4x &= 2(x+3)\\ 4x &= 2x+6\\ 2x &= 6\\ x &= 3.\end{aligned}$$

Therefore, $x = 3$ is a potential solution to the absolute value equation.

To see if $x = 3$ is a solution,

$$\begin{aligned}\left|\frac{4x}{x^2-x-12}\right| &= \left|\frac{2}{x-4}\right|\\ \left|\frac{4(3)}{3^2-3-12}\right| &\stackrel{?}{=} \left|\frac{2}{3-4}\right|\\ \left|-\frac{12}{6}\right| &\stackrel{?}{=} |-2|\\ 2 &= 2.\end{aligned}$$

Therefore, $x = 3$ is a solution to the absolute value equation.

Clearing the denominators in the second equation,

$$\begin{aligned}(x-4)(x+3)\cdot\frac{4x}{(x-4)(x+3)} &= -\frac{2}{x-4}\cdot(x-4)(x+3)\\ 4x &= -2(x+3)\\ 4x &= -2x-6\\ 6x &= -6\\ x &= -1.\end{aligned}$$

Therefore, $x = -1$ is a potential solution to the absolute value equation.

To see if $x = -1$ is a solution,

$$\begin{aligned}\left|\frac{4x}{x^2-x-12}\right| &= \left|\frac{2}{x-4}\right|\\ \left|\frac{4(-1)}{(-1)^2-(-1)-12}\right| &\stackrel{?}{=} \left|\frac{2}{-1-4}\right|\\ \left|\frac{-4}{-10}\right| &\stackrel{?}{=} \left|-\frac{2}{5}\right|\\ \frac{2}{5} &= \frac{2}{5}.\end{aligned}$$

Copyright © 2024 Math Learning Center

Therefore, $x = -1$ is also a solution to the absolute value equation.

2. **An airplane can fly 200 miles into a 30 mph headwind in the same amount of time it takes to fly 300 miles with a 30 mph tailwind. What is the speed of the airplane?**

   Solution:

   Let s be the average speed of the plane, in mph, throughout the flight. Since distance traveled is the product of speed and time, time is distance over speed, so

$$\frac{200}{s-30} = \frac{300}{s+30}.$$

   First, take note of the excluded values $s = -30$ and $s = 30$. To clear the denominators,

$$\begin{aligned}(s-30)(s+30)\cdot\frac{200}{s-30} &= \frac{300}{s+30}\cdot(s-30)(s+30)\\ 200(s+30) &= 300(s-30)\\ 200s+6000 &= 300s-9000\\ 100s &= 15000\\ s &= 150 \text{ mph.}\end{aligned}$$

3. **Jazmine trained for 3 hours on Saturday. She ran 8 miles and then biked 24 miles. Her biking speed is 4 mph faster than her running speed. What is her running speed?**

   Solution:

   Let s be her average running speed in mph. Since distance traveled is the product of speed and time, time is distance over speed, so

$$\frac{8}{s} + \frac{24}{s+4} = 3.$$

   First, take note of the excluded values $s = -4$ and $s = 0$. To clear the denominators,

$$\begin{aligned}s(s+4)\cdot\left(\frac{8}{s}+\frac{24}{s+4}\right) &= 3\cdot s(s+4)\\ 8(s+4)+24s &= 3s(s+4)\\ 8s+32+24s &= 3s^2+12s\\ 32s+32 &= 3s^2+12s\\ 0 &= 3s^2-20s-32\\ 0 &= (3s+4)(s-8).\end{aligned}$$

   Since negative speed does not make sense, $s = 8$ mph is her average running speed.

4. **Hamilton rode his bike downhill 12 miles on the river trail from his house to the ocean and then rode uphill to return home. His uphill speed was 8 miles per hour**

slower than his downhill speed. It took him 2 hours longer to get home than it took him to get to the ocean. Find Hamilton's downhill speed.

Solution:

Let s be his average downhill speed in mph. Since distance traveled is the product of speed and time, time is distance over speed, so

$$\frac{12}{s-8} = \frac{12}{s} + 2.$$

First, take note of the excluded values $s = 0$ and $s = 8$. To clear the denominators,

$$\begin{aligned} s(s-8) \cdot \frac{12}{s-8} &= \left(\frac{12}{s} + 2\right) \cdot s(s-8) \\ 12s &= 12(s-8) + 2s(s-8) \\ 12s = &= 12s - 96 + 2s^2 - 16s \\ 0 &= -96 + 2s^2 - 16s \\ 0 &= s^2 - 8s - 48 \\ 0 &= (s-12)(s+4). \end{aligned}$$

Since negative speed does not make sense, $s = 12$ mph is his average downhill speed.

Copyright © 2024 Math Learning Center