## 2024 Fall Math 140 Week-In-Review

Week 9: Sections 5.3 and 5.4

Some Key Words and Terms: Domain, Rational Functions, Holes, Vertical Asymptotes, Intercepts, Simplifying Rational Expressions, Difference Quotient, Power/Radical Functions, Converting Exponents, Conjugate, Rationalizing.

Domain:

- 3 domain restrictions
  - (1) denominator (+0)

  - 2 even root (inside 20)
    3 logs (inside >0)

Rational Function:

polynomial & ble this is a fraction (so has a denominator)

polynomial & we will always check for domain restrictions

- · domain restrictions for rational function can only be: a hole or VA

· to determine, we factor & reduce the rational function

Holes: an x-value is a hole if the term in the denominator completely cancels that gave that x-value:

$$f(x) = \frac{(x+1)(x-1)}{(x+1)(x-2)} \rightarrow \frac{x+1}{x-2}$$
 (x-1) completely concelled from the bottom, so

x=l is where a hale

Vertical Asymptotes: an x-value is a VA if the term in the denominator does not completely cancel that gave that x-value

$$f(x) = \frac{(x+1)(x-1)}{(x+1)(x-2)} \Rightarrow \frac{x+1}{x-2}$$
 (x-2) did not cancel from the bottom, so

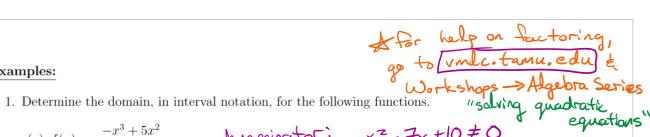
x=2 is where a VA \_\_\_ is at.

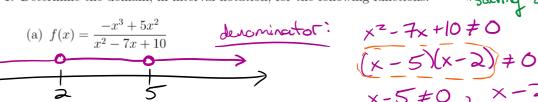
y-int: plug-in x=0 to function

x-int: ) we any function we set the whole function = 0 (top bottom

| Simplifying Rational Expressions: adding/subtracting/multiplying/dividing fractions   |
|---|
| add/subtract: 1) factoring & 2) common denominators   |
| multiply/divide: 1) factoring & @ converting division -> multiplication   |
| Difference Quotient:   f(x+h) - f(x) As be able to use with:  1) polynamials (multiplying energithing out & combine term  2) rationals (will always a common denominator)  3) radicals (will always use the conjugate of the top) |
| Power: (variable) (number: decimal, fraction, +/-,  |
| Radical: (Cirside) n is the "index": 1) odd (no restrictions) 2) even (check for restrictions)  |
| Converting Exponents:   |
| $\swarrow$ $\chi$  |
| In quent: (expression)  Detter for domain  Conjugate: derivatives (Moth 142)  Detter for domain   |
| (1)   |
| -xitil & -xitil are conjugates<br>$5-x \le 5+x$ are conjugates $(a)+(b)=(a)+(b)$<br>$(a+b)(a-b)=a^2-b^2$ helpful to remember  |
| Rationalizing: A you are most likely to see this with difference quotients  |
| Rationalizing: A you are most likely to see this with difference quotients to radical function &  |
| multiply the fraction (it not put over I) top & bottom by   |
| the conjugate of the radical reint  |
| $\frac{2}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} =$  |

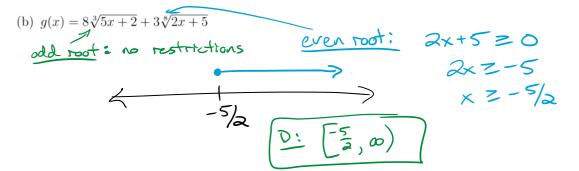
## **Examples:**





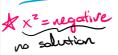
$$x^{2}-7x+10\neq0$$
  
 $(x-5)(x-2)\neq0$   
 $x-5\neq0$ ,  $x-2\neq0$   
 $x\neq5$ ,  $x\neq2$ 

D: (-0,2)U(2,5)U(5,00)/



(c) 
$$h(x) = \frac{2\sqrt{9-8x}}{x^2-5}$$

even root: 9-8x20

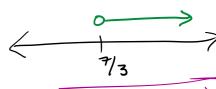


 $-\frac{8\times 2-9}{-8} = \frac{8}{-8} = \frac{1}{8} = \frac{1}{8$ 

(d) 
$$j(x) = \frac{5x^2 + 6x - 9}{\sqrt[4]{3x - 7}}$$



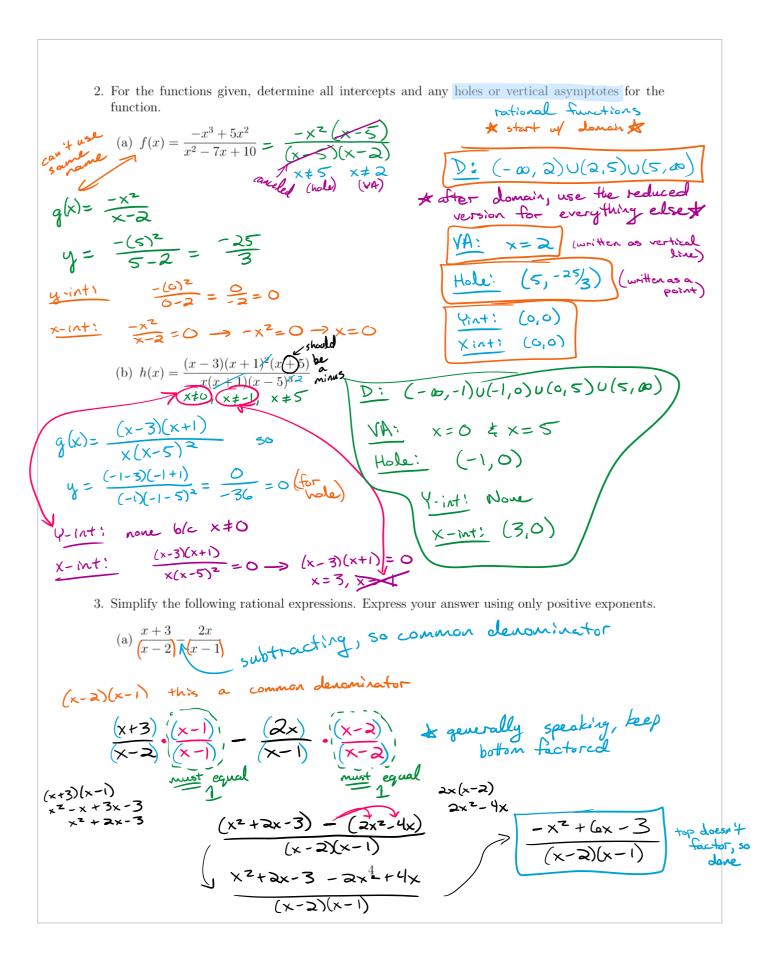
even root! 3x-720 denominator:  $4/3x-7 \neq 6$ 

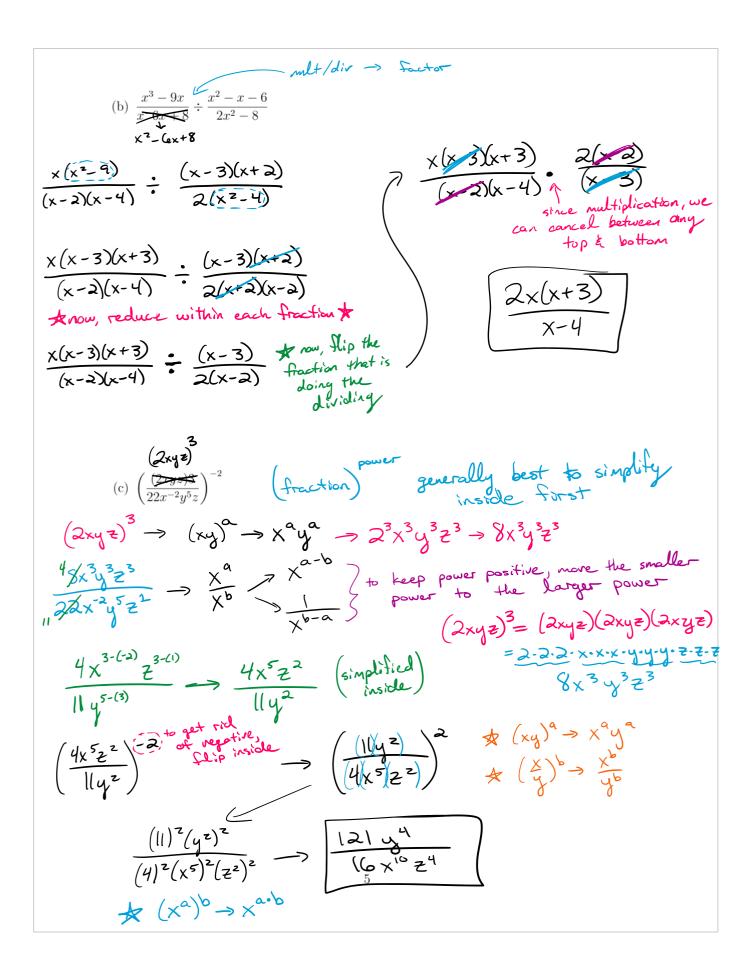


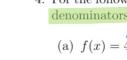
$$3x-7>0$$
 $3x>7$ 

(7/3, as)

, x>73







denominators.

(a) 
$$f(x) = 4\sqrt[3]{(x^2 - 6x)^7}$$
, rewrite

$$f(x) = 4\left(x^2 - 6x\right)^{\frac{7}{3}}$$

$$\forall x^a \rightarrow x^{a/6}$$

(b) 
$$g(x) = \frac{7}{\sqrt[6]{(8-3x)^5}} \longrightarrow \frac{7}{\sqrt[6]{(8-3x)^5}}$$

$$= \sqrt{7(8-3x)^5}$$

5. For the following functions, convert from power form to radical form. Express your answer without negative exponents.

negative exponents.

(a) 
$$f(x) = x^{1/2} - 3x^{-1/2} + (7x)^{-5/4}$$

$$f(x) = x^{1/2} - \frac{3}{x^{1/2}} + \frac{1}{(7x)^{5/4}}$$
(power to both)

$$5(x) = \sqrt[3]{x'} - \frac{3}{\sqrt[3]{x'}} + \frac{1}{\sqrt[4]{(7x)^5}}$$

$$\frac{\int (x) = \sqrt[3]{x^{1}} - \frac{3}{\sqrt[3]{x^{1}}} + \frac{1}{\sqrt[4]{(7x)^{5}}}}{(b) \ g(x) = x^{3/4}(x^{2} + 2)^{-3/2}} = \frac{\sqrt[3]{4}}{(x^{2} + 2)^{3/2}} = \sqrt[4]{x^{3}}$$

A with regative exponents, we with regative exponents, we make the positive before the positive padicals of 
$$f(x) = \sqrt{x} - \frac{3}{\sqrt{x}} + \frac{1}{\sqrt{(7x)^5}}$$

6. For the following expressions, determine the conjugate.

$$\begin{array}{ccc}
\text{(a) } x - 5 & \longrightarrow & \times + 5 \\
\text{a=x} & \text{b=5}
\end{array}$$

(b) 
$$2x - \sqrt{x} \longrightarrow 2x + \sqrt{x}$$
  
 $\alpha = 2x \quad b = \sqrt{x}$ 

(c) 
$$\sqrt{x+3} + \sqrt{11}$$
  $\longrightarrow$   $\sqrt{x+3} - \sqrt{11}$   
 $a = \sqrt{x+3}$   $b = \sqrt{11}$ 

$$a-b \rightarrow a+b$$
 $a+b \rightarrow a-b$ 

$$(x-5)(x+5) \rightarrow (x)^{2} - (5)^{2}$$

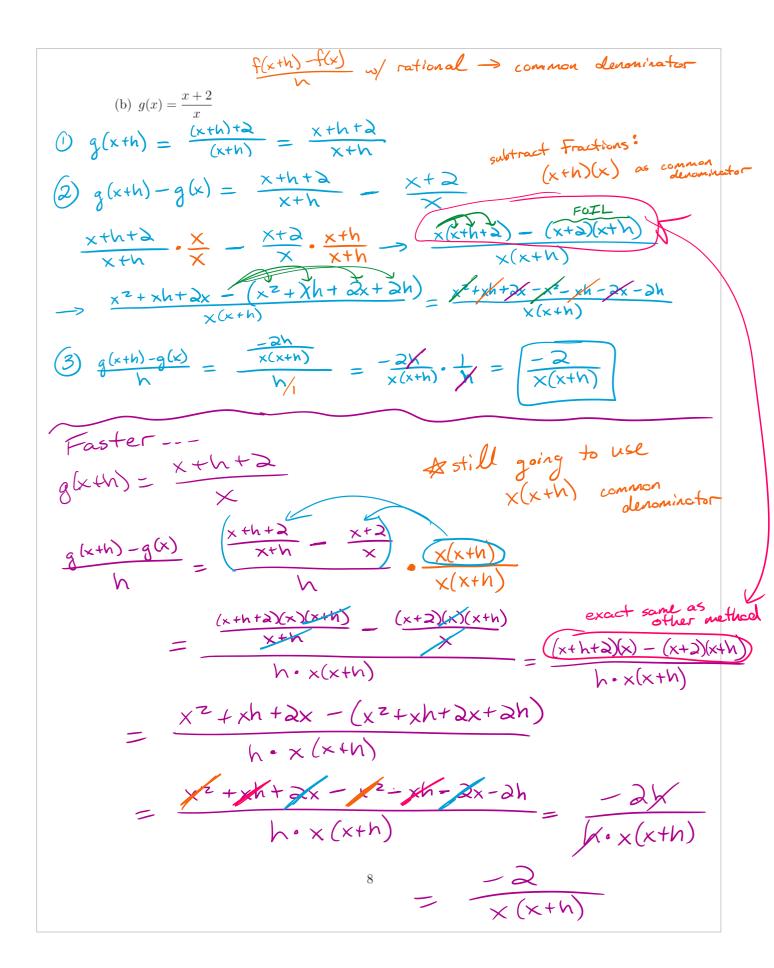
$$(2x-\sqrt{x})(2x+\sqrt{x}) \rightarrow (2x)^{2} - (\sqrt{x})^{2}$$

$$4x^{2} - x$$

2 f(x+h) - f(x) 7. For the given functions, setup and fully simplify the difference quotient (polynomial, multiply out & combine terms) (a)  $f(x) = 2x^2 - 5x$  $(x+h)^2 = (x+h)(x+h) = x^2 + 2xh + h^2$ (i)  $f(x+h) = 2(x+h)^2 - 5(x+h)$  $= 2(x^2+2xh+h^2)-5(x+h)$ = 2x2+4xh+2h2-5x-5h (not uncommon to have no like terms) (a)  $f(x+h) - f(x) = (2x^2 + 4xh + 2h^2 - 5x - 5h) - (2x^2 - 5x)$ = 2x2+4xh+2h2-5x-5h-2x2+5x = 4xh+2h2-5h (everything last here should have on "h") (3)  $\frac{f(x+h)-f(x)}{h} = \frac{4xh+2h^2-5h}{h}$  A last step is to factor h out of the top & cancel w/ the h on bottom  $=\frac{1}{12}\left(\frac{4x+2h-5}{2}\right)$   $=\frac{1}{12}\left(\frac{4x+2h-5}{2}\right)$ w/ difference quotient, once you cancel the h the started on bottom, you're done

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f(xth) f(x) wy rational -> common denominator



f(x+h) - f(x)(c)  $j(x) = \sqrt{2x-1}$  (x+h) - f(x) $0 j(x+h) = \sqrt{2(x+h)-1} = \sqrt{2x+2h-1}$ (3)  $j(x+h)-j(x) = \sqrt{2x+2h-1} - \sqrt{2x-1}$   $= \sqrt{2x+2h-1} - \sqrt{2x-1}$   $= \sqrt{2x+2h-1} + \sqrt{2x-1}$   $= \sqrt{2x+2h-1} + \sqrt{2x-1}$  $=\frac{\left(\sqrt{2x+2h-1}\right)^{2}-\left(\sqrt{2x-1}\right)^{2}}{\left(\sqrt{2x+2h-1}+\sqrt{2x-1}\right)}=\frac{\left(2x+2h-1\right)-\left(2x-1\right)}{h\left(\sqrt{2x+2h-1}+\sqrt{2x-1}\right)}$  $= \frac{2x + 2h - 1 - 2x + 1}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})} = \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x + 2h - 1 - 2x + 1}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$   $= \frac{2x}{h(\sqrt{2x + 2h - 1} + \sqrt{2x - 1})}$  $= \left| \frac{2}{\sqrt{2x+2h-1} + \sqrt{2x-1}} \right|$