# Math 150 - Week-In-Review 9

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## Problem Statements, sections 7.2-7.5

1. Evaluate the following:

a) $\sin(\frac{4\pi}{3}) = -\frac{\sqrt{3}}{2}$

b) $\cos(\frac{5\pi}{3}) = \frac{1}{2}$

a) $\tan(315°) = -\tan(45°) = -1$

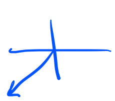
b) $\csc(120°) = \frac{1}{\sin(120°)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

2. Use the reference angle to find the indicated trigonometric value for the specified angles.

(a) $\sin\left(\frac{7\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

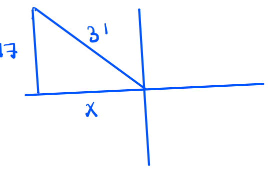
(b) $\cos\left(\frac{11\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

(c) $\tan\left(-\frac{2\pi}{3}\right) = +\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$



3. Find the exact value of the six trigonometric functions, given the following:

**hypotenuse** $= 31$, **side opposite the angle** $= 17$ , **Quadrant II**




$\sin(\theta) = \dfrac{opp.}{hyp.} = \dfrac{17}{31}$

$\csc(\theta) = \dfrac{31}{17}$

$\cos(\theta) = \dfrac{adj.}{hyp.} = -\dfrac{\sqrt{672}}{31}$

$\sec(\theta) = -\dfrac{31}{\sqrt{672}}$

$\tan\theta = \dfrac{opp.}{adj.} = -\dfrac{17}{\sqrt{672}}$

$\cot(\theta) = -\dfrac{\sqrt{672}}{17}$

$x^2 + (17)^2 = (31)^2$

$x^2 = (31)^2 - (17)^2$

$x^2 = 961 - 289$

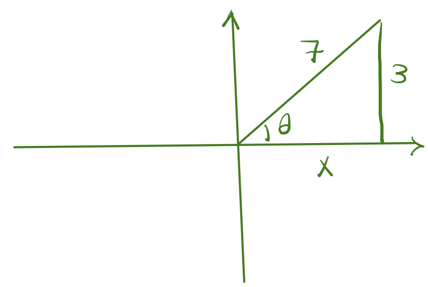
$x = -\sqrt{672}$

4. Given $\sin\theta = \dfrac{3}{7}$ and $\theta$ in QI, use the trigonometric identities to find the exact values of each:

a. $\cos(\theta) = \dfrac{\sqrt{40}}{7}$

b. $\cot(\theta) = \dfrac{\frac{\sqrt{40}}{7}}{\frac{3}{7}} = \dfrac{\sqrt{40}}{3}$

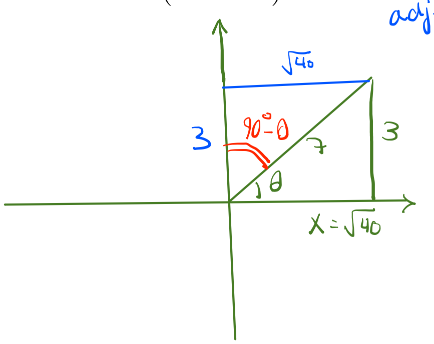
c. $\csc(\theta) = \dfrac{1}{\sin(\theta)} = \dfrac{7}{3}$




$x^2 + 9 = 49$

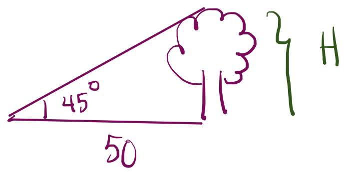
$x = \sqrt{40}$

d. $\tan(90° - \theta) = \dfrac{\text{opposite to } 90° - \theta}{\text{adj. to } 90° - \theta} = \dfrac{\sqrt{40}}{3}$




in other words $\tan(90° - \theta) = \cot(\theta)$

5. From a point on the ground 50 feet from the foot of a tree, the angle of elevation of the top of the tree is 45°. Find the height of the tree.



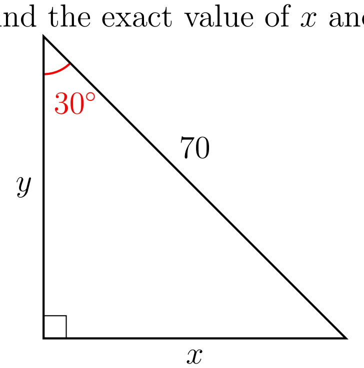

$$\tan(\theta) = \frac{\text{opp.}}{\text{adj}}$$

$$\tan(45°) = \frac{H}{50}$$

$$1 = \frac{H}{50}$$

$$\boxed{50 \text{ ft} = H}$$

6. Find the exact value of $x$ and $y$.




$$\sin(30°) = \frac{\text{opp.}}{\text{hyp.}} = \frac{x}{70}$$

$$\frac{\sqrt{3}}{2} = \frac{x}{70} \qquad x = \frac{70\sqrt{3}}{2}$$

$$\boxed{x = 35\sqrt{3}}$$

$$\cos(30°) = \frac{\text{adj.}}{\text{hyp.}} = \frac{y}{70}$$

$$\frac{1}{2} = \frac{y}{70} \Rightarrow \boxed{y = 35}$$

7. Given $y = -3\sin(4x - \pi) + 2$, describe the period, amplitude, and phase shift of the graph. Then graph the function.

Period: $\frac{2\pi}{B} = \frac{2\pi}{4} = \frac{\pi}{2}$

Amplitude: 3

Phase Shift: $-\frac{c}{B} = \frac{-(-\pi)}{4} = \frac{\pi}{4}$

$A = -3$
$B = 4$
$C = -\pi$
$D = 2$

**Period End points**

Start: $0 + \frac{\pi}{4} = \frac{\pi}{4}$

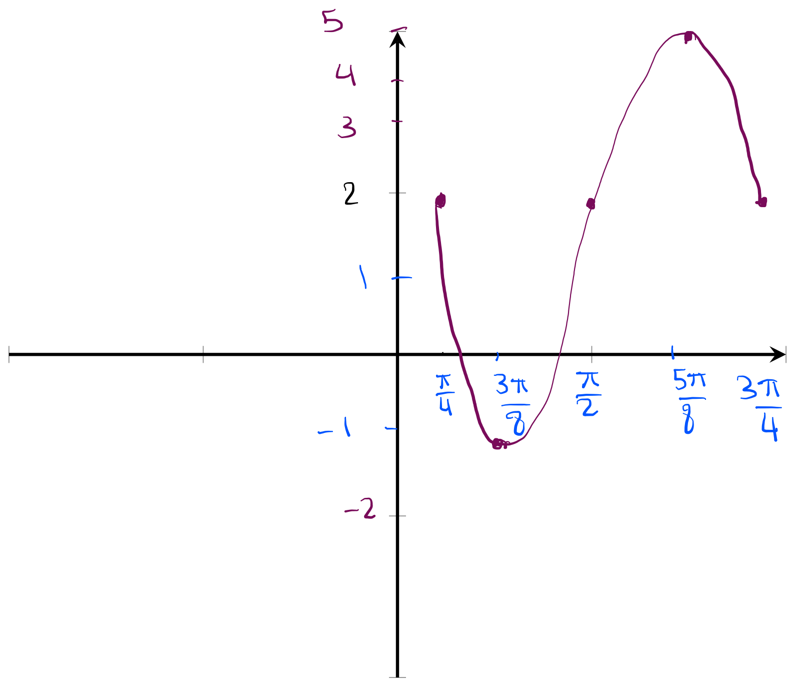
End: $\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

interval of full cycle

$\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

key points

| $x$ | $-3\sin(4x-\pi)+2$ |
|---|---|
| $\frac{\pi}{4}$ | $-3(0)+2 = 2$ |
| $\frac{3\pi}{8}$ | $-3(1)+2 = -1$ |
| $\frac{2\pi}{4} = \frac{\pi}{2}$ | $-3(0)+2 = 2$ |
| $\frac{5\pi}{8}$ | $-3(-1)+2 = 5$ |
| $\frac{3\pi}{4}$ | $-3(0)+2 = 2$ |



8. Given $y = \frac{1}{5}\cos\left(\frac{\pi}{2}x - 3\pi\right)$, describe the period, amplitude, and phase shift of the graph. Then graph the function.

$A = \frac{1}{5}$ $B = \frac{\pi}{2}$

$C = -3\pi$ $D = 0$

Period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{2}} = 4$

Amplitude: $\frac{1}{5}$

Phase Shift: $-\frac{C}{B} = \frac{-(-3\pi)}{\frac{\pi}{2}} = 6$

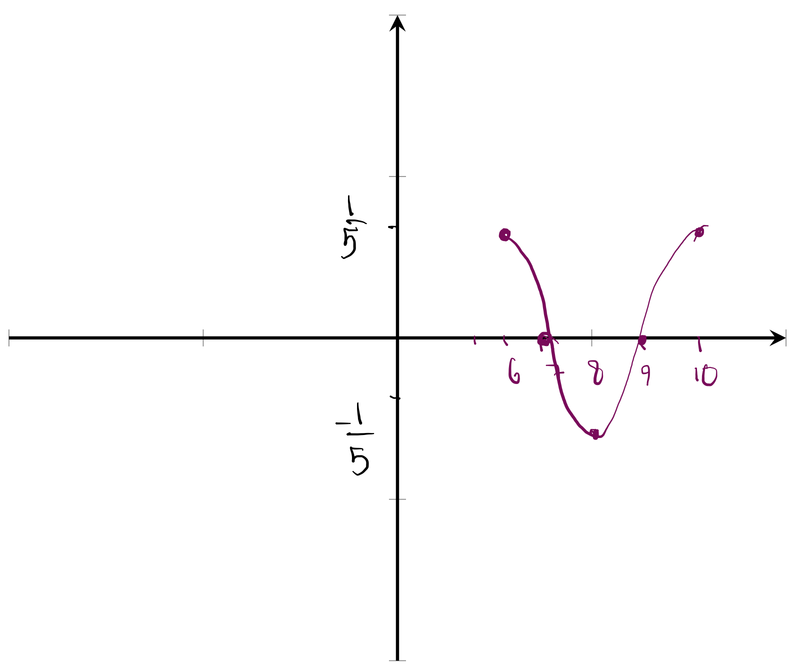
**Period End points**

Start: $0 + 6 = 6$

End: $4 + 6 = 10$

Key Points

| $x$ | $\frac{1}{5}\left(\cos\left(\frac{\pi}{2}x - 3\pi\right)\right)$ |
|---|---|
| 6 | $\frac{1}{5}(1) = \frac{1}{5}$ |
| 7 | 0 |
| 8 | $\frac{1}{5}(-1) = -\frac{1}{5}$ |
| 9 | 0 |
| 10 | $\frac{1}{5}(1) = \frac{1}{5}$ |



9. Find all Vertical asymptotes of the equation $f(x) = 2\tan\left(3x + \frac{\pi}{4}\right)$, then state the vertical asymptotes on the interval $[0, 2\pi)$.

$\cos\left(3x + \frac{\pi}{4}\right) = 0$

let $u = 3x + \frac{\pi}{4}$

$\cos(u) = 0$ when $u = \frac{\pi}{2} + k\pi$

$3x + \frac{\pi}{4} = \frac{\pi}{2} + k\pi$

$3x = \frac{\pi}{2} - \frac{\pi}{4} + k\pi$

$3x = \frac{\pi}{4} + k\pi$

$x = \frac{\pi}{12} + \frac{k\pi}{3}$

Solutions on $[0, 2\pi)$

$k=0 \quad x = \frac{\pi}{12}$

$k=1 \quad x = \frac{\pi}{12} + \frac{\pi}{3} = \frac{5\pi}{12}$

$k=2 \quad x = \frac{\pi}{12} + \frac{2\pi}{3} = \frac{9\pi}{12}$

$k=3 \quad x = \frac{\pi}{12} + \pi = \frac{13\pi}{12}$

$k=4 \quad x = \frac{\pi}{12} + \frac{4\pi}{3} = \frac{17\pi}{12}$

$k=5 \quad x = \frac{\pi}{12} + \frac{5\pi}{3} = \frac{21\pi}{12}$

10. Find Vertical asymptotes of the equation $f(x) = \csc\left(3\pi x - \frac{\pi}{6}\right) + 7$, then state the vertical asymptotes on the interval $[0, \frac{\pi}{6})$.

$\frac{1}{\sin(3\pi x - \frac{\pi}{6})}$

$\sin\left(3\pi x - \frac{\pi}{6}\right) = 0$

let $3\pi x - \frac{\pi}{6} = u$

$\sin u = 0$ when $u = k\pi$

So $3\pi x - \frac{\pi}{6} = k\pi$

$3\pi x = \frac{\pi}{6} + k\pi$

$x = \frac{1}{18} + \frac{k}{3}$

$\frac{3\pi}{18} \approx 9\ldots$

answers on $[0, \frac{\pi}{6}) =$

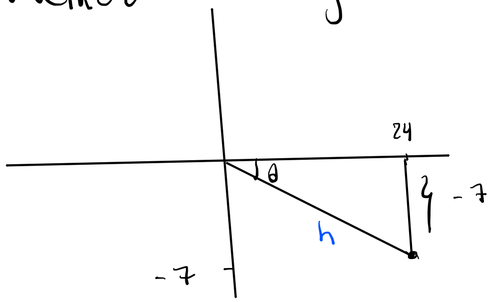
$k=0 \quad x = \boxed{\frac{1}{18}}$

$k=1 \quad x = \frac{1}{18} + \frac{1}{3} = \boxed{\frac{7}{18}}$

$k=2 \quad x = \frac{1}{18} + \frac{2}{3} = \cancel{\frac{13}{18}}$

11. Let $(24, -7)$ be a point on the terminal side of $\theta$. Find the sine, cosine, and tangent of $\theta$.

Method 1 : Triangle




$(24)^2 + (-7)^2 = h^2$

$5 = h$

$\sin(\theta) = \frac{-7}{25}$

$\cos(\theta) = \frac{24}{25}$

$\tan(\theta) = \frac{-7}{24}$

Method 2: Circle with radius r

$r^2 = \sqrt{(24)^2 + (-7)^2} = 25$

$\sin(\theta) = \frac{y}{r} = -\frac{7}{25}$

$\cos(\theta) = \frac{x}{r} = \frac{24}{25}$

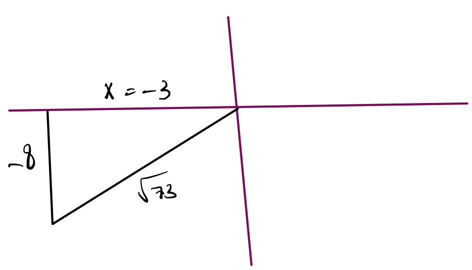
$\tan(\theta) = \frac{y}{x} = -\frac{7}{24}$

12. Suppose $\sin(\theta) = -\frac{8}{\sqrt{73}}$ and $\tan(\theta) > 0$. Find $\cot(\theta)$ and $\sec(\theta)$

opp

hyp.

the quadrant where $\sin(\theta) < 0$ & $\tan(\theta) > 0$ is Q III




$x^2 + (-8)^2 = (\sqrt{73})^2$

$x^2 + 64 = 73$

$x = -\sqrt{9}$

$x = -3$

$\cos(\theta) = \frac{3}{\sqrt{73}}$

$\tan(\theta) = \frac{8}{3}$

$\cot(\theta) = \frac{3}{8}$

$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{-\sqrt{73}}{3}$

13. Given $y = -2\cot(2x-5)$, describe the period, amplitude, and phase shift of the graph. Then graph the function.

Period: $\frac{\pi}{B} = \frac{\pi}{2}$

$A = -2 \quad B = 2 \quad C = -5 \quad D = 0$

Phase Shift: $-\frac{C}{B} = \frac{5}{2}$

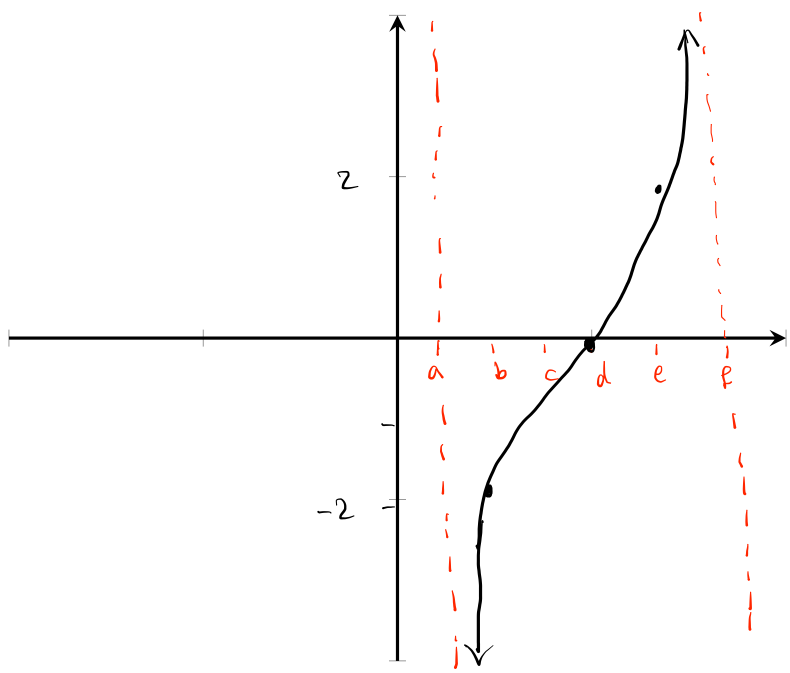
**Period End points**

Start: $0 + \frac{5}{2} = \frac{5}{2}$

End: $\frac{\pi}{2} + \frac{5}{2}$

Key points

| $x$ | $-2\cot(2x-5)$ |
|---|---|
| $a = \frac{5}{2}$ | undefined |
| $b = \frac{\pi}{8} + \frac{5}{2}$ | $-2(1) = -2$ |
| $c = \frac{\pi}{4} + \frac{5}{2}$ | $-2(0) = 0$ |
| $d = \frac{3\pi}{8} + \frac{5}{2}$ | $-2(-1) = 2$ |
| $e = \frac{\pi}{2} + \frac{5}{2}$ | undefined |



14. Given $y = \sec\left(3x + \frac{\pi}{6}\right) - 1$, describe the period, amplitude, and phase shift of the graph. Then graph the function.

$A = 1 \quad B = 3 \quad C = \frac{\pi}{6} \quad D = -1$

Period: $\frac{2\pi}{B} = \frac{2\pi}{3}$

Phase Shift: $-\frac{C}{B} = \frac{-\frac{\pi}{6}}{3} = -\frac{\pi}{18}$

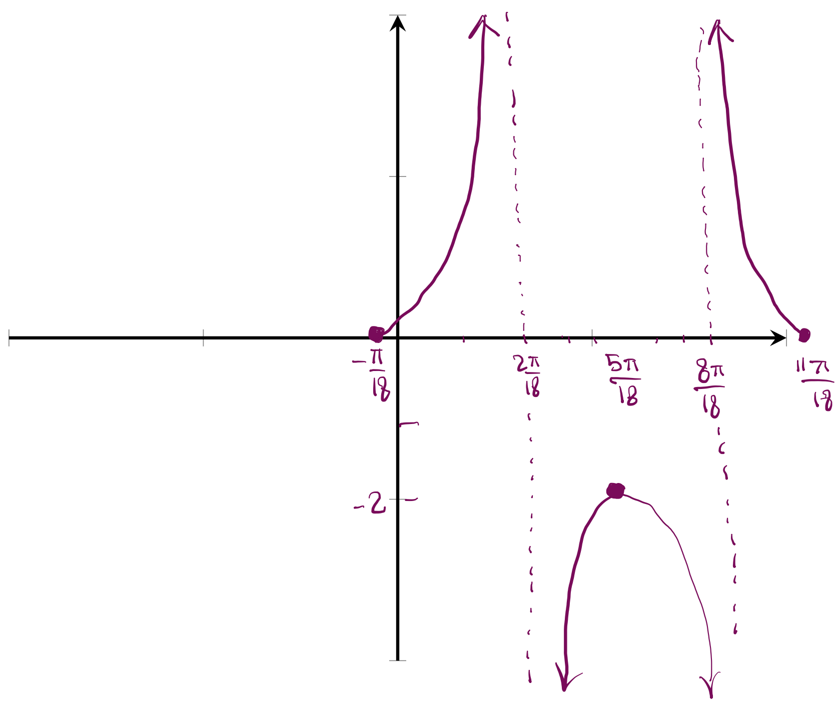
**Period End points**

Start: $0 - \frac{\pi}{18} = -\frac{\pi}{18}$

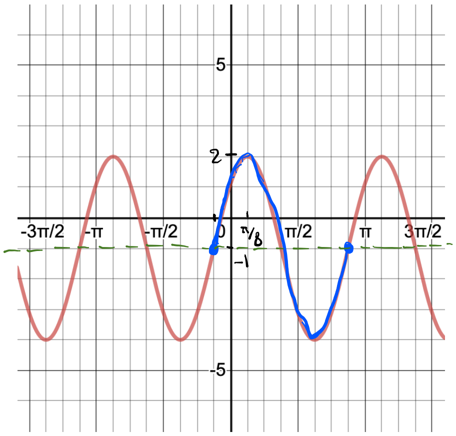
End: $\frac{6}{6} \cdot \frac{2\pi}{3} - \frac{\pi}{18} = \frac{11\pi}{18}$

key Points

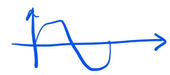
| $x$ | $\sec(3x + \frac{\pi}{6}) - 1$ |
|---|---|
| $-\frac{\pi}{18}$ | $(1) - 1 = 0$ |
| $\frac{\pi}{9} = \frac{2\pi}{18}$ | undefined |
| $\frac{5\pi}{18}$ | $(-1) - 1 = -2$ |
| $\frac{4\pi}{9} = \frac{8\pi}{18}$ | undefined |
| $\frac{11\pi}{18}$ | $(1) - 1 = 0$ |



15. Given the graph, write the equation of the sine function which matches the graph.




sin(x)



$A = 2 - (-1) = 3$

$D = -1$

we have one full cycle of sine

from $-\frac{\pi}{8}$ up to $\frac{7\pi}{8}$ (interval $\left[-\frac{\pi}{8}, \frac{7\pi}{8}\right]$)

So the length is $\frac{8\pi}{8} = \pi$

Period of this function is $\pi$

So $\frac{2\pi}{B} = \pi \Rightarrow B = 2$

Remains to find C. we know phase shift $= -\frac{C}{B}$

Comparing this graph with original sine, it has a phase (horizontal) shift $\frac{\pi}{8}$ left. Therefore

Since one full cycle $\left[-\frac{\pi}{8}, \frac{7\pi}{8}\right]$

$$-\frac{C}{B} = -\frac{\pi}{8}$$

$$-\frac{C}{2} = -\frac{\pi}{8} \Rightarrow C = \frac{\pi}{4}$$

$$\Rightarrow f(x) = 3\sin\left(2x + \frac{\pi}{4}\right) - 1$$

16. Write an equation for a function with the given characteristics. A sine curve with a period of $\frac{\pi}{4}$, an amplitude of 6, a right phase shift of $3\pi$.

We want period to be $\frac{\pi}{4}$ : $\frac{2\pi}{B} = \frac{\pi}{4} \Rightarrow B = \frac{2\pi}{\frac{\pi}{4}} = 8$

Phase shift $3\pi$ : $-\frac{C}{B} = -\frac{C}{8} = 3\pi$

$C = -24\pi$

Amplitude to be 6 : $A = 6$ (or even $A = -6$)

No vertical shifts mentioned: $D = 0$

$\Rightarrow \quad f(\theta) = 6\sin(8\theta - 24\pi)$